

Find The Quadratic Polynomial Whose Graph Passes Through The Points (0 , 0) , (-1 , 1) And (1 ,

Find The Quadratic Polynomial Whose Graph Passes Through The Points (0 , 0) , (-1 , 1) And (1 , in the first paragraph, we are often faced with the intriguing challenge of determining the specific quadratic function that passes through a given set of points. This process involves understanding the fundamental concepts of quadratic equations, the method of solving for coefficients, and applying algebraic techniques to find the unique polynomial that fits these points precisely. Whether you're a student studying algebra or someone looking to strengthen your understanding of quadratic functions, this guide will walk you through the comprehensive steps to find the quadratic polynomial passing through the points (0, 0), (-1, 1), and (1, ?). Let's delve into the essentials of quadratic equations and how to approach this classic problem.

Understanding Quadratic Polynomials and Their Graphs

What Is a Quadratic Polynomial?

A quadratic polynomial is a second-degree polynomial of the form:

- $f(x) = ax^2 + bx + c$

where a , b , and c are constants, and $a \neq 0$. The graph of this polynomial is a parabola that opens upward if $a > 0$ and downward if $a < 0$.

Why Find a Specific Quadratic Polynomial?

Determining a quadratic polynomial that passes through given points allows us to model real-world phenomena such as projectile trajectories, revenue projections, or any situation where a quadratic relationship exists. In this case, the points (0, 0), (-1, 1), and (1, ?) define the specific parabola we aim to find.

Step-by-Step Method to Find the Quadratic Polynomial

Step 1: Set Up the General Form

Begin with the general quadratic form:

- $f(x) = ax^2 + bx + c$

Our goal is to find the values of a , b , and c that satisfy the points provided.

Step 2: Use the Given Points to Create Equations

Since the polynomial passes through the points, substitute the x and y values into the general form:

1. For point $(0, 0)$:

- $f(0) = a(0)^2 + b(0) + c = 0$

- $\Rightarrow c = 0$

2. For point $(-1, 1)$:

- $f(-1) = a(-1)^2 + b(-1) + c = 1$

- $\Rightarrow a(1) - b + c = 1$

- Since $c = 0$, substitute:

- $a - b = 1$

3. For point $(1, ?)$:

- $f(1) = a(1)^2 + b(1) + c = ?$

- $\Rightarrow a + b + c = ?$

- With $c = 0$, this simplifies to:

- $a + b = ?$

Note: Since the third point's y -coordinate is not specified, we can analyze the problem further to determine it or use the given information to find the specific quadratic polynomial.

Step 3: Solve the System of Equations

From previous steps, we have:

- $c = 0$

- $a - b = 1$

The third point's y-value (let's call it y_3) gives:

- $a + b = y_3$

Adding the two equations:

- $(a - b) + (a + b) = 1 + y_3$

- $2a = 1 + y_3$

- $\Rightarrow a = (1 + y_3) / 2$

Similarly, subtracting the first from the second:

- $(a + b) - (a - b) = y_3 - 1$

- $2b = y_3 - 1$

- $\Rightarrow b = (y_3 - 1) / 2$

Therefore, the quadratic polynomial depends on the third point's y-value, y_3 . If you have the y-coordinate for the point $(1, y_3)$, you can substitute it to find explicit values for a and b .

Special Cases and Additional Considerations

Determining the Missing Point

In many real-world problems, all three points are known. If the y-value at $x=1$ is provided, you can directly compute a , b , and c . For instance, suppose the third point is $(1, 0)$, then $y_3=0$, and:

- $a = (1 + 0)/2 = 0.5$

- $b = (0 - 1)/2 = -0.5$

- $c = 0$

The quadratic polynomial becomes:

- $f(x) = 0.5x^2 - 0.5x$

This parabola passes through $(0, 0)$, $(-1, 1)$, and $(1, 0)$.

Note: Adjust the calculations based on the actual y-value of the third point.

Using Matrix Methods for a System of Equations

Alternatively, you can represent the equations in matrix form:

```
\[
\begin{bmatrix}
0^2 & 0 & 1 \\
(-1)^2 & -1 & 1 \\
1^2 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
=
\begin{bmatrix}
0 \\
1 \\
y_3
\end{bmatrix}
\]
```

and solve for a , b , c using methods such as substitution, elimination, or matrix inversion.

Practical Applications of Finding the Quadratic Polynomial

Physics and Engineering

Quadratic functions model projectile motion, such as the path of a ball thrown in the air. By knowing certain points along the trajectory, engineers can optimize launch angles and velocities.

Economics and Business

Quadratic models help forecast revenue, profit, or demand curves based on specific data points, enabling better decision-making.

Mathematics Education

Learning how to find a quadratic polynomial passing through given points strengthens algebraic manipulation skills and understanding of quadratic functions.

Conclusion: Mastering the Art of Finding Quadratic

Polynomials

In summary, finding a quadratic polynomial passing through specific points involves setting up the general form, substituting the known coordinates, creating a system of equations, and solving for the coefficients. The process highlights the importance of algebraic techniques and understanding quadratic relationships, whether for academic purposes or real-world applications. Remember, the key steps are:

- Start with the general form: $f(x) = ax^2 + bx + c$
- Use the given points to form equations
- Solve the system for a , b , c
- Write the specific quadratic polynomial

By mastering these steps, you can confidently determine the quadratic polynomial for any set of points, enhancing your mathematical toolkit and application skills. Whether you're analyzing scientific data, modeling real-life situations, or solving academic problems, understanding how to find the quadratic polynomial passing through given points is a fundamental skill in algebra and beyond.

Frequently Asked Questions

How do I find the quadratic polynomial passing through the points (0, 0), (-1, 1), and (1, 1)?

To find the quadratic polynomial, assume it is of the form $y = ax^2 + bx + c$. Use the given points to set up equations and solve for a , b , and c .

What are the steps to determine the quadratic polynomial passing through (0, 0), (-1, 1), and (1, 1)?

Step 1: Plug each point into $y = ax^2 + bx + c$. Then, solve the resulting system of equations to find the coefficients a , b , and c .

What is the quadratic polynomial that passes through the points (0, 0), (-1, 1), and (1, 1)?

The quadratic polynomial is $y = x^2$.

Why does the quadratic polynomial $y = x^2$ pass through the

points (0, 0), (-1, 1), and (1, 1)?

Because substituting $x = 0, -1$, and 1 into $y = x^2$ yields $y = 0, 1$, and 1 respectively, matching all three points.

Can multiple quadratic polynomials pass through three points? How do I know which one is correct?

Generally, three non-collinear points determine a unique quadratic polynomial. By solving the system, you find the specific polynomial passing through those points.

If a quadratic passes through (0, 0), (-1, 1), and (1, 1), what is its equation?

Its equation is $y = x^2$, as it satisfies all three points.

How does knowing the points help in constructing the quadratic polynomial?

The points provide specific (x, y) pairs that the polynomial must satisfy, allowing you to set up equations to solve for the coefficients of the quadratic.

What is the importance of the point (0, 0) in finding the quadratic polynomial?

Since the polynomial passes through $(0, 0)$, it implies that $c = 0$ in $y = ax^2 + bx + c$, simplifying the process of finding the polynomial.

Additional Resources

Find The Quadratic Polynomial Whose Graph Passes Through The Points (0 , 0), (-1 , 1), And (1 ,

Understanding how to determine a quadratic polynomial that passes through specific points is a foundational concept in algebra and analytical geometry. This task not only deepens comprehension of quadratic functions but also demonstrates practical problem-solving techniques with real-world applications, from physics to engineering. In this article, we will explore the systematic approach to finding such a quadratic polynomial when given three points, focusing on the points $(0, 0)$, $(-1, 1)$, and $(1,)$. We will guide you through the theory, the step-by-step process, and the reasoning behind each move, making the subject accessible yet detailed enough for learners and professionals alike.

Introduction to Quadratic Polynomials

Before diving into the problem, it's essential to understand what a quadratic polynomial is and its general form.

What Is a Quadratic Polynomial?

A quadratic polynomial is a second-degree polynomial, which can be expressed in the standard form:

$$y = ax^2 + bx + c$$

where:

- a , b , and c are constants,
- $a \neq 0$,
- x is the independent variable,
- y is the dependent variable or the output of the function.

Quadratic functions are characterized by their parabolic graphs, which open upwards when $a > 0$ and downwards when $a < 0$.

Significance of Knowing the Polynomial

Knowing the quadratic polynomial that passes through specific points allows us to:

- Predict the behavior of the function beyond the given points,
- Understand the shape and position of the parabola,
- Solve real-world problems involving maximum or minimum values (vertex),
- Use the polynomial in modeling scenarios where the relationship between variables is quadratic.

Understanding the Given Points

Given points are:

- $(0, 0)$
- $(-1, 1)$
- $(1,)$

The last point's y -coordinate is missing, represented by "". To proceed, we need to determine this third point's value or understand how to find the quadratic polynomial with the first two points and then analyze the third point's relationship.

Points Analysis:

- The point $(0, 0)$ indicates the parabola passes through the origin, which simplifies some calculations.
- The point $(-1, 1)$ provides additional information about the parabola's shape and position.
- The point $(1,)$: The value of y at $x=1$ is unknown, but we can consider various possibilities or treat it as a variable to find the polynomial that satisfies certain conditions.

For the purpose of this article, let's assume the third point is $(1, y_3)$, with y_3 being an unknown that we will determine based on the context or for illustrative purposes.

Note: If the exact value of y at $x=1$ (i.e., y_3) is provided later, the process becomes

more concrete and straightforward.

Formulating the System of Equations

To find the quadratic polynomial, we substitute each point into the general form:

$$y = ax^2 + bx + c$$

and derive a system of equations.

Substituting the Known Points

1. Point $(0, 0)$:

$$0 = a \times 0^2 + b \times 0 + c$$

$$\Rightarrow c = 0$$

This simplifies our quadratic to:

$$y = ax^2 + bx$$

2. Point $(-1, 1)$:

$$1 = a \times (-1)^2 + b \times (-1)$$

$$1 = a - b$$

$$\Rightarrow 1 = a - b$$

3. Point $(1, y_3)$:

$$y_3 = a \times 1^2 + b \times 1$$

$$y_3 = a + b$$

Thus, the polynomial simplifies to:

$$y = ax^2 + bx$$

with the constraints:

$$a - b = 1,$$

$$y_3 = a + b.$$

Note: In scenarios where y_3 is known, these equations can be solved directly for a and b .
If y_3 is unknown, the polynomial is expressed in terms of y_3 .

Solving for the Coefficients

Our primary goal is to determine a and b , given the constraints.

Step 1: Express a in terms of b :

From $a - b = 1$,

$$a = 1 + b$$

Step 2: Express y_3 in terms of b :

$$y_3 = a + b = (1 + b) + b = 1 + 2b$$

Step 3: Consider the third point:

- If y_3 is known, then:

$$y_3 = 1 + 2b$$

$$\Rightarrow b = \frac{y_3 - 1}{2}$$

- Consequently,

$$a = 1 + b = 1 + \frac{y_3 - 1}{2} = \frac{2 + y_3 - 1}{2} = \frac{1 + y_3}{2}$$

Example: Assuming y_3 is known

Suppose the third point is $(1, 2)$. Then,

$$y_3 = 2$$

and

$$b = \frac{2 - 1}{2} = \frac{1}{2}$$

$$a = \frac{1 + 2}{2} = \frac{3}{2}$$

Resulting quadratic polynomial:

$$y = \frac{3}{2}x^2 + \frac{1}{2}x$$

which passes through the points $(0, 0)$, $(-1, 1)$, and $(1, 2)$.

Note: If the third point's y -coordinate is unknown, the polynomial can be expressed parametrically with y_3 , or additional constraints are needed to determine a unique polynomial.

Verifying the Polynomial

Once the coefficients are determined, it's vital to verify whether the polynomial truly passes through all given points.

Step 1: Plug in the known points

- For $(0, 0)$:

$$y = a \times 0^2 + b \times 0 = 0 \quad \checkmark$$

- For $(-1, 1)$:

$$y = a \times 1 - b$$

Using the earlier example with $a = \frac{3}{2}$, $b = \frac{1}{2}$:

$$y = \frac{3}{2} \times 1 - \frac{1}{2} = \frac{3}{2} - \frac{1}{2} = 1 \quad \checkmark$$

- For $(1, 2)$:

$$y = a \times 1 + b = \frac{3}{2} + \frac{1}{2} = 2 \quad \checkmark$$

All points are satisfied, confirming the correctness of the polynomial.

Step 2: Graphical interpretation

Plotting the points and the quadratic curve provides visual confirmation. The parabola should pass through all three points, aligning with the algebraic results.

Note: If the third point's y is different, adjust the calculations accordingly to find new coefficients.

Applications and Broader Implications

The ability to determine a quadratic polynomial passing through specific points has numerous practical applications:

- Physics: Modeling projectile trajectories where initial and specific point data are known.
- Economics: Fitting quadratic models to data points for optimization.
- Engineering: Designing curves that meet certain positional constraints.
- Computer Graphics: Generating parabolic shapes with precise control points.

Extending the Method

The approach outlined can be extended to:

- Find quadratic polynomials passing through more than three points (least squares method).
- Determine quadratic functions with additional constraints (e.g., vertex location).
- Generalize to higher-degree polynomials for more complex data fitting.

Conclusion

Finding the quadratic polynomial passing through three points involves understanding the standard form, substituting known points to form a system of equations, and solving for the coefficients. When one of the points' (x, y) -coordinate is unknown, the process can incorporate parameters, allowing for flexible modeling based on available data. By verifying the solution both algebraically and graphically, one ensures the polynomial accurately represents the intended relationship.

This process is fundamental in algebra and calculus, serving as a cornerstone for more advanced topics such as polynomial interpolation, curve fitting, and optimization. Whether in academic settings or applied fields, mastering the technique of deriving quadratic polynomials from data points enhances analytical skills and provides a robust tool for real-world problem-solving.

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