Find The Slope Of The Line Tangent To The Polar Curve At The Given Point. R= 3 Sin Theta (3/2, Pi/6)

Find The Slope Of The Line Tangent To The Polar Curve At The Given Point. R= 3 Sin Theta (3/2, Pi/6)

Understanding how to find the slope of the tangent line to a polar curve at a specific point is a fundamental aspect of calculus and analytical geometry. When dealing with polar equations like R = 3 Sin Theta, the process involves converting the polar coordinates to Cartesian coordinates or directly differentiating using the polar form. This article provides a comprehensive guide to calculating the slope of the tangent line at a given point on the curve R = 3 Sin Theta, specifically at the point where the radius and angle are (3/2, Pi/6). We will explore the necessary concepts, step-by-step procedures, and tips to master this technique.

Understanding Polar Curves and Their Derivatives

Before diving into the calculation, it is essential to understand what polar curves are and how their derivatives relate to slopes of tangent lines.

What Is a Polar Curve?

A polar curve is a graph of a relation between a radius R and an angle θ, expressed as $R = f(\theta)$. Each point on the curve is represented in polar coordinates (R, θ), where R is the distance from the origin, and θ is the angle measured from the positive x-axis.

Example:
The given curve $R = 3 \sin \theta$ is a classic example of a sinusoidal polar curve, often forming a circle or a petal shape depending on the equation.

Converting to Cartesian Coordinates

To analyze the slope at a point, one common approach is to convert the polar equation into Cartesian coordinates using the relationships:
- $x = R \cos \theta$
- $y = R \sin \theta$

However, for calculating the slope directly, it’s often more efficient to use the derivative formulas specific to polar curves.

Derivative of a Polar Curve

The slope $\frac{dy}{dx}$ of the tangent line to a polar curve at a point can be calculated using the derivatives of R with respect to θ:

$$
\frac{dy}{dx} = \frac{R'(\theta) \sin \theta + R(\theta) \cos \theta}{R'(\theta) \cos \theta - R(\theta) \sin \theta}
$$

Where:
- $R'(\theta) = \frac{dR}{d\theta}$

This formula allows us to compute the tangent slope directly from the polar equation without converting to Cartesian coordinates explicitly.

Step-by-Step Guide to Find the Slope at a Specific Point

Let's apply these concepts to find the slope of the tangent line to the curve $R = 3 \sin \theta$ at the point where $\theta = \pi/6$.

Step 1: Identify the Given Point in Polar Coordinates

The point is given as $(\frac{3}{2}, \pi/6)$.
- $R = 3 \sin \theta$
- At $\theta = \pi/6$,

$$
R = 3 \sin \left(\frac{\pi}{6} \right) = 3 \times \frac{1}{2} = \frac{3}{2}
$$

This confirms the point $\left(\frac{3}{2}, \pi/6 \right)$ lies on the curve.

Step 2: Compute $R'(\theta)$

Differentiate $R = 3 \sin \theta$ with respect to θ:

$$
R'(\theta) = 3 \cos \theta
$$

At $\theta = \pi/6$:

$$
R' \left(\frac{\pi}{6} \right) = 3 \times \cos \left(\frac{\pi}{6} \right) = 3 \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}
$$

Step 3: Apply the Derivative Formula for $\frac{dy}{dx}$

Using the formula:

$$
\frac{dy}{dx} = \frac{R'(\theta) \sin \theta + R(\theta) \cos \theta}{R'(\theta) \cos \theta - R(\theta) \sin \theta}
$$

Substitute the known values:

- $R'(\theta) = \frac{3\sqrt{3}}{2}$
- $R(\theta) = \frac{3}{2}$
- $\sin \left(\frac{\pi}{6} \right) = \frac{1}{2}$
- $\cos \left(\frac{\pi}{6} \right) = \frac{\sqrt{3}}{2}$

Calculate numerator:

$$
\text{Numerator} = \left(\frac{3\sqrt{3}}{2} \right) \times \frac{1}{2} + \frac{3}{2} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4} + \frac{3\sqrt{3}}{4} = \frac{6\sqrt{3}}{4} = \frac{3\sqrt{3}}{2}
$$

Calculate denominator:

$$
\text{Denominator} = \left(\frac{3\sqrt{3}}{2} \right) \times \frac{\sqrt{3}}{2} - \frac{3}{2} \times \frac{1}{2} = \frac{3\sqrt{3} \times \sqrt{3}}{4} - \frac{3}{4} = \frac{3 \times 3}{4} - \frac{3}{4} = \frac{9}{4} - \frac{3}{4} = \frac{6}{4} = \frac{3}{2}
$$

Finally, compute the slope:

$$
\frac{dy}{dx} = \frac{\frac{3\sqrt{3}}{2}}{\frac{3}{2}} = \frac{3\sqrt{3}/2}{3/2} = \frac{3\sqrt{3}}{2} \times \frac{2}{3} = \sqrt{3}
$$

Result:
The slope of the tangent line at the point $\left(\frac{3}{2}, \pi/6 \right)$ on the curve $R=3 \sin \theta$ is $\boxed{\sqrt{3}}$.

Summary of Key Steps

- Identify the point on the curve in polar coordinates.
- Compute the derivative $R'(\theta)$.
- Plug values into the slope formula for $\frac{dy}{dx}$.
- Simplify to find the slope.

Additional Tips for Finding Tangent Slopes in Polar Coordinates

- Always verify the point lies on the curve before calculating derivatives.
- Be comfortable differentiating $R(\theta)$ with respect to θ.
- Use the derivative formula for $\frac{dy}{dx}$ in polar form for efficiency.
- Remember that the slope can be positive, negative, or undefined depending on the curve's behavior.

Applications of Finding the Tangent Slope

Understanding how to find the tangent slope in polar coordinates is useful in various fields:
- Physics: analyzing motion along curved paths.
- Engineering: designing components with specific curvature properties.
- Mathematics: exploring properties of polar graphs, symmetry, and calculus.

Conclusion

Calculating the slope of the tangent line to a polar curve at a given point involves understanding the relationship between polar and Cartesian coordinates, differentiating the polar equation, and applying the appropriate derivative formula. For the specific curve $R=3 \sin \theta$ at the point $(3/2, \pi/6)$, the slope is $\sqrt{3}$. Mastery of these techniques enhances your ability to analyze complex curves and their tangents, which is a vital skill in calculus and related disciplines.

Whether you're solving problems in mathematics, physics, or engineering, the ability to find tangent slopes in polar coordinates opens the door to deeper insights into the behavior of curves and their geometric properties.

Frequently Asked Questions

How do you find the slope of the tangent line to a polar curve at a given point?

To find the slope of the tangent line to a polar curve at a specific point, differentiate the curve equations $y = r \sin \theta$ and $x = r \cos \theta$ with respect to θ, then compute $dy/dx = (dy/d\theta) / (dx/d\theta)$ at the given θ.

What is the given polar curve in the problem?

The given polar curve is $r = 3 \sin \theta$.

What is the coordinates of the point in Cartesian form for r=3

sin θ at θ=π/6?

At θ=π/6, r = 3 sin(π/6) = 3 1/2 = 1.5. Therefore, the Cartesian coordinates are x = r cos θ = 1.5 cos(π/6) ≈ 1.5 (√3/2) ≈ 1.5 0.866 ≈ 1.299, y = r sin θ = 1.5 sin(π/6) = 1.5 0.5 = 0.75.

How do you compute dy/dθ and dx/dθ for the polar curve r=3 sin θ?

Using r = 3 sin θ, we have: dy/dθ = dr/dθ sin θ + r cos θ, and dx/dθ = dr/dθ cos θ - r sin θ. Since dr/dθ = 3 cos θ, substitute to find these derivatives.

What are the derivatives dy/dθ and dx/dθ at θ=π/6?

At θ=π/6, dr/dθ = 3 cos(π/6) = 3 (√3/2) ≈ 2.598. Then, dy/dθ = 2.598 0.5 + 1.5 (√3/2) ≈ 1.299 + 1.299 = 2.598. Similarly, dx/dθ = 2.598 (√3/2) - 1.5 0.5 ≈ 2.598 0.866 - 0.75 ≈ 2.25 - 0.75 = 1.5.

How is the slope of the tangent line calculated from these derivatives?

The slope m of the tangent line is given by dy/dx = (dy/dθ) / (dx/dθ). Using the derivatives calculated, m ≈ 2.598 / 1.5 ≈ 1.732.

What is the approximate numerical value of the slope at θ=π/6?

The approximate slope of the tangent line at θ=π/6 is about 1.732.

What is the significance of the slope value in understanding the tangent line?

The slope indicates the steepness and direction of the tangent line at the given point on the polar curve, helping in graphing and analysis of the curve's behavior.

Can the process be generalized for other polar curves and points?

Yes, the method of differentiating r with respect to θ, finding dy/dθ and dx/dθ, and then computing dy/dx applies to any polar curve and point, with appropriate derivatives based on the specific r(θ).

What is the final answer for the slope of the tangent line at the point (1.5, π/6) on r=3 sin θ?

The slope of the tangent line at the point corresponding to θ=π/6 is approximately 1.732.

Additional Resources

Find The Slope Of The Line Tangent To The Polar Curve At The Given Point: R = 3 Sin θ (3/2, π/6)

Introduction

In the realm of advanced calculus and analytical geometry, the exploration of curves defined in polar coordinates stands as a fundamental pursuit. Among these, the task of determining the slope of a tangent line at a specific point on a polar curve is a classic yet intricate problem, blending the elegance of geometric intuition with the rigor of calculus. This article delves into the detailed process of finding the slope of the tangent line to the polar curve described by the equation $r = 3 \sin \theta$ at the particular point where $r = \frac{3}{2}$ and $\theta = \frac{\pi}{6}$.

This investigation is not merely an academic exercise but also a crucial step in understanding the behavior of such curves, which have applications ranging from physics to engineering, and serve as a powerful tool for visualizing complex relationships in a coordinate system that extends beyond the familiar Cartesian framework.

Understanding Polar Coordinates and the Curve Equation

What is a Polar Curve?

A polar curve is a graph of a function $r = f(\theta)$ where each point on the curve is represented by its distance r from the origin and the angle θ it makes with the positive x-axis. Unlike Cartesian equations involving x and y, polar equations specify the position of points in terms of radius and angle, providing unique insights especially for curves with symmetry or periodic properties.

The Specific Curve: $r = 3 \sin \theta$

The curve under examination is defined by:

$$
r = 3 \sin \theta
$$

This is a classic example of a lemniscate-like or cardioid-type curve, depending on the range of θ. It exhibits symmetry about the y-axis and is commonly used in physics and engineering to model wave patterns, antenna radiation patterns, and more.

The Geometric Significance of the Given Point

Coordinates in Cartesian Coordinates

To analyze the tangent at a specific point, it's often helpful to convert polar coordinates to Cartesian coordinates:

$$
x = r \cos \theta
$$
$$
y = r \sin \theta
$$

Given:

$$
r = \frac{3}{2}
$$
$$
\theta = \frac{\pi}{6}
$$

Calculating:

$$
x = \frac{3}{2} \cos \left(\frac{\pi}{6} \right) = \frac{3}{2} \times \frac{\sqrt{3}}{2} = \frac{3 \sqrt{3}}{4}
$$
$$
y = \frac{3}{2} \sin \left(\frac{\pi}{6} \right) = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}
$$

Thus, the point in Cartesian coordinates is:

$$
\left(\frac{3 \sqrt{3}}{4}, \frac{3}{4} \right)
$$

This conversion not only confirms the location but also provides a reference for geometric interpretation.

Deriving the Slope of the Tangent Line

Theoretical Framework

In polar coordinates, the slope of the tangent line $\frac{dy}{dx}$ at a point on the curve can be obtained via the derivatives of r with respect to θ. The relation is given by:

$$
\frac{dy}{dx} = \frac{r'(\theta) \sin \theta + r(\theta) \cos \theta}{r'(\theta) \cos \theta - r(\theta) \sin \theta}
$$

where:

- $r'(\theta) = \frac{dr}{d\theta}$.

This formula emerges from the parametric form of the curve, considering $x = r \cos \theta$, $y = r \sin \theta$, and applying the chain rule.

Step-by-Step Derivation

1. Calculate $r'(\theta)$

Given:

$$
r(\theta) = 3 \sin \theta
$$

Differentiating:

$$
r'(\theta) = 3 \cos \theta
$$

2. Evaluate $r(\theta)$ and $r'(\theta)$ at $\theta = \frac{\pi}{6}$

$$
r \left(\frac{\pi}{6} \right) = 3 \times \frac{1}{2} = \frac{3}{2}
$$

$$
r' \left(\frac{\pi}{6} \right) = 3 \times \frac{\sqrt{3}}{2} = \frac{3 \sqrt{3}}{2}
$$

3. Substitute into the slope formula

$$
\frac{dy}{dx} = \frac{ \left(\frac{3 \sqrt{3}}{2} \right) \times \frac{1}{2} + \frac{3}{2} \times \frac{\sqrt{3}}{2} }{ \left(\frac{3 \sqrt{3}}{2} \right) \times \frac{\sqrt{3}}{2} - \frac{3}{2} \times \frac{1}{2} }
$$

Calculating numerator:

$$
\text{Numerator} = \frac{3 \sqrt{3}}{2} \times \frac{1}{2} + \frac{3}{2} \times \frac{\sqrt{3}}{2} = \frac{3 \sqrt{3}}{4} + \frac{3 \sqrt{3}}{4} = \frac{6 \sqrt{3}}{4} = \frac{3 \sqrt{3}}{2}
$$

Calculating denominator:

$$
\text{Denominator} = \frac{3 \sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{3}{2} \times \frac{1}{2} = \frac{3 \sqrt{3} \times \sqrt{3}}{4} - \frac{3}{4}
$$

$$
= \frac{3 \times 3}{4} - \frac{3}{4} = \frac{9}{4} - \frac{3}{4} = \frac{6}{4} = \frac{3}{2}
$$

Final calculation:

$$
\boxed{
\frac{dy}{dx} = \frac{\frac{3 \sqrt{3}}{2}}{\frac{3}{2}} = \frac{3 \sqrt{3} / 2}{3/2} = \frac{3 \sqrt{3}}{2} \times \frac{2}{3} = \sqrt{3}
}
$$

Interpreting the Result

The slope of the tangent line at the specified point is:

$$
\boxed{
\frac{dy}{dx} = \sqrt{3}
}
$$

This indicates that at $r = \frac{3}{2}$ and $\theta = \frac{\pi}{6}$, the tangent to the polar curve is inclined at an angle whose tangent is $\sqrt{3}$, corresponding to an angle of approximately 60°.

Significance and Applications

Geometric Insights

The calculation reveals that the curve's tangent at the given point is quite steep, reflecting the curvature's local geometry. Such precise slope determination is essential in understanding the behavior of the curve, especially in applications requiring exact tangential properties.

Practical Applications

- Physics: Analyzing wavefronts, electromagnetic radiation patterns, or particle trajectories often involves working in polar coordinates and their tangents.
- Engineering: Design of antennas, mechanical linkages, and robotics frequently utilizes polar equations to determine movement paths and contact points.
- Mathematics Education: This problem exemplifies the application of calculus in non-Cartesian

systems, enriching students’ understanding of derivatives and coordinate transformations.

Conclusion

The process of finding the slope of the tangent line to a polar curve at a specific point encapsulates a beautiful intersection of calculus, geometry, and algebra. By systematically deriving the derivatives, evaluating at the given point, and applying the appropriate formula, we have shown that the slope of the tangent to the curve $r = 3 \sin \theta$ at $r = \frac{3}{2}$, $\theta = \frac{\pi}{6}$ is $\sqrt{3}$.

This detailed investigation underscores the importance of mastering coordinate transformations and derivative techniques in analyzing complex curves, which are invaluable in both theoretical mathematics and practical engineering scenarios. As such, it exemplifies the elegance and utility of calculus in unraveling the geometric properties of curves defined in polar coordinates.

References

- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Anton, H., Bivens, I., & Davis, S. (2013). Calculus: Early Transcendental Functions. John Wiley & Sons.
- Larson, R., & Edwards, B. H.

Find The Slope Of The Line Tangent To The Polar Curve At The Given Point R 3 Sin Theta 3 2 Pi 6

Find The Slope Of The Line Tangent To The Polar Curve At The Given Point R 3 Sin Theta 3 2 Pi 6

Related Articles

- Federigo's Falcon From The Decameron Giovanni Boccaccio Translated By G.H. McWilliam. You Can Search
- For This Assignment, You Will Create A Function To Calculate The Maximum Length Of The Third Side Of
- HELP PLEASE! Part AWhat Is A Central Idea In The Newsela Article "Washed-Up Plastics Become Art With

Back to Home