Find The Slope Of The Tangent Line To The Given Polar Curve At The Point Specified By The Value Of .

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Understanding how to find the slope of the tangent line to a polar curve at a specific point is a fundamental skill in calculus, especially in the study of curves defined in polar coordinates. Unlike Cartesian coordinates, where the slope is straightforward to compute using derivatives of y with respect to x, polar coordinates require a different approach because the relationship between r and θ involves a different set of derivatives. This article will guide you through the process of determining the slope of the tangent line at a particular point on a polar curve, emphasizing the steps, formulas, and conceptual understanding needed for a comprehensive grasp of the topic.

Understanding Polar Coordinates and Curves

What Are Polar Coordinates?

Polar coordinates provide a way to specify a point in the plane using a radius r and an angle θ. Unlike Cartesian coordinates (x, y), where a point is defined by its horizontal and vertical distances from the origin, in polar coordinates, the position is determined relative to the origin:

- r (the radius) – the distance from the origin to the point.
- θ (the angle) – the counterclockwise angle from the positive x-axis to the line segment connecting the origin to the point.

The relationship between Cartesian and polar coordinates is given by:

$$
x = r \cos \theta, \quad y = r \sin \theta
$$

Polar Curves and Their Equations

A polar curve is described by an equation involving r and θ, typically written as:

$$
r = f(\theta)
$$

This function describes how the radius varies as the angle θ changes, thus defining the shape of the curve.

Finding the Slope of the Tangent Line in Polar Coordinates

Why Is the Slope Not Simply $\frac{dy}{dx}$ in Polar Coordinates?

In Cartesian coordinates, the slope of the tangent line at a point is simply $\frac{dy}{dx}$. However, in polar coordinates, the relationship between r and θ complicates this because both x and y depend on r and θ. Therefore, to find the slope of the tangent line at a point on a polar curve, we need to express $\frac{dy}{dx}$ in terms of derivatives with respect to θ.

Deriving the Formula for the Slope

Given $r = f(\theta)$, the coordinates are:

$$
x = r \cos \theta = f(\theta) \cos \theta
$$

$$
y = r \sin \theta = f(\theta) \sin \theta
$$

To find the slope $\frac{dy}{dx}$, we differentiate y and x with respect to θ:

$$
\frac{dy}{d\theta} = \frac{d}{d\theta} \left[f(\theta) \sin \theta \right] = f'(\theta) \sin \theta + f(\theta) \cos \theta
$$

$$
\frac{dx}{d\theta} = \frac{d}{d\theta} \left[f(\theta) \cos \theta \right] = f'(\theta) \cos \theta - f(\theta) \sin \theta
$$

Therefore, the slope of the tangent line at a point corresponding to θ is:

$$
\boxed{
\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta}
}
$$

This formula allows us to compute the slope of the tangent line at any point θ on the curve, provided $r = f(\theta)$.

Step-by-Step Procedure to Find the Slope at a Given Point

1. Identify the Given θ-Value

Begin by noting the specific value of θ at which you want to find the tangent's slope. This value is often given or can be found by solving the equation $r = f(\theta)$ for the point of interest.

2. Compute r at θ

Calculate $r = f(\theta)$. This gives the distance from the origin to the point on the curve.

3. Find $f'(\theta)$

Differentiate $f(\theta)$ with respect to θ. This derivative, $f'(\theta)$, measures how r changes as θ varies.

4. Calculate $\frac{dy}{d\theta}$ and $

$\frac{dx}{d\theta}$

Use the formulas:

$$
\frac{dy}{d\theta} = f'(\theta) \sin \theta + f(\theta) \cos \theta
$$
$$
\frac{dx}{d\theta} = f'(\theta) \cos \theta - f(\theta) \sin \theta
$$

Substitute the known values of $f(\theta)$, $f'(\theta)$, and θ.

5. Compute the Slope $\frac{dy}{dx}$

Divide $\frac{dy}{d\theta}$ by $\frac{dx}{d\theta}$:

$$
\frac{dy}{dx} = \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta}
$$

This value is the slope of the tangent line to the polar curve at the specified point.

Examples Illustrating the Process

Example 1: The Cardioid $r = 1 + \cos \theta$

Suppose we want to find the slope of the tangent line at $\theta = \frac{\pi}{3}$.

Step 1: $\theta = \frac{\pi}{3}$

Step 2: Calculate r:

$$
r = 1 + \cos \frac{\pi}{3} = 1 + \frac{1}{2} = \frac{3}{2}
$$

Step 3: Find $f'(\theta)$:

$$

$$
f'(\theta) = - \sin \theta
$$
$$
f' \left(\frac{\pi}{3} \right) = - \sin \frac{\pi}{3} = - \frac{\sqrt{3}}{2}
$$

Step 4: Compute $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$:

$$
\frac{dy}{d\theta} = - \frac{\sqrt{3}}{2} \times \sin \frac{\pi}{3} + \frac{3}{2} \times \cos \frac{\pi}{3}
$$
$$
= - \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{3}{2} \times \frac{1}{2} = - \frac{3}{4} + \frac{3}{4} = 0
$$

Similarly,

$$
\frac{dx}{d\theta} = - \frac{\sqrt{3}}{2} \times \cos \frac{\pi}{3} - \frac{3}{2} \times \sin \frac{\pi}{3}
$$
$$
= - \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{3}{2} \times \frac{\sqrt{3}}{2} = - \frac{\sqrt{3}}{4} - \frac{3 \sqrt{3}}{4} = - \frac{4 \sqrt{3}}{4} = - \sqrt{3}
$$

Step 5: Calculate the slope:

$$
\frac{dy}{dx} = \frac{0}{- \sqrt{3}} = 0
$$

Interpretation: The tangent line at $\theta = \frac{\pi}{3}$ is horizontal.

Example 2: The Lemniscate $r^2 = 4 \cos 2 \theta$

This example involves implicit differentiation, but the core idea remains similar. For simplicity, focus on points where the curve is defined, and follow the procedure to find r, derivatives, and then the slope.

Frequently Asked Questions

How do you find the slope of the tangent line to a polar curve at a specific point?

To find the slope of the tangent line at a point on a polar curve r(θ), you differentiate r with respect to θ to find dr/dθ, then use the formula dy/dx = (r' (θ) sin θ + r(θ) cos θ) / (r' (θ) cos θ - r(θ) sin θ).

What is the significance of the point specified by a value of θ when finding the tangent slope in polar coordinates?

The value of θ determines the exact location on the polar curve where the tangent slope is calculated, as each θ corresponds to a specific point (r(θ), θ) on the curve.

Can you explain the step-by-step process to compute the slope of the tangent line at a given θ?

Yes. First, find r(θ). Next, compute dr/dθ. Then, plug these into the derivative formula for dy/dx in polar coordinates: dy/dx = (r' (θ) sin θ + r(θ) cos θ) / (r' (θ) cos θ - r(θ) sin θ). Simplify to find the slope.

What are common challenges when calculating the tangent slope of a polar curve at a specific point?

Common challenges include correctly differentiating r(θ), handling points where the denominator of dy/dx becomes zero (vertical tangent), and ensuring the correct θ value is used for the point of interest.

How does the value of r(θ) affect the slope of the tangent line in polar curves?

The radius r(θ) influences the position of the point on the curve, and together with dr/dθ, it determines the slope of the tangent line at that point according to the derivative formula.

Are there specific types of polar curves where finding the tangent slope is easier or more difficult?

Linear or simple curves like circles and lemniscates often make calculations straightforward, while complex or highly non-linear curves can make differentiation and simplification more challenging.

Can the slope of the tangent line in polar coordinates be vertical or horizontal? How is this identified?

Yes. A vertical tangent occurs when the denominator of dy/dx is zero, while a horizontal tangent occurs when the numerator is zero. These can be checked by setting the derivative equal to zero or undefined.

How do you interpret the slope of the tangent line in the context of a polar curve's geometry?

The slope indicates the direction of the curve at the point, showing whether the curve is increasing, decreasing, or turning, and helps in understanding the curve's local behavior.

What tools or software can assist in calculating the slope of the tangent to a polar curve at a given point?

Mathematical software like Wolfram Alpha, Desmos, GeoGebra, or graphing calculators can help compute derivatives and visualize the tangent line to a polar curve at specified points.

Additional Resources

Find The Slope Of The Tangent Line To The Given Polar Curve At The Point Specified By The Value Of

Understanding how to find the slope of the tangent line to the given polar curve at the point specified by the value of θ (theta) is a fundamental skill in calculus, especially when dealing with curves expressed in polar coordinates. This task involves translating the polar function into a form where derivatives can be computed, and then applying the principles of calculus to determine the slope at a specific point. Whether you're a student tackling a calculus problem or a professional analyzing complex curves, mastering this process is key to understanding the behavior of polar curves.

Introduction to Polar Curves and Their Significance

Polar coordinates provide a powerful way to describe curves that are symmetric or naturally expressed in terms of angles and distances from a fixed point, usually called the pole (origin). A typical polar curve is given as:

$$ r = f(\theta) $$

where:
- r is the distance from the origin,
- θ is the angle measured from the positive x-axis.

These curves include familiar shapes like circles, spirals, cardioids, and lemniscates. Analyzing their tangent lines involves understanding how the curves change as θ varies.

Why Find the Slope of the Tangent Line?

Determining the slope of the tangent line at a specific point on a polar curve helps in understanding:
- The instantaneous rate of change of the radius with respect to the angle.
- The direction in which the curve is moving at that point.
- The behavior of the curve near that point (e.g., increasing, decreasing, points of inflection).

- Applications like physics (velocity and acceleration in polar motion), engineering, and computer graphics.

The Challenge: Converting Polar to Cartesian for Derivative Calculation

While the polar function $r = f(\theta)$ is straightforward, calculating derivatives directly requires some transformations. The key approach involves expressing the Cartesian coordinates (x, y) in terms of r and θ:

$$
x = r \cos \theta
$$

$$
y = r \sin \theta
$$

Given this, the slope of the tangent line at a particular θ can be found using derivatives:

$$
\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}
$$

\]

This quotient rule, applied to the parametric equations, allows us to compute the slope at the specified θ.

Step-by-Step Guide to Find the Slope

1. Identify the Given Polar Equation and the Point

Suppose you are given:
- A polar curve $r = f(\theta)$,
- A specific value θ_0 at which you need to find the slope.

First, evaluate r at θ_0:

$$
r_0 = f(\theta_0)
$$

This gives the point in polar coordinates:

$$
\text{Point} = (r_0, \theta_0)
$$

---

2. Express x and y in terms of θ

Using the relationships:

$$
x = r \cos \theta
$$
$$
y = r \sin \theta
$$

Substitute $r = f(\theta)$:

$$
x(\theta) = f(\theta) \cos \theta
$$
$$
y(\theta) = f(\theta) \sin \theta
$$

---

3. Compute Derivatives $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$

Differentiate both $x(\theta)$ and $y(\theta)$ with respect to θ:

$$
\frac{dx}{d\theta} = \frac{d}{d\theta} \left[f(\theta) \cos \theta \right] = f'(\theta) \cos \theta - f(\theta) \sin \theta
$$

$$
\frac{dy}{d\theta} = \frac{d}{d\theta} \left[f(\theta) \sin \theta \right] = f'(\theta) \sin \theta + f(\theta) \cos \theta
$$

where $f'(\theta)$ is the derivative of $f(\theta)$ with respect to θ.

---

4. Calculate the Slope $\frac{dy}{dx}$ at θ_0

Using the derivatives:

$$
\boxed{
\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta}
}
$$

\]

Evaluate this expression at \(\theta = \theta_0 \) to find the slope of the tangent line at that point.

Practical Example

Suppose the polar curve is:

\[
r = 2 \sin \theta
\]

and we want to find the slope of the tangent line at \(\theta = \frac{\pi}{4} \).

Step 1: Evaluate \(r \):

\[
r_0 = 2 \sin \frac{\pi}{4} = 2 \times \frac{\sqrt{2}}{2} = \sqrt{2}
\]

Step 2: Compute derivatives:

\[
f(\theta) = 2 \sin \theta \Rightarrow

$$
f'(\theta) = 2 \cos \theta
$$

Step 3: Find $\frac{dy}{dx}$:

$$
\frac{dy}{dx} = \frac{2 \cos \theta \sin \theta + 2 \sin \theta \cos \theta}{2 \cos \theta \cos \theta - 2 \sin \theta \sin \theta}
$$

Simplify numerator:

$$
2 \cos \theta \sin \theta + 2 \sin \theta \cos \theta = 4 \sin \theta \cos \theta
$$

Simplify denominator:

$$
2 \cos^2 \theta - 2 \sin^2 \theta = 2 (\cos^2 \theta - \sin^2 \theta) = 2 \cos 2\theta
$$

At $\theta = \frac{\pi}{4}$:

$$
\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}, \quad \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}
$$

\]

Calculate numerator:

$$
4 \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = 4 \times \frac{1}{2} = 2
$$

Calculate denominator:

$$
2 \cos \left(2 \times \frac{\pi}{4} \right) = 2 \cos \frac{\pi}{2} = 2 \times 0 = 0
$$

Thus, the slope is:

$$
\frac{dy}{dx} = \frac{2}{0}
$$

which indicates the tangent line is vertical at this point.

Additional Considerations and Tips

- Handling points where denominator is zero:

When the denominator $f'(\theta) \cos \theta - f(\theta) \sin \theta$ is zero, the tangent line is vertical, and the slope approaches infinity. Conversely, if numerator is zero and denominator is non-zero, the tangent line is horizontal.

- Using parametric derivatives: The approach hinges on expressing the curve parametrically and differentiating accordingly.

- Numerical approximation: For complicated functions where derivatives are difficult to compute analytically, numerical methods can estimate derivatives at specific points.

- Graphical interpretation: Always plot or visualize the curve near the point to understand the behavior of the tangent line.

Summary

Finding the slope of the tangent line to the given polar curve at the point specified by the value of θ involves a systematic process:

1. Recognize the polar equation \(r =

$f(\theta)$.
2. Convert to parametric Cartesian coordinates $x = r \cos \theta$, $y = r \sin \theta$.
3. Derive $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$.
4. Calculate $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$.
5. Evaluate at the specified θ to find the slope.

Mastering this process allows you to analyze the geometric properties of polar curves comprehensively. Whether solving textbook problems or applying in scientific contexts, knowing how to find the tangent line's slope enhances your understanding of the curve's behavior and its applications.

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