# Find The Solution To The Differential Equation (i) Y" + 3y' - 4y = 0, Y(0) = 0, Y'(0) = 3, Y"(0) = 6

**Find The Solution To the Differential Equation (i) Y" + 3y' - 4y = 0, Y(0) = 0, Y'(0) = 3, Y"(0) = 6** is a fundamental problem in differential equations, often encountered in various fields such as engineering, physics, and applied mathematics. Solving such differential equations not only helps in understanding the behavior of dynamic systems but also enhances analytical and problem-solving skills. This comprehensive guide aims to walk you through the step-by-step process of solving the differential equation, analyzing initial conditions, and obtaining the explicit solution. Whether you're a student, educator, or professional, mastering these methods is essential for tackling complex differential equations with confidence.

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## Understanding the Differential Equation

Before diving into the solution process, it's crucial to understand the structure and type of the differential equation at hand.

### Type of Differential Equation

The given differential equation is:

$$Y'' + 3Y' - 4Y = 0$$

This is a second-order linear homogeneous differential equation with constant coefficients. Such equations are fundamental in mathematical modeling because their solutions describe a variety of physical phenomena, such as mechanical vibrations, electrical circuits, and thermal processes.

### Initial Conditions

The problem also specifies initial conditions:

- $Y(0) = 0$
- $Y'(0) = 3$
- $Y''(0) = 6$

These conditions are essential for determining the particular solution from the general solution. Note that since this is a second-order differential equation, typically two initial conditions are sufficient; however, here, three are provided, which suggests an overdetermined set or that the problem may involve higher derivatives for validation.

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# Step-by-Step Solution of the Differential Equation

Solving this differential equation involves several systematic steps:

## Step 1: Find the Characteristic Equation

For differential equations with constant coefficients, the first step is to write the characteristic (or auxiliary) equation:

$$ r^2 + 3r - 4 = 0 $$

This quadratic equation will give us the roots, which determine the form of the general solution.

## Step 2: Solve the Characteristic Equation

Using the quadratic formula:

$$ r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

where $a=1$, $b=3$, and $c=-4$:

$$ r = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-4)}}{2(1)} = \frac{-3 \pm \sqrt{9 + 16}}{2} = \frac{-3 \pm \sqrt{25}}{2} $$

$$ r = \frac{-3 \pm 5}{2} $$

Thus, the roots are:

- $r_1 = \frac{-3 + 5}{2} = 1$
- $r_2 = \frac{-3 - 5}{2} = -4$

## Step 3: Write the General Solution

Since the roots are real and distinct, the general solution to the differential equation is:

$$ Y(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t} = C_1 e^{t} + C_2 e^{-4t} $$

where $C_1$ and $C_2$ are constants determined by initial conditions.

## Step 4: Find Derivatives of the General Solution

Calculate the first and second derivatives:

$$
Y'(t) = C_1 e^{t} - 4 C_2 e^{-4t}
$$

$$
Y''(t) = C_1 e^{t} + 16 C_2 e^{-4t}
$$

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# Applying Initial Conditions to Find Constants

To determine $C_1$ and $C_2$, substitute the initial conditions into the expressions for $Y(t)$, $Y'(t)$, and $Y''(t)$.

## Calculating Using $Y(0) = 0$

$$
Y(0) = C_1 e^{0} + C_2 e^{0} = C_1 + C_2 = 0
$$

$$
\Rightarrow C_2 = -C_1
$$

## Calculating Using $Y'(0) = 3$

$$
Y'(0) = C_1 e^{0} - 4 C_2 e^{0} = C_1 - 4 C_2
$$

Substitute $C_2 = -C_1$:

$$
Y'(0) = C_1 - 4 (-C_1) = C_1 + 4 C_1 = 5 C_1
$$

Set equal to 3:

\[

$$
5 C_1 = 3 \Rightarrow C_1 = \frac{3}{5}
$$

Then:

$$
C_2 = -\frac{3}{5}
$$

## Calculating Using $Y''(0) = 6$

$$
Y''(0) = C_1 e^{0} + 16 C_2 e^{0} = C_1 + 16 C_2
$$

Substitute known $C_2 = -C_1$:

$$
Y''(0) = C_1 + 16 (-C_1) = C_1 - 16 C_1 = -15 C_1
$$

Set equal to 6:

$$
-15 C_1 = 6 \Rightarrow C_1 = -\frac{6}{15} = -\frac{2}{5}
$$

But earlier, from $Y'(0)$, we found $C_1 = \frac{3}{5}$. The inconsistency suggests that the initial conditions are not independent or the problem is overdetermined. Typically, for a second-order differential equation, only two initial conditions are necessary; having three initial conditions implies the third condition is a check or that the conditions are compatible.

To reconcile this, we note that the initial conditions must be consistent. Since the initial conditions specify both $Y(0)$, $Y'(0)$, and $Y''(0)$, but our solution is only second order, this indicates an overdetermined set.

In practice, only the first two initial conditions are necessary to find $C_1$ and $C_2$. The third condition can be used to verify the solution once $C_1$ and $C_2$ are determined.

Therefore, the consistent solution considering the initial conditions $Y(0) = 0$ and $Y'(0) = 3$ is:

$$
C_1 = \frac{3}{5}, \quad C_2 = -\frac{3}{5}
$$

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# Final Solution to the Differential Equation

Putting it all together, the explicit solution is:

$$
Y(t) = \frac{3}{5} e^{t} - \frac{3}{5} e^{-4t}
$$

This function satisfies the differential equation and initial conditions (assuming the primary initial conditions are $Y(0) = 0$ and $Y'(0) = 3$).

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# Additional Insights and Applications

Understanding how to solve second-order linear homogeneous differential equations with constant coefficients is crucial for modeling real-world systems.

## Key Points to Remember

- The characteristic equation provides the roots that determine the solution form.
- Distinct real roots lead to exponential solutions.
- Initial conditions are used to find the specific constants in the general solution.
- Compatibility of initial conditions is essential; overdetermined conditions may require revisiting the problem setup.

## Applications of Differential Equation Solutions

- Mechanical vibrations: Analyzing oscillations in mechanical systems.
- Electrical circuits: Determining current and voltage over time.
- Thermal analysis: Modeling heat conduction.
- Population dynamics: Predicting growth or decline trends.

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# Conclusion: Mastering the Solution Process for Differential Equations

Solving the differential equation $Y'' + 3Y' - 4Y = 0$ with initial conditions involves understanding the nature of the equation, deriving the characteristic roots, constructing the general solution, and applying initial conditions to find particular constants. This systematic approach not only provides the explicit solution but also deepens your understanding of differential equations' role in modeling complex systems.

To enhance your skills further, practice solving similar equations with different coefficients and initial conditions. With consistent practice, you'll develop a strong intuition for differential equations, enabling you to tackle advanced problems confidently.

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Keywords for SEO Optimization:

- Differential equations solutions
- Homogeneous second-order differential equations
- Solving second-order linear differential equations
- Differential equations with initial conditions
- Characteristic equation method
- Exponential solutions of differential equations
- Initial value problems in differential equations
- Engineering applications of differential equations
- Physics

# Frequently Asked Questions

## What is the characteristic equation for the differential equation Y'' + 3Y' - 4Y = 0?

The characteristic equation is $r^2 + 3r - 4 = 0$.

## How do you solve the characteristic equation to find the general solution?

Solve $r^2 + 3r - 4 = 0$ by factoring or quadratic formula to find roots, then form the general solution based on these roots.

## What are the roots of the characteristic equation $r^2 + 3r - 4 = 0$?

The roots are $r = 1$ and $r = -4$.

## What is the general solution to the differential equation based on the roots?

The general solution is $Y(t) = C_1e^{t} + C_2e^{-4t}$.

### How do we use initial conditions $Y(0)=0$, $Y'(0)=3$, and $Y''(0)=6$ to find the constants $C_1$ and $C_2$?

Substitute $t=0$ into Y, Y', and Y'' expressions, then solve the resulting system of equations for $C_1$ and $C_2$.

### What is the expression for the first derivative Y' in terms of $C_1$ and $C_2$?

$Y'(t) = C_1e^{t} - 4C_2e^{-4t}$.

### What is the expression for the second derivative Y'' in terms of $C_1$ and $C_2$?

$Y''(t) = C_1e^{t} + 16C_2e^{-4t}$.

### What is the particular solution for Y(t) after applying the initial conditions?

The solution is $Y(t) = 3e^{t} - 0.75e^{-4t}$.

## Additional Resources

Differential Equations: Unlocking the Solution to a Second-Order Homogeneous Equation

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In the realm of mathematical analysis, differential equations serve as fundamental tools for modeling a myriad of phenomena—from physics and engineering to economics and biology. Among these, second-order linear homogeneous differential equations stand out due to their broad applicability and rich solution structures. Today, we explore a compelling example:

$Y'' + 3Y' - 4Y = 0$, with initial conditions $Y(0) = 0$, $Y'(0) = 3$, and $Y''(0) = 6$.

This problem exemplifies the classic approach to solving such equations, blending theoretical elegance with practical utility. Let's delve into this problem step-by-step, uncovering the methods, reasoning, and insights along the way.

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## Understanding the Differential Equation and Its Context

### The Nature of the Equation

The differential equation in question is:

$$ Y'' + 3Y' - 4Y = 0 $$

This is a second-order linear homogeneous differential equation with constant coefficients. Its form indicates that the solution can be approached systematically by characteristic equations, a standard method in differential equations theory.

Why is this important? Because equations with constant coefficients often model physical systems such as mechanical vibrations, electrical circuits, or thermal processes where the relation between variables doesn't change over time or space.

### Initial Conditions and Their Significance

The initial conditions provided are:

- $Y(0) = 0$
- $Y'(0) = 3$
- $Y''(0) = 6$

These conditions specify the state of the system at $t = 0$, enabling us to determine the particular solution among infinitely many general solutions. The inclusion of the second derivative initial condition suggests an interest in the system’s acceleration or curvature at the initial point, which is less common but crucial in certain physical contexts like beam deflections or dynamic systems.

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## Step 1: Deriving the Characteristic Equation

The cornerstone of solving second-order linear ODEs with constant coefficients is the characteristic equation.

Given:

$$ Y'' + 3Y' - 4Y = 0 $$

Assuming a solution of the form:

$$ Y(t) = e^{rt} $$

where $r$ is a constant to be determined.

Substituting into the differential equation:

$$ r^2 e^{rt} + 3r e^{rt} - 4 e^{rt} = 0 $$

Dividing through by $e^{rt}$ (which is never zero):

$$ r^2 + 3r - 4 = 0 $$

This quadratic equation:

$$ r^2 + 3r - 4 = 0 $$

is the characteristic equation. Solving it provides the roots that define the general solution.

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## Step 2: Solving the Characteristic Equation

The quadratic formula:

$$ r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

where $a=1$, $b=3$, $c=-4$.

Calculating discriminant:

$$ \Delta = 3^2 - 4(1)(-4) = 9 + 16 = 25 $$

Thus:

$$ r = \frac{-3 \pm \sqrt{25}}{2} = \frac{-3 \pm 5}{2} $$

This yields two roots:

- $r_1 = \frac{-3 + 5}{2} = 1$
- $r_2 = \frac{-3 - 5}{2} = -4$

Implication: The roots are real and distinct, which simplifies the general solution considerably.

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## Step 3: Constructing the General Solution

For distinct real roots, the general solution to the differential equation is:

$$ Y(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t} = C_1 e^{t} + C_2 e^{-4t} $$

where $C_1$ and $C_2$ are constants to be determined by initial conditions.

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# Step 4: Applying Initial Conditions

To find $C_1$ and $C_2$, we leverage the initial conditions. First, compute $Y(t)$, $Y'(t)$, and $Y''(t)$, then evaluate at $t=0$.

Function:

$$Y(t) = C_1 e^{t} + C_2 e^{-4t}$$

First derivative:

$$Y'(t) = C_1 e^{t} - 4 C_2 e^{-4t}$$

Second derivative:

$$Y''(t) = C_1 e^{t} + 16 C_2 e^{-4t}$$

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## Applying $Y(0) = 0$:

$$Y(0) = C_1 e^{0} + C_2 e^{0} = C_1 + C_2 = 0$$

Equation 1:

$$C_1 + C_2 = 0$$

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## Applying $Y'(0) = 3$:

$$Y'(0) = C_1 e^{0} - 4 C_2 e^{0} = C_1 - 4 C_2 = 3$$

Equation 2:

$$C_1 - 4 C_2 = 3$$

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## Applying $Y''(0) = 6$:

$$Y''(0) = C_1 e^{0} + 16 C_2 e^{0} = C_1 + 16 C_2 = 6$$

Equation 3:

$$C_1 + 16 C_2 = 6$$

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## Step 5: Solving for $C_1$ and $C_2$

From Equation 1:

$$C_1 = - C_2$$

Substitute into Equation 2:

$$- C_2 - 4 C_2 = 3$$

$$-5 C_2 = 3$$

$$C_2 = - \frac{3}{5}$$

Then:

$$C_1 = - C_2 = - \left( - \frac{3}{5} \right) = \frac{3}{5}$$

Now verify with Equation 3:

$$C_1 + 16 C_2 = \frac{3}{5} + 16 \left( - \frac{3}{5} \right) = \frac{3}{5} - \frac{48}{5} = - \frac{45}{5} = -9$$

But this conflicts with the initial condition $Y''(0) = 6$. This indicates an inconsistency, suggesting that the initial conditions are incompatible with the general solution derived from the differential equation.

Implication: The initial conditions specified $Y(0) = 0$, $Y'(0) = 3$, and $Y''(0) = 6$ do not satisfy the differential equation's structure when expressed via the standard solution.

Resolution: Since the third initial condition contradicts the general solution's derivative at zero, it indicates a need for a more comprehensive approach that considers the differential equation's structure, possibly involving nonhomogeneous terms or considering initial conditions as approximate or part of an overdetermined system.

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## Alternative Approach: The Power Series Solution

Given the inconsistency, an alternative is to approach the problem via power series expansion or Laplace transforms, which can handle initial conditions more flexibly.

However, in the typical setting, for a second-order homogeneous linear differential equation, initial conditions should be compatible with the general solution. Since they are not, the problem may be designed to illustrate the importance of consistent initial data or to show the solution process up to the point of inconsistency.

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## Summary and Final Remarks

- The characteristic equation for $Y'' + 3Y' - 4Y = 0$ is quadratic, yielding roots $r = 1$ and $r = -4$.
- The general solution is:

$$Y(t) = C_1 e^{t} + C_2 e^{-4t}$$

- Initial conditions are used to solve for $C_1$ and $C_2$, but in this case, they lead to an inconsistency, highlighting the importance of verifying initial data's compatibility with the differential equation.

- When initial conditions are consistent, this method provides a straightforward way to find particular solutions, which can then be used to analyze physical systems or to further explore the behavior of solutions over time.

- For more complex initial conditions or when inconsistencies arise, advanced methods like Laplace transforms, power series solutions, or numerical techniques can be employed.

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## Implications for Practitioners

This detailed analysis underscores a critical lesson: the importance

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