Find The Solution To The Initial-value Problem 6 Dxdy=(2x+1)(y 22y8) With Y(0)=3 Using The Method Of

Find The Solution To The Initial-value Problem 6 Dxdy=(2x+1)(y 22y8) With Y(0)=3 Using The Method Of is a common question encountered in calculus, particularly within the scope of solving differential equations. This problem involves solving a first-order differential equation with an initial condition using a specific method. In this article, we will explore the detailed steps involved in solving such an initial-value problem (IVP), including the relevant techniques, mathematical concepts, and practical tips to arrive at an accurate solution.

Understanding the Initial-Value Problem and Its Components

What Is an Initial-Value Problem?

An initial-value problem (IVP) consists of a differential equation coupled with an initial condition that specifies the value of the unknown function at a particular point. In our case, the differential equation is:

6 Dxdy = (2x + 1)(y 22y8)

and the initial condition is:

Y(0) = 3

This means that at x = 0, the value of y is 3.

Deciphering the Differential Equation

The notation "6 Dxdy" suggests a differential equation involving derivatives of y with respect to x. Usually, in standard notation, this would be written as:

$6\ dy/dx = (2x + 1)(y - 22\ y^8)$

Note: The original equation appears to contain some notation or formatting issues, especially "y 22 y8". For clarity and correctness, we'll interpret it as:

$6\ dy/dx = (2x + 1)\ (y - 22\ y^8)$

Alternatively, if the original problem involves different notation, please clarify. Assuming this is the intended form, the goal is to solve for y as a function of x, satisfying the initial condition y(0) = 3.

Rearranging and Simplifying the Differential Equation

Step 1: Write the Differential Equation Clearly

Assuming the form:

6 dy/dx = (2x + 1) (y − 22 y^8)

Divide both sides by 6:

dy/dx = [(2x + 1) / 6] (y − 22 y^8)

Step 2: Recognize the Type of Differential Equation

The resulting differential equation is:

dy/dx = f(x, y) = [(2x + 1) / 6] (y − 22 y^8)

This is a first-order ordinary differential equation, specifically a nonlinear equation due to the y^8 term.

Choosing the Appropriate Method to Solve the IVP

Method Overview

Given the structure of the differential equation, the most suitable approach is to attempt a separation of variables, if possible, or a substitution that simplifies the equation.

Since the equation involves y and y^8, a substitution related to y^{1} or y^{k} might be effective. Alternatively, considering the form, a substitution involving u = y^{n} could help.

Step 1: Check for Separable Form

Rearranged as:

dy/dx = [(2x + 1)/6] y − [(2x + 1)/6] 22 y^8

This suggests the equation is not directly separable because of the mixed y terms with different powers.

Step 2: Use a Substitution to Simplify the Equation

Notice that the equation involves y and y^8. A common approach is to substitute:

$u = y^{m}$

Choosing m so that the resulting differential equation becomes linear or separable.

Since y^8 appears, let's consider $u = y^{k}$. To cancel the y^8 term, choose:

$u = y^{1} = y$ (so no change), or $u = y^{n}$ for some n.

Alternatively, note that the equation involves $y - 22 y^8$, which suggests factoring or substitution.

Transforming the Equation Using Substitution

Step 1: Substitution $u = y^{m}$

Let's choose $u = y^{1}$, which leaves the equation unchanged, or try $u = y^{k}$ to see if it simplifies.

Alternatively, observe that the ratio $y / y^8 = y^{-7}$.

Suppose we divide both sides of the original differential equation by y:

$dy/dx / y = [(2x + 1) / 6] (1 - 22 y^7)$

This yields:

$(1/y) dy/dx = [(2x + 1)/6] (1 - 22 y^7)$

Now, define:

$v = y^{-6}$

Then:

$dv/dx = -6 y^{-7} dy/dx$

Rearranged as:

$dy/dx = - (1/6) y^{7} dv/dx$

Substituting into the previous expression:

$(1/y) dy/dx = - (1/6) y^{6} dv/dx (1/y) = - (1/6) y^{5} dv/dx$

But this approach complicates the problem further.

Instead, it's more straightforward to consider the substitution:

$u = y^{7}$

Then:

$du/dx = 7 y^{6} dy/dx$

Express dy/dx as:

$dy/dx = (1/7 y^{6}) du/dx$

Now, rewrite the original differential equation:

$6 dy/dx = (2x + 1)(y - 22 y^{8})$

Substitute dy/dx:

$6 (1/7 y^{6}) du/dx = (2x + 1)(y - 22 y^{8})$

Express y and y^{8} in terms of u:

Since $u = y^{7} \Rightarrow y = u^{1/7}$

Then:

$y = u^{1/7}$

and $y^{8} = y^{7} y = u u^{1/7} = u^{(1 + 7)/7} = u^{8/7}$

Similarly, $y - 22 y^{8} = u^{1/7} - 22 u^{8/7}$

Expressed in terms of u:

$(2x + 1) (u^{1/7} - 22 u^{8/7})$

Now, rewrite the entire differential equation:

$(6 / 7 y^{6}) du/dx = (2x + 1) (u^{1/7} - 22 u^{8/7})$

But $y^{6} = (u^{1/7})^{6} = u^{6/7}$

So:

$(6 / 7) u^{-6/7} du/dx = (2x + 1) (u^{1/7} - 22 u^{8/7})$

Multiply both sides by $u^{6/7}$:

$(6 / 7) du/dx = (2x + 1) [u^{1/7} u^{6/7} - 22 u^{8/7} u^{6/7}]$

Calculate the exponents:

$u^{1/7} u^{6/7} = u^{7/7} = u^{1}$

$u^{8/7} u^{6/7} = u^{14/7} = u^{2}$

Thus, the equation becomes:

$(6 / 7) du/dx = (2x + 1) (u - 22 u^{2})$

This is a Bernoulli differential equation, which is solvable via the substitution $v = u^{1 - n}$.

Since the equation is:

$du/dx = (7/6) (2x + 1) (u - 22 u^{2})$

Rewrite as:

$du/dx + P(x)\ u = Q(x)\ u^{n}$

Where:

$P(x) = -\ (7/6)\ (2x + 1)\ 1$

$Q(x) = (7/6)\ (2x + 1)\ 22$

and $n = 2$

This is a Bernoulli equation with $n = 2$.

Solving the Bernoulli Equation

Step 1: Write the Bernoulli Equation

The Bernoulli form:

$du/dx + P(x)\ u = Q(x)\ u^{n}$

Here,

$du/dx + P(x)\ u = Q(x)\ u^{2}$

with

$P(x) = -\ (7/6)(2x + 1)$

$Q(x) = (7/6)(2x + 1)\ 22$

Step 2: Substitution $v = u^{1 - n} = u^{-1}$

Differentiate v:

$dv/dx = -\ u^{-2}\ du/dx$

Rearranged:

$du/dx = -\ u^{2}\ dv/dx$

Substitute into the Bernoulli equation:

$-\ u^{2}\ dv/dx + P(x)\ u = Q(x)\ u^{2}$

Divide through by u^{2}:

$-\ dv/dx + P(x)\ u^{-1} = Q(x)$

But $u^{-1} = v$

Thus,

Frequently Asked Questions

What is the initial-value problem given in the question?

The initial-value problem is 6 Dxdy = $(2x + 1)(y - 2y^8)$ with $Y(0) = 3$.

How do you interpret the differential equation 6 Dxdy = $(2x + 1)(y - 2y^8)$?

It appears to be a typo; likely the intended form is a differential equation involving dy/dx, possibly $6\, dy/dx = (2x + 1)(y - 2y^8)$, which can be solved using the method of separation of variables.

What method is suggested for solving this initial-value problem?

The method suggested is the method of separation of variables, which involves rewriting the differential equation to separate variables y and x.

How do you separate variables in the differential equation?

Rearranged as $dy/dx = [(2x + 1)/6]\ (y - 2y^8)$, then rewrite to isolate y terms on one side and x terms on the other for integration.

What is the initial condition provided, and how is it used?

The initial condition is $Y(0) = 3$, which is used to find the constant of integration after integrating the separated variables.

What are the key steps to solving this initial-value problem?

The steps include rewriting the differential equation in separable form, integrating both sides, applying the initial condition to solve for the constant, and then expressing the solution for y in terms of x.

Are there any potential difficulties in solving this problem?

Yes, the nonlinear terms like $y - 2y^8$ may lead to complex integration, and ensuring correct separation and integration is crucial.

What is the significance of the initial condition $Y(0)=3$ in the solution?

It helps determine the particular solution among the family of solutions by solving for the constant of integration.

Can this problem be solved analytically, and what tools might assist in solving it?

Yes, it can be solved analytically through separation of variables and integration, possibly aided by substitution if needed; symbolic computation tools can also assist.

Additional Resources

Find The Solution To The Initial-value Problem 6 Dxdy = (2x + 1)(y^2 - 2y + 8) With y(0) = 3 Using The Method Of

When tackling differential equations, the phrase "find the solution to the initial-value problem" often marks the beginning of a journey into the core techniques of calculus. In this particular problem, we're presented with a differential equation involving both variables and an initial condition:

$$ 6 \, dx \, dy = (2x + 1)(y^2 - 2y + 8) \quad \text{with} \quad y(0) = 3 $$

Our goal is to find the explicit solution $y(x)$ that satisfies the initial condition. To do this, we'll employ the method of separation of variables, a powerful technique for solving first-order differential equations that can be rewritten in the form $\frac{dy}{dx} = f(x)g(y)$.

Understanding the Differential Equation

Before proceeding with solution techniques, it's essential to understand what the problem entails:

- The equation involves a product of functions in x and y.
- The initial condition specifies the value of y at $x = 0$.

Rearranged, the differential equation becomes:

$$ 6 \, dx \, dy = (2x + 1)(y^2 - 2y + 8) $$

Dividing both sides by 6:

$$ dx \, dy = \frac{(2x + 1)(y^2 - 2y + 8)}{6} $$

But to clarify the structure, note that in differential form notation, this can be expressed as:

$$ dy/dx = \frac{(2x + 1)(y^2 - 2y + 8)}{6} $$

Step 1: Expressing as a Separable Equation

The core idea behind the method of separation of variables is to rewrite the differential equation so that all y-terms are on one side and all x-terms are on the other.

Given:

\[dy/dx = \frac{(2x + 1)}{6}(y^2 - 2y + 8) \]

We observe that the right side factors into a product of a function of \(x \) and a function of \(y \). This suggests the potential for separation of variables:

\[\frac{dy}{(y^2 - 2y + 8)} = \frac{(2x + 1)}{6} \, dx \]

Step 2: Simplify the \(y \)-dependent expression

Focus on the denominator:

\[y^2 - 2y + 8 \]

Complete the square to facilitate integration:

\[
\begin{aligned}
y^2 - 2y + 8 &= (y^2 - 2y + 1) + 7 \\
&= (y - 1)^2 + 7
\end{aligned}
\]

Hence, the differential equation becomes:

\[\frac{dy}{(y - 1)^2 + 7} = \frac{(2x + 1)}{6} \, dx \]

Step 3: Integrate both sides

The problem now reduces to integrating:

\[\int \frac{dy}{(y - 1)^2 + 7} = \int \frac{2x + 1}{6} \, dx \]

Step 4: Perform the integrations

Integration with respect to \(y \):

The integral:

\[\int \frac{dy}{(y - 1)^2 + 7} \]

is a standard form, akin to the arctangent integral:

\[\int \frac{du}{u^2 + a^2} = \frac{1}{a} \arctan \left(\frac{u}{a} \right) + C \]

Let \(u = y - 1 \), so \(du = dy \).

Given \(a^2 = 7 \), so \(a = \sqrt{7} \).

Thus:

\[\int \frac{dy}{(y - 1)^2 + 7} = \frac{1}{\sqrt{7}} \arctan \left(\frac{y

- 1}{\sqrt{7}} \right) + C \]

Integration with respect to \(x \):

\[
\int \frac{2x + 1}{6} \, dx = \frac{1}{6} \int (2x + 1) \, dx
\]

which simplifies to:

\[
\frac{1}{6} \left(x^2 + x \right) + C
\]

Step 5: Combine the results

Putting it all together:

\[
\frac{1}{\sqrt{7}} \arctan \left(\frac{y - 1}{\sqrt{7}} \right) =
\frac{1}{6} (x^2 + x) + C
\]

Multiply both sides by \(\sqrt{7} \):

\[
\arctan \left(\frac{y - 1}{\sqrt{7}} \right) = \frac{\sqrt{7}}{6} (x^2 + x)
+ C'
\]

where \(C' = C \times \sqrt{7} \), an arbitrary constant.

Step 6: Apply the initial condition \(y(0) = 3 \)

At \(x = 0 \), \(y = 3 \):

\[
\arctan \left(\frac{3 - 1}{\sqrt{7}} \right) = \frac{\sqrt{7}}{6} (0 + 0) +
C'
\]

Calculating:

\[
\arctan \left(\frac{2}{\sqrt{7}} \right) = C'
\]

Therefore, the particular solution is:

\[
\arctan \left(\frac{y - 1}{\sqrt{7}} \right) = \frac{\sqrt{7}}{6} (x^2 + x)
+ \arctan \left(\frac{2}{\sqrt{7}} \right)
\]

Final explicit solution:

Expressed explicitly for y:

$$
\boxed{
y(x) = 1 + \sqrt{7} \tan \left[\frac{\sqrt{7}}{6} (x^2 + x) + \arctan \left(\frac{2}{\sqrt{7}} \right) \right]
}
$$

This formula provides the solution to the initial-value problem using the method of separation of variables.

Summary of the Solution Process

1. Recognize the structure of the differential equation:

- The original differential equation was expressed as a product involving dx and dy.
- Rearranged into a form suitable for separation of variables.

2. Simplify the y-dependent part:

- Completed the square on $y^2 - 2y + 8$ to facilitate integration.

3. Separate variables:

- Placed y terms on one side and x terms on the other.

4. Integrate both sides:

- Used the standard arctangent integral for the y-side.
- Integrated the polynomial on the x-side.

5. Apply initial condition:

- Solved for the constant of integration.

6. Express the solution explicitly:

- Solved for y in terms of x.

Additional Tips for Solving Similar Problems

- Always check if the differential equation can be separated into $dy/dx = f(x)g(y)$.
- Complete the square when the denominator involves quadratic expressions to identify standard integral forms.
- Use substitution to simplify integrals involving quadratic or rational expressions.
- Apply initial conditions immediately after integration to find the particular solution.
- Remember the properties of inverse trigonometric functions, especially when solving integrals leading to arctangent forms.

Conclusion

The process of solving the initial-value problem involving differential equations often hinges on recognizing the structure and applying appropriate methods like separation of variables. In this case, by rewriting the differential equation, completing the square, and integrating using standard forms, we arrived at a clear, explicit solution. The key is to methodically manipulate the equation, simplify where possible, and leverage known integral formulas, all while ensuring the initial condition is satisfied for the particular solution. This approach exemplifies the power of classical methods in differential equations, providing a robust framework for tackling similar problems in calculus and applied mathematics.

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