Find The Solution To The Linear System Using Gaussian Elimination.x-2y=4 4x +2y=6 A. (2,1) B. (-1,2)

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Understanding how to solve a system of linear equations is fundamental in algebra, especially when dealing with multiple variables and equations. Among various methods, Gaussian elimination stands out for its systematic approach to reducing complex systems into simpler forms, ultimately leading to the solutions. In this article, we will explore the process of finding solutions to the given system of equations using Gaussian elimination. We will also verify the provided options (A. (2,1) and B. (-1,2)) to determine which one satisfies the system.

Introduction to the System of Linear Equations

Before diving into Gaussian elimination, it's essential to understand the given system:

1. Equation 1: x - 2y = 4
2. Equation 2: 4x + 2y = 6

This system involves two variables, x and y, and two equations. Our goal is to find the values of x and y that satisfy both equations simultaneously.

Understanding Gaussian Elimination

Gaussian elimination is a method for solving linear systems by converting the system's augmented matrix into row-echelon form and then performing back substitution to find the solutions.

Key steps involved:

- Step 1: Write the augmented matrix of the system.
- Step 2: Use row operations to eliminate variables systematically.

- Step 3: Convert the matrix into row-echelon form.
- Step 4: Use back substitution to find the solutions for variables.

Representing the System as an Augmented Matrix

The first step is to express the system as an augmented matrix:

$$
\begin{bmatrix}
1 & -2 & | & 4 \\
4 & 2 & | & 6 \\
\end{bmatrix}
$$

Here:

- The first row corresponds to Equation 1.
- The second row corresponds to Equation 2.
- The vertical bar separates the coefficients from the constants.

Applying Gaussian Elimination Step-by-Step

Let's now perform row operations to simplify this matrix.

Step 1: Make the leading coefficient of the first row 1 (already done)

The first row's leading coefficient (pivot) is 1, which is ideal.

Step 2: Use the first row to eliminate the x-term in the second row

- Multiply Row 1 by 4 to align with the second row's x coefficient:

\[
4 \times R_1: \quad [4 \quad -8 \quad | \quad 16]
\]

- Subtract this from Row 2:

\[
R_2 - 4 \times R_1: \quad [4 - 4 \quad 2 - (-8) \quad | \quad 6 - 16] = [0 \quad 10 \quad | \quad -10]
\]

Now, the matrix becomes:

\[
\begin{bmatrix}
1 & -2 & | & 4 \\
0 & 10 & | & -10 \\
\end{bmatrix}
\]

Step 3: Simplify the second row

Divide Row 2 by 10 to make the leading coefficient 1:

\[
R_2 \div 10: \quad [0 \quad 1 \quad | \quad -1]
\]

The matrix now is:

\[
\begin{bmatrix}
1 & -2 & | & 4 \\
0 & 1 & | & -1 \\
\end{bmatrix}
\]

Step 4: Back substitution

- From Row 2: y = -1
- Substitute y = -1 into Row 1:

$$
x - 2(-1) = 4 \\
x + 2 = 4 \\
x = 4 - 2 = 2
$$

Solution: $x = 2$, $y = -1$

Verifying the Solutions with the Given Options

The options provided are:

- Option A: (2, 1)
- Option B: (-1, 2)

Let's verify each.

Verification of Option A: (2, 1)

- Equation 1:

$$
x - 2y = 4 \\
2 - 2(1) = 2 - 2 = 0 \neq 4
$$

- Equation 2:

$$
4x + 2y = 6 \\
4(2) + 2(1) = 8 + 2 = 10 \neq 6
$$

Thus, Option A does not satisfy the system.

Verification of Option B: (-1, 2)

- Equation 1:

$$
x - 2y = 4 \\
-1 - 2(2) = -1 - 4 = -5 \neq 4
$$

- Equation 2:

$$
4x + 2y = 6 \\
4(-1) + 2(2) = -4 + 4 = 0 \neq 6
$$

Option B does not satisfy the system either.

Conclusion: The Correct Solution

From the Gaussian elimination process, we found the solution:

$$
\boxed{
x = 2, \quad y = -1
}
$$

which is not among the provided options. This indicates a possible discrepancy in the options or their intended solutions. Nonetheless, the systematic Gaussian elimination confirms that (2, -1) satisfies the system.

Additional Insights into Solving Linear Systems

To deepen understanding, here are some key points about solving systems like these:

- Unique solutions: When the system yields a consistent and non-dependent set of equations, the solution is unique.
- No solution: If the system simplifies to a contradiction (e.g., 0 = non-zero), then no solutions exist.
- Infinite solutions: If equations reduce to a dependent system (e.g., same line), infinitely many solutions exist.

Practical Applications of Gaussian Elimination

Gaussian elimination is widely used across various fields such as:

- Engineering: Analyzing circuit networks.
- Computer Science: Solving large systems in algorithms.
- Economics: Optimizing resource allocations.
- Physics: Solving systems of equations in mechanics and thermodynamics.

Its efficiency and systematic approach make it a fundamental method in linear algebra.

Summary and Final Thoughts

In this article, we explored how to solve a system of linear equations using Gaussian elimination. Starting with the given equations:

- $x - 2y = 4$
- $4x + 2y = 6$

We represented the system as an augmented matrix, performed row operations to reach row-echelon form, and used back substitution to find the solution: $x = 2$, $y = -1$. Testing the provided options revealed that neither matched the solution, highlighting the importance of systematic methods over assumptions.

Understanding Gaussian elimination equips students and professionals with a powerful tool for solving complex systems efficiently. Whether in academics or real-world applications, mastering this technique is essential for success in linear algebra and related fields.

Meta Description:
Learn how to solve a linear system using Gaussian elimination with step-by-step guidance on equations x - 2y = 4 and 4x + 2y = 6. Verify solutions and understand the process for accurate results.

Keywords:
Gaussian elimination, linear system, solving equations, system of equations, algebra, matrices, linear algebra solutions

Frequently Asked Questions

What is the first step in solving the linear system using Gaussian elimination for the equations x - 2y = 4 and 4x + 2y = 6?

The first step is to write the system as an augmented matrix and then use row operations to eliminate variables. Specifically, you can start by aligning the coefficients in matrix form:
[[1, -2, | 4], [4, 2, | 6]].

How do you eliminate one variable to solve the system using Gaussian elimination?

You can multiply the first row by a suitable factor and subtract it from the second row to eliminate one variable. For example, multiply the first row by 4 and subtract from the second to eliminate x and solve for y.

What is the solution to the system after applying Gaussian elimination?

After performing the row operations, the solution to the system is y = 1, and substituting back into one of the original equations gives x = 2. Therefore, the solution is (2, 1).

Which answer choice correctly represents the solution to the system?

The correct answer choice is A. (2, 1).

Can you verify the solution (2, 1) by substituting into both equations?

Yes, substituting x=2 and y=1:
- First equation: 2 - 2(1) = 2 - 2 = 0 ≠ 4, so check the calculation. Actually, the first equation is x - 2y = 4.
Substituting: 2 - 2(1) = 2 - 2 = 0, which does not equal 4.
This indicates a mistake—let's verify carefully.
Original equations:
- x - 2y = 4

- 4x + 2y = 6

Substitute x=2, y=1:

- 2 - 2(1) = 2 - 2 = 0 (not 4)
- 4(2) + 2(1) = 8 + 2 = 10 (not 6)

So the point (2,1) does not satisfy the equations.

Therefore, the correct solution is not (2,1). Let's re-solve the system to find the correct solution.

Additional Resources

Find The Solution To The Linear System Using Gaussian Elimination is a fundamental topic in algebra and linear algebra that provides a systematic approach to solving systems of linear equations. In this article, we will explore how to apply Gaussian elimination to a specific system of equations, analyze the solution process, and determine the correct solution from the given options. This method is widely used in various fields, including engineering, computer science, and mathematics, due to its efficiency and reliability in solving linear systems.

Understanding the Given System of Equations

The problem presents a system of two linear equations:

1. $x - 2y = 4$
2. $4x + 2y = 6$

Along with options for solutions:

- A. (2, 1)
- B. (-1, 2)

Before applying Gaussian elimination, it's essential to understand the structure of these equations and what a solution entails.

Analyzing the Equations

The system can be viewed as a set of simultaneous equations where the goal is to find the values of x and y that satisfy both equations simultaneously.

- The first equation $x - 2y = 4$ can be rewritten as $x = 4 + 2y$.

- The second equation $4x + 2y = 6$ can be used to express x or y after substitution.

Applying Gaussian Elimination to the System

Gaussian elimination involves converting the system into an augmented matrix, performing row operations to reach row-echelon form, and then back-substituting to find the solution.

Step 1: Write the Augmented Matrix

The system's augmented matrix is:

$$
\begin{bmatrix}
1 & -2 & | & 4 \\
4 & 2 & | & 6 \\
\end{bmatrix}
$$

This matrix represents the coefficients and constants of the equations.

Step 2: Use Row Operations to Achieve Upper Triangular Form

- The goal is to eliminate the x term from the second row.

- First, keep Row 1 as is:

$$
R_1: [1, -2, | 4]
$$

- To eliminate $4x$ in Row 2, perform:

$$
R_2 \rightarrow R_2 - 4 \times R_1
$$

Calculating:

$$
R_2: [4 - 4 \times 1, \quad 2 - 4 \times (-2), \quad 6 - 4 \times 4]
$$

$$
R_2: [4 - 4, \quad 2 + 8, \quad 6 - 16]
$$

$$
R_2: [0, 10, | -10]
$$

The matrix now looks like:

$$
\begin{bmatrix}
1 & -2 & | & 4 \\
0 & 10 & | & -10 \\
\end{bmatrix}
$$

Step 3: Back-Substitution to Find Variables

- From the second row:

$$
10 y = -10 \Rightarrow y = -1
$$

- Substitute $y = -1$ into the first equation:

$$
x - 2(-1) = 4 \Rightarrow x + 2 = 4 \Rightarrow x = 2
$$

Thus, the solution is (x, y) = (2, -1).

Comparing the Solution with the Provided Options

The derived solution $(2, -1)$ does not match either option directly:

- Option A: (2, 1)
- Option B: (-1, 2)

Therefore, neither option appears to be the exact solution. However, let's verify if any of these options satisfy the original equations.

Verification of Options

Option A: (2, 1)

- First equation: $x - 2y = 4$

$$
2 - 2 \times 1 = 2 - 2 = 0 \neq 4
$$

- Second equation: $4x + 2y = 6$

$$
4 \times 2 + 2 \times 1 = 8 + 2 = 10 \neq 6
$$

So, Option A doesn't satisfy the system.

Option B: (-1, 2)

- First equation: $-1 - 2 \times 2 = -1 - 4 = -5 \neq 4$

- Second equation: $4 \times -1 + 2 \times 2 = -4 + 4 = 0 \neq 6$

Option B also doesn't satisfy the system.

Analysis and Conclusion

The calculations clearly indicate that the true solution to the system is \((2, -1)\), which is not among the provided options. This suggests that there might be a typo or mistake in the options listed, or perhaps the question aims to test the ability to perform Gaussian elimination and verify solutions rather than match the options exactly.

Key points to note:

- The Gaussian elimination process efficiently reduces the system to its simplest form, making solutions straightforward.
- Verifying options against the original equations is a crucial step to confirm their validity.
- In this case, neither option A nor B fits the system, indicating that the correct solution is \((2, -1)\).

Features and Pros/Cons of Gaussian Elimination

Features:

- Systematic approach to solving linear systems
- Can be extended to larger systems with multiple variables
- Facilitates understanding of matrix operations and linear algebra

Pros:

- Reliable and straightforward for small to medium systems
- Facilitates algorithmic implementation in computational tools
- Can detect inconsistent or dependent systems

Cons:

- Can be computationally intensive for very large systems
- Requires careful row operations to avoid errors
- Not always the most efficient for a single small system compared to substitution or graphing

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Final Remarks

Using Gaussian elimination to solve linear systems is an essential skill in mathematics and engineering. It provides a clear, step-by-step method to find solutions, especially when systems become more complex. In this particular case, the solution \((2, -1)\) emerges as the correct answer, even though it isn't listed among the options. This exemplifies the importance of performing calculations rather than relying solely on multiple-choice options. Mastery of Gaussian elimination enhances problem-solving skills and deepens understanding of linear algebra fundamentals.

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