Find The Unit Tangent Vector T(t).r(t) = E2ti + Cos(t)j Sin(3t)k, P(l, 1, 0)Find A Set Of Parametric

Find The Unit Tangent Vector T(t).r(t) = E2ti + Cos(t)j Sin(3t)k, P(l, 1, 0)Find A Set Of Parametric

Understanding how to find the unit tangent vector $\mathbf{T}(t)$ for a given vector function $\mathbf{r}(t)$ is fundamental in the study of calculus and vector analysis. The process involves differentiating the vector function, calculating its magnitude, and normalizing the derivative to obtain a unit vector that is tangent to the curve at any point t. This article provides a comprehensive step-by-step guide to finding the unit tangent vector for the specific vector function:

$$
\mathbf{r}(t) = 2t\, \mathbf{i} + \cos(t)\, \mathbf{j} + \sin(3t)\, \mathbf{k}
$$

and explores how to determine parametric equations corresponding to the given point $P(1, 1, 0)$.

Understanding the Vector Function $\mathbf{r}(t)$

Before diving into calculations, it’s crucial to understand the components of the vector function:

$$
\mathbf{r}(t) = 2t\, \mathbf{i} + \cos(t)\, \mathbf{j} + \sin(3t)\, \mathbf{k}
$$

This function describes a curve in three-dimensional space, with each component depending on the parameter t:

- The x-component: $x(t) = 2t$
- The y-component: $y(t) = \cos(t)$
- The z-component: $z(t) = \sin(3t)$

The goal is to find the unit tangent vector $\mathbf{T}(t)$ at any point along this curve and then determine the parametric equations that pass through a specific point $P(1, 1, 0)$.

Step-by-Step Process to Find the Unit Tangent Vector $\mathbf{T}(t)$

1. Compute the Derivative $\mathbf{r}'(t)$

The derivative of $\mathbf{r}(t)$ with respect to t gives the tangent vector (not necessarily unit length):

$$
\mathbf{r}'(t) = \frac{d}{dt} (2t\, \mathbf{i} + \cos(t)\, \mathbf{j} + \sin(3t)\, \mathbf{k})
$$

Calculating each component:

- $\frac{d}{dt} (2t) = 2$
- $\frac{d}{dt} (\cos(t)) = -\sin(t)$
- $\frac{d}{dt} (\sin(3t)) = 3 \cos(3t)$

Thus,

$$
\mathbf{r}'(t) = 2\, \mathbf{i} - \sin(t)\, \mathbf{j} + 3 \cos(3t)\, \mathbf{k}
$$

---

2. Find the Magnitude $|\mathbf{r}'(t)|$

The magnitude of the derivative vector is:

$$
|\mathbf{r}'(t)| = \sqrt{(2)^2 + (-\sin(t))^2 + (3 \cos(3t))^2}
$$

Simplifying:

$$
|\mathbf{r}'(t)| = \sqrt{4 + \sin^2(t) + 9 \cos^2(3t)}
$$

This expression will be used to normalize $\mathbf{r}'(t)$.

---

3. Derive the Unit Tangent Vector \(\mathbf{T}(t) \)

The unit tangent vector is obtained by dividing the derivative \(\mathbf{r}'(t) \) by its magnitude:

\[
\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \left(\frac{2}{|\mathbf{r}'(t)|} \right) \mathbf{i} + \left(\frac{-\sin(t)}{|\mathbf{r}'(t)|} \right) \mathbf{j} + \left(\frac{3 \cos(3t)}{|\mathbf{r}'(t)|} \right) \mathbf{k}
\]

This vector points in the direction of the curve at \(t \) with unit length.

Finding the Parameter \(t \) Corresponding to Point \(P(1, 1, 0) \)

To find the value(s) of \(t \) where the curve passes through \(P(1, 1, 0) \), set \(\mathbf{r}(t) \) equal to \(P \):

\[
\begin{cases}
x(t) = 2t = 1 \\
y(t) = \cos(t) = 1 \\
z(t) = \sin(3t) = 0
\end{cases}
\]

Step 1: Solve for \(t \) from \(x(t) \)

\[
2t = 1 \Rightarrow t = \frac{1}{2}
\]

Step 2: Verify \(y(t) = \cos(t) = 1 \)

\[
\cos\left(\frac{1}{2}\right) \neq 1

$$

Since $\cos(0) = 1$, and $\cos(t) = 1$ only when $t = 2\pi k$, $k \in \mathbb{Z}$. But $t = \frac{1}{2} \neq 2\pi k$. Therefore, the point $P(1, 1, 0)$ does not lie on the curve at $t = \frac{1}{2}$.

Step 3: Find t from $y(t) = \cos(t) = 1$

$$
\cos(t) = 1 \Rightarrow t = 2\pi k, \quad k \in \mathbb{Z}
$$

Substituting back into $x(t)$:

$$
x(t) = 2t = 2(2\pi k) = 4\pi k
$$

We want $x(t) = 1$, so:

$$
4 \pi k = 1 \Rightarrow k = \frac{1}{4\pi}
$$

Since k must be an integer, $\frac{1}{4\pi}$ is not an integer, indicating that the point P does not lie exactly on the curve at these parameters.

Constructing a Set of Parametric Equations Passing Through $P(1, 1, 0)$

Given the previous calculations, the point $P(1, 1, 0)$ does not correspond to a simple value of t in the original parameterization. Therefore, to find a set of parametric equations passing through P, we can consider the inverse problem: find t that satisfy the point's coordinates approximately or set t as a parameter that allows the curve to pass through the point.

1. Express t in terms of x

Since:

$$
x(t) = 2t
$$

Set $x(t) = 1$:

$$
t = \frac{1}{2}
$$

2. Express $y(t)$ and $z(t)$ in terms of this t

At $t = 0.5$:

$$
y(t) = \cos(0.5) \approx 0.8776
$$
$$
z(t) = \sin(3 \times 0.5) = \sin(1.5) \approx 0.9975
$$

Since these do not match $y=1$ and $z=0$, the point P is not on the original curve at $t=0.5$. However, if the goal is to generate a family of parametric equations passing through P, then we can define:

$$
\boxed{
\begin{cases}
x(s) = 1 + a s \\
y(s) = 1 + b s \\
z(s) = 0 + c s
\end{cases}
}
$$

where a, b, c are constants chosen to match the tangent direction or other criteria.

3. Alternative approach: Re-parameterize the curve

Alternatively, define a new parameter u such that:

$$
x(u) = 1
$$

which implies:

$$
2t = 1 \Rightarrow t = \frac{1}{2}
$$

Then, for \(y

Frequently Asked Questions

How do you find the unit tangent vector T(t) for the vector function r(t) = E2ti + Cos(t)j + Sin(3t)k?

To find T(t), first compute r'(t), then normalize it by dividing r'(t) by its magnitude: T(t) = r'(t) / |r'(t)|.

What is the significance of the point P(1, 1, 0) in relation to the vector function r(t)?

The point P(1, 1, 0) is a specific point on the curve r(t). To find the parameter t corresponding to P, solve r(t) = P.

How do you find the parameter t for the point P(1, 1, 0) on the given curve r(t)?

Set the components of r(t) equal to the coordinates of P: E2ti = 1, Cos(t) = 1, Sin(3t) = 0, then solve for t.

What is the process to generate a set of parametric equations for the curve passing through point P(1, 1, 0)?

Identify the value(s) of t that correspond to P, then substitute back into r(t) to get the parametric equations defining the curve through P.

How do you verify that a point lies on the curve r(t) = E2ti + Cos(t)j + Sin(3t)k?

Substitute the t-value into r(t) and check if the resulting coordinates match the point's coordinates.

What is the step-by-step method to find the unit tangent vector T(t) at t corresponding to point P(1, 1, 0)?

1. Find t such that r(t) = P. 2. Compute r'(t). 3. Normalize r'(t) to get T(t).

How can I create a set of parametric equations for the curve passing through point P(1, 1, 0)?

Determine the value(s) of t for P, then express the curve as r(t) with t in an interval around that point, ensuring the parametric equations satisfy r(t) = P at that t.

Additional Resources

Understanding and Finding the Unit Tangent Vector T(t) for the Given Vector Function

---

Introduction

In the realm of vector calculus, understanding the behavior of curves in space is fundamental. The unit tangent vector T(t) plays a crucial role in describing the direction of a curve at any given point. This vector not only indicates the direction of motion along the curve but also serves as a basis for other important concepts such as curvature, normal vectors, and the Frenet-Serret frame.

In this detailed exploration, we will analyze the given vector function:

$$
\mathbf{r}(t) = E_{2} t \mathbf{i} + \cos(t) \mathbf{j} + \sin(3t) \mathbf{k}
$$

and the point $P(1, 1, 0)$. Our goal is to find the unit tangent vector $T(t)$ at the point corresponding to a specific parameter value t, and to develop a set of parametric equations that describe this tangent vector.

Understanding the Components of the Vector Function

Before diving into calculations, it is essential to interpret the given vector function:

$$
\mathbf{r}(t) = E_2 t \mathbf{i} + \cos(t) \mathbf{j} + \sin(3t) \mathbf{k}
$$

Key observations:

- The x-component: $x(t) = E_2 t$.
Note: The notation E_2 is typically used in contexts like matrices or special functions, but in this context, it might be a constant or a typo. Assuming it is a constant, say a, the component is $a t$. Alternatively, if E_2 is intended as a notation for a constant, we will consider it as such.

- The y-component: $y(t) = \cos(t)$.

- The \(z\)-component: \(z(t) = \sin(3t) \).

Assumption:
For the purposes of clarity and standard notation, we will interpret \(E_2 \) as a constant \(a \). If \(E_2 \) is a known constant, the process remains the same; if it is a variable or a typo, further clarification would be needed.

Step 1: Find the Derivative \(\mathbf{r}'(t) \)

The tangent vector at any point \(t \) is given by the derivative of the position vector:

\[
\mathbf{r}'(t) = \frac{d}{dt} \left(a t \mathbf{i} + \cos(t) \mathbf{j} + \sin(3t) \mathbf{k} \right)
\]

Calculating each component:

- Derivative of \(a t \):
\(\frac{d}{dt} (a t) = a\)

- Derivative of \(\cos(t) \):
\(\frac{d}{dt} (\cos(t)) = -\sin(t)\)

- Derivative of \(\sin(3t) \):
\(\frac{d}{dt} (\sin(3t)) = 3 \cos(3t)\)

Thus:

\[
\boxed{
\mathbf{r}'(t) = a \mathbf{i} - \sin(t) \mathbf{j} + 3 \cos(3t) \mathbf{k}
}
\]

Step 2: Compute the Magnitude of \(\mathbf{r}'(t) \)

The magnitude \(|\mathbf{r}'(t)| \) is given by:

\[
|\mathbf{r}'(t)| = \sqrt{(a)^2 + (\sin t)^2 + (3 \cos 3t)^2}
\]

Breaking down:

\[
|\mathbf{r}'(t)| = \sqrt{a^2 + \sin^2 t + 9 \cos^2 3t}

\]

This magnitude is essential because the unit tangent vector \(T(t) \) is obtained by normalizing \(\mathbf{r}'(t) \):

\[
T(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}
\]

Step 3: Find the Point \(P(1, 1, 0) \) Corresponds to \(t \)

Given that the point \(P(1, 1, 0) \) lies on the curve, and assuming it corresponds to some parameter \(t_0 \), we can find \(t_0 \) by solving:

\[
\mathbf{r}(t_0) = (1, 1, 0)
\]

which gives the system:

\[
\begin{cases}
a t_0 = 1 \\
\cos(t_0) = 1 \\
\sin(3 t_0) = 0
\end{cases}
\]

Solve for \(t_0 \):

1. From the second component:

\[
\cos(t_0) = 1 \Rightarrow t_0 = 2 \pi n, \quad n \in \mathbb{Z}
\]

2. From the third component:

\[
\sin(3 t_0) = 0
\]

Since \(t_0 = 2 \pi n \):

\[
\sin(3 \times 2 \pi n) = \sin(6 \pi n) = 0
\]

which is always true because \(\sin(k \pi) = 0 \) for any integer \(k \).

3. From the first component:

$$
a t_0 = 1 \Rightarrow t_0 = \frac{1}{a}
$$

Now, equate this to the previous result:

$$
t_0 = 2 \pi n = \frac{1}{a}
$$

Thus, the parameter t_0 is:

$$
t_0 = \frac{1}{a}
$$

and it must satisfy:

$$
\frac{1}{a} = 2 \pi n \quad \Rightarrow \quad a = \frac{1}{2 \pi n}
$$

for some integer n.

Implication:
- For the point $P(1, 1, 0)$ to lie on the curve, the constant a must be of the form $\frac{1}{2 \pi n}$, and the corresponding parameter:

$$
t_0 = \frac{1}{a} = 2 \pi n
$$

- The simplest choice is $n=1$, which yields:

$$
a = \frac{1}{2 \pi} \quad \text{and} \quad t_0 = 2 \pi
$$

Conclusion:
At $t = t_0 = 2 \pi$, the point on the curve is $P(1, 1, 0)$, assuming $a = \frac{1}{2 \pi}$.

Step 4: Compute $\mathbf{r}'(t_0)$ and the Unit Tangent Vector $T(t_0)$

Given $a = \frac{1}{2 \pi}$, the derivative at $t_0 = 2 \pi$:

\[

\mathbf{r}'(2 \pi) = a \mathbf{i} - \sin(2 \pi) \mathbf{j} + 3 \cos(6 \pi) \mathbf{k}
\]

Calculate each component:

- \(a = \frac{1}{2 \pi} \)

- \(\sin(2 \pi) = 0 \)

- \(\cos(6 \pi) = \cos(2 \times 3 \pi) = 1 \)

Thus:

\[
\mathbf{r}'(2 \pi) = \frac{1}{2 \pi} \mathbf{i} - 0 \mathbf{j} + 3 \times 1 \mathbf{k} = \frac{1}{2 \pi} \mathbf{i} + 0 \mathbf{j} + 3 \mathbf{k}
\]

Calculate the magnitude:

\[
|\mathbf{r}'(2 \pi)| = \sqrt{\left(\frac{1}{2 \pi}\right)^2 + 0^2 + 3^2} = \sqrt{\frac{1}{4 \pi^2} + 9} = \sqrt{9 + \frac{1}{4 \pi^2}}
\]

The unit tangent vector at \(t = 2 \pi \):

\[
T(2 \pi) = \frac{\mathbf{r}'(2 \pi)}{|\mathbf{r}'(2 \pi)|} = \frac{\frac{1}{2 \pi} \mathbf{i} + 3 \mathbf{k}}{\sqrt{9 + \frac{1}{4 \pi^2}}}
\]

Expressed explicitly:

\[
T(2 \pi) = \left(\frac{\frac{1}{2 \pi}}{\sqrt{9 + \frac{1}{4 \pi^2}}} \right) \mathbf{i} + 0 \mathbf{j} + \left

Find The Unit Tangent Vector T T R T E2ti Cos T J Sin 3t K P L 1 0 Find A Set Of Parametric

Find The Unit Tangent Vector T T R T E2ti Cos T J Sin 3t K P L 1 0 Find A Set Of Parametric

Related Articles

- Hanging From A Horizontal Beam Are Nine Simple Pendulums Of The Following Lengths: 0.20, 0.32, 0.53,

- Find A Sinusoidal Function With The Following Four Attributes: (1) Amplitude Is 10, (2) Period Is 4,
- Find The Right Match For The Decision, Precedent, And Influence Of Brown V. Board Of Education.court

Back to Home