Find The Value Of T In The Interval [0, 2n) That Satisfies The Given Equation. Tan T = 3, Csct <0

Find The Value Of T In The Interval [0, 2n) That Satisfies The Given Equation. Tan T = 3, Csct < 0

Introduction

In trigonometry, solving equations that involve the tangent function and other trigonometric ratios is a fundamental skill. These problems often require understanding the behavior of trigonometric functions within specific intervals and applying the properties of the unit circle.

The problem at hand asks us to find the value of T in the interval $[0, 2\pi)$ that satisfies the equations:

- Tan T = 3
- Csc T < 0

This type of problem combines the solution of a basic tangent equation with an inequality involving cosecant, which is the reciprocal of sine. To approach this problem systematically, we will explore the properties of tangent and cosecant functions, analyze their signs within the interval, and determine the specific values of T that satisfy both conditions.

Understanding the Trigonometric Functions Involved

Before diving into the solution, it is essential to understand the behavior and properties of the functions involved:

1. The Tangent Function (Tan T)

- Defined as Tan T = sin T / cos T
- Periodicity: Period is π, meaning Tan T repeats every π radians.
- Domain restrictions: Tan T is undefined where cos T = 0, i.e., at $T = \pi/2 + k\pi$, where k is an integer.
- Range: $(-\infty, +\infty)$

2. The Cosecant Function (Csc T)

- Defined as Csc T = 1 / sin T
- Sign behavior:

- $\csc T > 0$ when $\sin T > 0$
- $\csc T < 0$ when $\sin T < 0$
- Domain restrictions: $\csc T$ is undefined where $\sin T = 0$, i.e., at $T = k\pi$

Step-by-Step Solution Approach

To find the value(s) of T satisfying both $\tan T = 3$ and $\csc T < 0$, follow these steps:

Step 1: Find all T where $\tan T = 3$ in the interval $[0, 2\pi)$

Step 2: Determine the sign of $\sin T$ at these solutions to check the inequality $\csc T < 0$

Step 3: Verify which solutions satisfy $\csc T < 0$ (i.e., $\sin T < 0$)

Step 1: Solving $\tan T = 3$ in $[0, 2\pi)$

Since $\tan T = 3$, T is one of the angles where the tangent value equals 3.

General solution:

- $T = \arctan(3) + k\pi$, where k is an integer, because the tangent function repeats every π.

Calculate arctangent(3):

Using a calculator or inverse tangent function:

- $\arctan(3) \approx 1.249$ radians

Find the solutions within $[0, 2\pi)$:

- For $k = 0$:

$T_1 \approx 1.249$ radians

- For $k = 1$:

$T_2 \approx 1.249 + \pi \approx 1.249 + 3.142 \approx 4.391$ radians

- For $k = 2$:

$T \approx 1.249 + 2\pi \approx 1.249 + 6.283 \approx 7.532$ radians

But $7.532 > 2\pi$ (~6.283), so discard.

- For $k = -1$:

$T \approx 1.249 - \pi \approx 1.249 - 3.142 \approx -1.893$ radians, which is outside $[0, 2\pi)$ (negative), so discard for the interval.

Final solutions for Tan T = 3 in $[0, 2\pi)$:

- $T \approx 1.249$ radians
- $T \approx 4.391$ radians

Step 2: Analyzing the Sign of sin T at these solutions

Recall:

- Csc T < 0 ⇒ sin T < 0

Now, evaluate the sine at these solutions:

For $T \approx 1.249$ radians:

- Since 1.249 radians ≈ 71.6°, which is in the first quadrant where sin T > 0.

- Therefore, Csc T > 0 at this point, not satisfying Csc T < 0.

For $T \approx 4.391$ radians:

- Convert to degrees:

4.391 radians ≈ 251.6°

- This angle is in the third quadrant, where sin T < 0.

- Therefore, Csc T < 0 at $T \approx 4.391$ radians.

Step 3: Final answer

The only solution within [0, 2π) where Tan T = 3 and Csc T < 0 is:

- T ≈ 4.391 radians

Additional Considerations

Validity within the interval:

- The interval is [0, 2π), which corresponds to [0, 6.283) radians.
- Our solution T ≈ 4.391 radians lies within this interval.

Exact form:

- The exact solution for T is:

T = arctangent(3) + π ≈ 1.249 + 3.142 ≈ 4.391 radians

Note on other solutions:

- The other solution T ≈ 1.249 radians does not satisfy Csc T < 0 because sin T > 0 there.
- No other solutions in [0, 2π) satisfy both conditions.

Summary

Step	Description	Result
1	Find solutions of Tan T = 3 in [0, 2π)	T ≈ 1.249 radians, T ≈ 4.391 radians
2	Determine sign of sin T at these points	sin T > 0 at 1.249 rad, sin T < 0 at 4.391 rad
3	Check which satisfy Csc T < 0	Only T ≈ 4.391 radians satisfies Csc T < 0

Conclusion

The value of T in the interval [0, 2π) that satisfies the equations:

- Tan T = 3
- Csc T < 0

is approximately:

> $T \approx 4.391$ radians

or in exact terms:

> $T = \text{arctangent}(3) + \pi$

Additional Tips for Solving Similar Problems

- Always analyze the signs of the involved functions within the specified interval.
- Remember the periodicity of trigonometric functions: Tan T repeats every π, Sin T and Csc T repeat every 2π.
- Use inverse functions to find principal solutions, then adjust using periodicity to find all solutions within the interval.
- Always verify the solution by checking the sign of the sine or cosine to ensure it satisfies the inequality.

Final Remarks

Understanding the behavior of trigonometric functions and their signs in different quadrants is crucial when solving equations involving inequalities. This problem exemplifies the importance of combining algebraic solutions with sign analysis to find the correct solutions within a given interval. Whether for academic purposes or practical applications in physics and engineering, mastering these techniques enhances problem-solving skills and deepens comprehension of trigonometric concepts.

Keywords for SEO optimization:
trigonometry solutions, tangent equation, interval $[0, 2\pi)$, solve $\tan T = 3$, cosecant inequality, $\csc T < 0$, trigonometric equations, unit circle, sine and cosine signs, periodicity of tangent, solving trigonometric inequalities, angle solutions, inverse tangent, exact solutions, approximate solutions

Frequently Asked Questions

What is the general solution for T when tan T = 3 within the interval [0,

2π)?

The general solution is T = arctangent(3) + nπ, where n is an integer. Within [0, 2π), the solutions are T = arctangent(3) and T = arctangent(3) + π.

How do we determine the specific value of T in [0, 2π) given that csc T < 0 and tan T = 3?

Since tan T = 3 (positive), and csc T < 0 (meaning sin T < 0), T must be in the third or fourth quadrant where sine is negative. Check the solutions to see which fall into these quadrants within [0, 2π).

What are the approximate solutions for T in [0, 2π) satisfying tan T = 3 and csc T < 0?

The solutions are T ≈ 1.249 radians (arctangent of 3) and T ≈ 4.391 radians (π + arctangent of 3). Since csc T < 0, T in the third or fourth quadrant, so the relevant solution is T ≈ 4.391 radians.

Why is the value of T in the interval [0, 2π) for tan T = 3 and csc T < 0 only approximately 4.391 radians?

Because tan T = 3 is positive, T must be in quadrants I or III, but since csc T < 0 (sin T < 0), T must be in quadrants III or IV. The only solution in [0, 2π) with these conditions is T ≈ 4.391 radians, located in quadrant III.

How can we verify that the found T satisfies both tan T = 3 and csc T < 0?

Calculate sin T and cos T at T ≈ 4.391 radians. Since sin T is negative and tan T is positive (meaning sin T and cos T have the same sign), verify that sin T < 0, cos T < 0, and tan T ≈ 3 to confirm the solution.

Additional Resources

Trigonometry Insights: Finding the Value of T in the Interval [0, 2π) for Tan T = 3 with Csct T < 0

Introduction

When exploring the depths of trigonometry, understanding how to locate specific angles that satisfy given equations is fundamental. This task becomes especially intriguing when constraints are introduced, such as the sign of a trigonometric function. Today, we delve into the problem:

> Find the value of T in the interval $[0, 2\pi)$ that satisfies the equation $\tan T = 3$, with the additional condition $\csc T < 0$.

This problem requires a comprehensive grasp of tangent and cosecant functions, their properties, and how their signs influence the solutions within the specified interval. Let's approach this systematically, dissecting each component to provide clarity and insight.

Understanding the Core Functions

The Tangent Function ($\tan T$)

- Definition: $\tan T = \frac{\sin T}{\cos T}$
- Periodicity: The tangent function has a fundamental period of π, meaning it repeats every π radians.
- Range: $\tan T$ takes on all real values $(-\infty, +\infty)$, with vertical asymptotes where $\cos T = 0$.

Given the equation $\tan T = 3$, we're looking for angles T where the tangent function equals 3. Since the tangent function is continuous and monotonic within its principal domain $(- \pi/2, \pi/2)$, solutions can be found using inverse tangent, respecting the periodicity to find all solutions within $[0, 2\pi)$.

The Cosecant Function ($\csc T$)

- Definition: $\csc T = \frac{1}{\sin T}$
- Significance: The sign of $\csc T$ directly depends on $\sin T$:
- $\csc T > 0$ when $\sin T > 0$
- $\csc T < 0$ when $\sin T < 0$

The additional condition $\csc T < 0$ constrains solutions to where $\sin T$ is negative, i.e., the third and fourth quadrants in the interval $[0, 2\pi)$.

Step 1: Solving $\tan T = 3$

The primary step involves solving for T:

\[

T = \arctan(3)
\]

Using a calculator or computational tool:

\[
T_0 = \arctan(3) \approx 1.249 \text{ radians}
\]

Since tangent has a period of \(\pi\), the general solutions within \([0, 2\pi)\) are:

\[
T = T_0 + k\pi, \quad k \in \mathbb{Z}
\]

Let's determine the specific solutions within the interval:

- For \(k=0\):

\[
T = 1.249 \text{ radians}
\]

- For \(k=1\):

\[
T = 1.249 + \pi \approx 1.249 + 3.142 = 4.391 \text{ radians}
\]

- For \(k=2\):

\[
T = 1.249 + 2\pi \approx 1.249 + 6.283 = 7.532 \text{ radians}
\]

But \(7.532 > 2\pi\), so it lies outside \([0, 2\pi)\). Therefore, the solutions for \(T\) satisfying \(\tan T = 3\) within \([0, 2\pi)\) are approximately:

\[
\boxed{
T \approx 1.249 \text{ radians} \quad \text{and} \quad T \approx 4.391 \text{ radians}
}
\]

Step 2: Applying the Condition $\csc T < 0$

Recall that:

$$
\csc T = \frac{1}{\sin T}
$$

and that $\csc T < 0$ implies $\sin T < 0$.

In the interval $[0, 2\pi)$, $\sin T < 0$ in the third $(\pi, 3\pi/2)$ and fourth quadrants $(3\pi/2, 2\pi)$.

Let's analyze the approximate solutions:

- At $T \approx 1.249$ radians:

Since 1.249 radians is approximately 71.6°, which is in the first quadrant where $\sin T > 0$. Therefore:

$$
\sin T > 0 \Rightarrow \csc T > 0
$$

This solution does not satisfy $\csc T < 0$.

- At $T \approx 4.391$ radians:

Since 4.391 radians is approximately 251.6°.

- $\pi \approx 3.142$ radians.

- $4.391 - \pi \approx 1.249$ radians,

which is the same as the first solution, but shifted by π.

In this quadrant, 4.391 radians is in the third quadrant $(\pi, 3\pi/2)$, where:

$$
\sin T < 0 \Rightarrow \csc T < 0
$$

Thus, this solution satisfies the condition $\csc T < 0$.

Final Solution Set

Considering all factors, the only T in $[0, 2\pi)$ that satisfies both:

- $\tan T = 3$
- $\csc T < 0$

is approximately:

$$
\boxed{
T \approx 4.391 \text{ radians}
}
$$

or, in degrees,

$$
T \approx 251.6^\circ
$$

This angle lies in the third quadrant, where both $\tan T$ is positive (since $\tan T$ is positive in quadrants I and III) and $\sin T$ is negative, satisfying all conditions.

Additional Considerations: Exact Expression and Domain Checks

While the approximate value is sufficient for most purposes, for exact solutions:

$$
T = \arctan(3) + \pi
$$

since $\arctan(3)$ is the principal value in $(- \pi/2, \pi/2)$, and adding π shifts the solution into the third quadrant.

Expressed precisely:

$$
\boxed{
T = \arctan(3) + \pi
}
$$

This is the only solution within $[0, 2\pi)$ satisfying both the equation and the sign condition.

Summary of Key Steps

- Solved $\tan T = 3$ to find principal solutions.
- Used tangent's periodicity to find all solutions in $[0, 2\pi)$.
- Analyzed the sign of $\sin T$ (since $\csc T = 1/\sin T$) to impose $\csc T < 0$.
- Identified that only the solution in the third quadrant meets this sign condition.
- Expressed the solution precisely as $T = \arctan(3) + \pi$.

Final Thoughts and Practical Tips

- Understanding Quadrants: Always consider the quadrant when analyzing the sign of sine, cosine, and tangent functions.
- Periodicity: Remember that functions like $\tan T$ and $\cot T$ have periods of π, while $\sin T$ and $\cos T$ have periods of 2π.
- Inverse Functions: Use inverse tangent carefully, noting its principal value range, and adjust with added multiples of π to find all solutions.
- Checking Conditions: Always verify the sign conditions after finding the solutions, as they can eliminate otherwise valid solutions.

This comprehensive approach ensures a thorough understanding of the problem and equips you with methods to tackle similar trigonometric equations with constraints effectively.

References for Further Reading

- Trigonometric Identities and Properties – Khan Academy, Trigonometry Section
- Inverse Trigonometric Functions – Paul's Online Math Notes
- Quadrant Sign Analysis – Mathematics textbooks and online resources

---

In conclusion, the key to solving for T in the interval $[0, 2\pi)$ with $\tan T = 3$ and $\csc T < 0$ is understanding the periodic nature of the tangent function and the sign implications of the cosecant function. The isolated solution $T = \arctan(3) + \pi$ in the third quadrant satisfies all conditions, highlighting the elegant interplay between function values and their signs in trigonometry.

Find The Value Of T In The Interval 0 2n That Satisfies The Given Equation Tan T 3 Csct Lt 0

Find The Value Of T In The Interval 0 2n That Satisfies The Given Equation Tan T 3 Csct Lt 0

Related Articles

- For Her Job, Natasha Spent A Total Of N Minutes Processing Id Card Applications And Driver's License
- Four Racers Are Racing On A Track. Raheem Covers 26%26% Of The Track, Lila Covers 3838 Of The Track,
- Gilbert's Weekly Pay In Dollars, D. Can Be Represented By The Function D = 13.5h, Where H Represents

Back to Home