Find The Vector Equation For The Line Of Intersection Of The Planes 2x - 4y + Z = 0 And 2x + Z = 0 R

Find The Vector Equation For The Line Of Intersection Of The Planes 2x - 4y + Z = 0 And 2x + Z = 0 R

Understanding how to find the vector equation of the line where two planes intersect is a fundamental skill in vector calculus and three-dimensional geometry. When two planes intersect, they form a straight line, and determining its vector equation involves leveraging the properties of vectors, normals, and parametric equations. In this comprehensive guide, we will explore the step-by-step process to find the vector equation of the line of intersection of the given planes:

- Plane 1: $2x - 4y + Z = 0$
- Plane 2: $2x + Z = 0$ R (assuming 'R' indicates the standard form or a notation for the second plane)

By the end of this article, you will understand how to derive the parametric (vector) form of the intersection line, interpret the geometric significance of the normals, and apply these techniques to similar problems.

Understanding the Geometric Context

Planes and Their Intersection

- Planes in 3D Space: A plane in three dimensions is a flat, two-dimensional surface extending infinitely in all directions within space, described by a linear equation in x, y, and z.
- Line of Intersection: The intersection of two planes, if not parallel or coincident, is a straight line. This line can be represented parametrically or via a vector equation.

Normals to the Planes

- Each plane has a normal vector perpendicular to its surface.
- The normals are derived directly from the coefficients of the plane equations.
- The direction of the line of intersection is perpendicular to the normals of both planes, which can be found via the cross product of the normals.

Step-by-Step Solution to Find the Intersection

Line

Step 1: Write the Equations Clearly

The given planes can be rewritten as:

1. Plane 1: 2x - 4y + Z = 0
2. Plane 2: 2x + Z = 0 R (assuming this is 2x + Z = 0)

Note: The notation 'R' may be a typo or extraneous; for clarity, we will interpret the second plane as 2x + Z = 0.

Step 2: Identify the Normal Vectors

Normal vectors are derived from the coefficients of the plane equations:

- Normal vector of Plane 1 (N_1):
\[
\mathbf{N}_1 = \langle 2, -4, 1 \rangle
\]
- Normal vector of Plane 2 (N_2):
\[
\mathbf{N}_2 = \langle 2, 0, 1 \rangle
\]

Step 3: Find the Direction Vector of the Line

The line of intersection is perpendicular to both normal vectors, so:

\[
\boxed{
\mathbf{d} = \mathbf{N}_1 \times \mathbf{N}_2
}
\]

Calculating the cross product:

\[
\mathbf{d} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & -4 & 1 \\
2 & 0 & 1 \\
\end{vmatrix}
\]

Expanding the determinant:

\[
\mathbf{d} = \mathbf{i}((-4)(1) - (1)(0)) - \mathbf{j}(2(1) - 1(2)) + \mathbf{k}(2(0) - (-4)(2))

\]

Simplify each component:

- For \(\mathbf{i}\):

\[
(-4)(1) - (1)(0) = -4 - 0 = -4
\]

- For \(\mathbf{j}\):

\[
2(1) - 1(2) = 2 - 2 = 0
\]

- For \(\mathbf{k}\):

\[
2(0) - (-4)(2) = 0 + 8 = 8
\]

Therefore, the direction vector:

\[
\boxed{
\mathbf{d} = \langle -4, 0, 8 \rangle
}
\]

For simplicity, divide through by 4:

\[
\mathbf{d} = \langle -1, 0, 2 \rangle
\]

This vector indicates the direction of the line of intersection.

Step 4: Find a Point on the Line

To write the full vector equation, we need a specific point \((x_0, y_0, z_0)\) on the line. To find such a point, we can assign a value to one variable and solve the system of equations for the others.

Let’s choose \(y = t \) as a parameter and find corresponding \(x \) and \(z \).

Alternatively, better is to solve the system directly:

From the plane equations:

1. \(2x - 4y + z = 0 \)
2. \(2x + z = 0 \)

Express \(z \) from the second equation:

$$
z = -2x
$$

Substitute into the first:

$$
2x - 4y + (-2x) = 0 \Rightarrow 2x - 4y - 2x = 0 \Rightarrow -4y = 0 \Rightarrow y = 0
$$

Now, with $y = 0$, substitute back into the second:

$$
z = -2x
$$

Choose an arbitrary value for x, say $x = 0$:

- Then $z = 0$

So, a point on the line is $(0, 0, 0)$. Alternatively, pick $x = 1$:

- Then $z = -2 \times 1 = -2$

So, another point is $(1, 0, -2)$.

Constructing the Vector Equation of the Line

Using the Point and Direction Vector

- Point on the line: $\mathbf{r}_0 = (0, 0, 0)$
- Direction vector: $\mathbf{d} = \langle -1, 0, 2 \rangle$

The vector equation of the line is:

$$
\boxed{
\mathbf{r} = \mathbf{r}_0 + \lambda \mathbf{d} = (0, 0, 0) + \lambda \langle -1, 0, 2 \rangle
}
$$

Expressed component-wise:

$$
\boxed{
\begin{cases}
x = -\lambda \\
y = 0 \\
z = 2\lambda
\end{cases}
}

\]

Alternatively, as a parametric vector form:

\[
\boxed{
\mathbf{r}(\lambda) = \langle -\lambda, 0, 2\lambda \rangle
}
\]

Or, explicitly:

\[
\boxed{
\textbf{Line Equation:} \quad \mathbf{r} = \langle 0, 0, 0 \rangle + \lambda \langle -1, 0, 2 \rangle, \quad \lambda \in \mathbb{R}
}
\]

Summary and Final Remarks

- The process involves identifying the normals of the planes, computing their cross product to find the direction vector, and solving the system to find a specific point on the line.
- The vector equation provides a complete description of the line of intersection in parametric form.
- This method applies generally to the intersection of any two planes, making it a crucial technique in vector geometry.

Additional Tips and Common Mistakes to Avoid

- **Always verify the point**: Ensure the point you choose satisfies both plane equations.
- **Normalize the direction vector**: Simplify the direction vector for easier interpretation and calculation.
- **Check for parallelism**: If the cross product of normals is zero, the planes are parallel or coincident, and the intersection is either nonexistent or the entire plane.
- **Understand the geometric meaning**: The cross product of the normals gives the direction of the intersection line, which is perpendicular to both normals.

Applications of Finding the Intersection Line

- Engineering: Designing structures where intersecting planes form edges.
- Computer Graphics: Calculating lines of intersection for rendering 3D scenes.
- Robotics: Path planning involving planes and obstacles.
- Mathematics Education: Developing spatial visualization skills and understanding vector calculus concepts.

Conclusion

Finding the vector equation for the line of intersection of two planes is a fundamental problem in three-dimensional geometry. By determining the normals, calculating their cross product, and finding a point on the line, you can derive

Frequently Asked Questions

How do I find the vector equation of the line of intersection of the planes 2x - 4y + z = 0 and 2x + z = 0?

First, find the normal vectors of both planes: n1 = (2, -4, 1) and n2 = (2, 0, 1). Then, compute their cross product to get the direction vector of the line. Next, find a point common to both planes by solving the system of equations. Finally, write the parametric form using the point and direction vector.

What is the step-by-step process to derive the vector equation for the intersection line of two planes?

Step 1: Identify the normal vectors of both planes. Step 2: Compute the cross product of these normals to find the line's direction vector. Step 3: Solve the two plane equations simultaneously to find a point on the line. Step 4: Write the line's vector equation using the point and the direction vector.

How do I find a specific point on the line of intersection of the given planes?

Set one variable to a convenient value (e.g., z = 0) and solve the system of equations: 2x - 4y + z = 0 and 2x + z = 0. Solving these will give you the corresponding x and y, providing a specific point on the line.

What is the cross product of the normals of the planes 2x - 4y + z = 0 and 2x + z = 0?

Normal vectors are n1 = (2, -4, 1) and n2 = (2, 0, 1). Their cross product is

((-4)(1) - (1)(0), (1)(2) - (2)(1), (2)(0) - (-4)(2)) = (-4 - 0, 2 - 2, 0 + 8) = (-4, 0, 8). Simplified, the direction vector is proportional to (-4, 0, 8).

How do I write the parametric equations once I have a point and a direction vector?

Suppose the point is P(x0, y0, z0) and the direction vector is v = (a, b, c). Then, the parametric equations are: x = x0 + at, y = y0 + bt, z = z0 + ct, where t is a parameter.

Can I verify the line of intersection by substituting the parametric equations into the original plane equations?

Yes, after finding the parametric equations, substitute them back into both plane equations to ensure they satisfy both simultaneously. This confirms the correctness of the intersection line.

What is the final vector equation for the line of intersection of the given planes?

The line passes through a specific point (for example, when z=0, x=0, y=0). Using the cross product result, a possible direction vector is (-4, 0, 8). The vector equation is: r = (x0, y0, z0) + t(-4, 0, 8). For instance, choosing the point (0, 0, 0), the equation is r = (0, 0, 0) + t(-4, 0, 8).

Additional Resources

Find The Vector Equation For The Line Of Intersection Of The Planes 2x - 4y + Z = 0 And 2x + Z = 0

The task of finding the vector equation of the line of intersection between two planes is a fundamental problem in vector calculus and analytical geometry. It involves understanding the geometric relationship between planes and lines in three-dimensional space, translating algebraic equations into vector form, and ultimately deriving a parametric representation of the intersection line. This process is not only a core component of advanced mathematics but also essential in fields like engineering, physics, computer graphics, and robotics, where spatial reasoning and precise geometric modeling are critical.

In this comprehensive exploration, we'll analyze the problem step-by-step, starting from the basic understanding of the given planes, moving through the algebraic procedures to find the intersection line, and culminating in deriving a clear, concise vector equation. We will also discuss the geometric interpretation and significance of the results.

Understanding the Problem: Intersection of Planes

Planes in Three-Dimensional Space

A plane in three-dimensional space can be expressed via a linear equation of the form:

$$ Ax + By + Cz + D = 0 $$

where (A, B, C) represents the normal vector to the plane, and D is a scalar constant. The normal vector is perpendicular to every vector lying on the plane.

Given the two planes:

1. $2x - 4y + Z = 0$
2. $2x + Z = 0$

we recognize that each can be represented as a linear equation with associated normal vectors:

- Plane 1: $2x - 4y + Z = 0$

Normal vector: $\mathbf{n}_1 = (2, -4, 1)$

- Plane 2: $2x + Z = 0$

Normal vector: $\mathbf{n}_2 = (2, 0, 1)$

The intersection of two planes, if not parallel or coincident, is a line. The goal is to determine this line's vector equation, which requires finding a point on the line and a direction vector along the line.

Mathematical Approach to Finding the Intersection Line

Step 1: Express the Planes in Standard Form

Rearranged equations:

- Plane 1: $2x - 4y + z = 0$
- Plane 2: $2x + z = 0$

This makes the system clearer for algebraic manipulation.

Step 2: Find a Common Point on the Line

Since the line lies on both planes, any point (x, y, z) satisfying both equations is on the intersection line.

From Plane 2:

$$ 2x + z = 0 \Rightarrow z = -2x $$

Substitute this into Plane 1:

$$ 2x - 4y + (-2x) = 0 \Rightarrow 2x - 4y - 2x = 0 \Rightarrow -4y = 0 \Rightarrow y=0 $$

Now, with $y = 0$, and $z = -2x$, the point on the line can be parametrized by choosing a variable for x.

Let:

$$ x = t $$

then:

$$ y = 0 $$
$$ z = -2t $$

Thus, a parametric point on the line is:

$$ (x, y, z) = (t, 0, -2t) $$

or, more explicitly:

$$ \mathbf{r}_0 = (0, 0, 0) \quad \text{when} \quad t=0 $$

which confirms that the origin is a point on the intersection line.

Step 3: Find the Direction Vector of the Line

The direction vector $\mathbf{d}$ of the line can be found as the cross product of the normals of the two planes:

$$ \mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2 $$

Calculating this cross product:

$$
\mathbf{d} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & -4 & 1 \\
2 & 0 & 1 \\
\end{vmatrix}
$$

Expanding:

$$
\mathbf{d} = \mathbf{i}((-4)(1) - (1)(0)) - \mathbf{j} (2 \times 1 - 1 \times 2) + \mathbf{k} (2 \times 0 - (-4) \times 2)
$$

Simplify each component:

- $\mathbf{i}$ component:

$$ (-4)(1) - (1)(0) = -4 - 0 = -4 $$

- $\mathbf{j}$ component:

$$ 2 \times 1 - 1 \times 2 = 2 - 2 = 0 $$

- $\mathbf{k}$ component:

$$ 2 \times 0 - (-4) \times 2 = 0 + 8 = 8 $$

Thus,

$$
\mathbf{d} = (-4, 0, 8)
$$

which can be simplified by dividing by 4:

$$
\mathbf{d} = (-1, 0, 2)
$$

This vector indicates the direction of the intersection line.

Formulating the Vector Equation of the Line

Step 1: Establish a Point on the Line

From earlier, the point $\mathbf{r}_0$ can be taken as the point corresponding to $t=0$:

$$ (0, 0, 0) $$

which satisfies both equations:

- $2(0) - 4(0) + 0 = 0$
- $2(0) + 0 = 0$

Hence, the point $(0, 0, 0)$ lies on the line.

Step 2: Write the Vector Equation

The vector equation of the line is given by:

$$ \mathbf{r}(t) = \mathbf{r}_0 + t \mathbf{d} $$

Substituting the point and direction vector:

$$ \boxed{
\mathbf{r}(t) = (0, 0, 0) + t (-1, 0, 2)
} $$

Expressed component-wise:

$$
\boxed{
\begin{cases}
x = -t \\
y = 0 \\
z = 2t
\end{cases}
}
$$

This parametric form succinctly describes every point on the line.

Alternative Representation: Symmetric Equations

From the parametric equations, derive the symmetric form:

$$
x = -t \Rightarrow t = -x
$$
$$
z = 2t \Rightarrow z = 2(-x) = -2x
$$
$$
y = 0
$$

Thus, the symmetric equations are:

$$
\boxed{
x = -t, \quad y=0, \quad z= 2t
}
$$

or equivalently:

$$
\frac{x}{-1} = \frac{z}{2}, \quad y=0
$$

which simplifies to:

\[
x = -\frac{z}{2}, \quad y=0
\]

This indicates the line lies in the plane \(y=0 \) and passes through the origin with the relation between \(x \) and \(z \).

Geometric Interpretation and Significance

Understanding the line of intersection between two planes in three-dimensional space offers insights into spatial relationships, constraints, and geometric configurations. The derived vector equation reveals several key aspects:

- Directionality: The line's direction vector \((-1, 0, 2) \) indicates that as \(t \) increases, the line moves in a direction that decreases \(x \) and increases \(z \). The zero component in \(y \) suggests the line resides entirely within the plane \(y=0 \).

- Symmetry and Orientation: The relation \(z = -2x \) showcases the slope of the line in the \(xz \)-plane, illustrating a linear relationship between the \(x \) and \(z \) coordinates.

- Positioning: Since the point \((0, 0, 0) \) lies on both planes, the line passes through the origin, which is often a convenient reference point for further geometric or algebraic analysis.

This geometric understanding is crucial in practical applications. For example, in computer graphics, such intersection lines are used in rendering scenes; in engineering, they are relevant in analyzing the intersection of structural components; and in physics, they can represent constraints in three-dimensional systems.

Summary and Conclusions

The process of finding the vector equation of the line of intersection of two planes involves a systematic approach combining algebraic manipulation, vector calculus, and geometric interpretation. Starting from the equations:

- \(

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