Find The Vertex, Focus, And Directrix Of The Parabola. Y2 6y 3x 3 = 0 Vertex (x, Y) = Focus (x, Y) =

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Understanding the properties of parabolas is fundamental in algebra and coordinate geometry. When analyzing quadratic equations, especially those representing conic sections, identifying the vertex, focus, and directrix provides crucial insights into the parabola's shape and position. In this article, we will explore how to find these key features for the specific parabola given by the equation:

$$ Y^2 + 6y - 3x + 3 = 0 $$

and how to interpret these elements to understand the parabola's geometry.

Understanding the Standard Form of a Parabola

Before diving into calculations, it’s essential to recall the standard forms of parabolas and what each parameter represents.

Horizontal and Vertical Parabolas

- Vertical Parabola: Its standard form is $$ (x - h)^2 = 4p(y - k) $$
- Horizontal Parabola: Its standard form is $$ (y - k)^2 = 4p(x - h) $$

where:

- (h, k) is the vertex of the parabola.
- p is the distance from the vertex to the focus (and also from the vertex to the directrix).

The sign of p determines the parabola's opening direction:

- For $(x - h)^2 = 4p(y - k)$:

- If $p > 0$, the parabola opens upward.
- If $p < 0$, it opens downward.

- For $(y - k)^2 = 4p(x - h)$:

- If $p > 0$, it opens to the right.

- If $p < 0$, it opens to the left.

Rearranging the Given Equation into Standard Form

Given the equation:

$$ Y^2 + 6y - 3x + 3 = 0 $$

Our goal is to manipulate it into a standard form to identify the vertex, focus, and directrix.

Step 1: Grouping Terms

Rearranged:

$$ Y^2 + 6y = 3x - 3 $$

Notice that the left side involves $Y^2 + 6y$, which suggests completing the square for the Y terms.

Step 2: Completing the Square for the Y-terms

- The expression $Y^2 + 6y$ can be rewritten as:

$$ Y^2 + 6y = (Y^2 + 6y + 9) - 9 = (Y + 3)^2 - 9 $$

(here, since $(Y + 3)^2 = Y^2 + 6Y + 9$, but our middle term is $6y$, so note that the variable is uppercase Y, but the same applies.)

- Wait, the original equation uses uppercase Y, but it's standard to use lowercase y for the variable; assuming uniform notation, we proceed with Y.

- The middle coefficient is 6, so half of 6 is 3, and the square of 3 is 9. Therefore, the completed square:

$$ (Y + 3)^2 - 9 $$

- Substituting back:

$$ (Y + 3)^2 - 9 = 3x - 3 $$

Step 3: Isolate x

Add 9 to both sides:

$$ (Y + 3)^2 = 3x - 3 + 9 $$

$$ (Y + 3)^2 = 3x + 6 $$

Divide both sides by 3:

$$ \frac{(Y + 3)^2}{3} = x + 2 $$

Rewrite as:

$$ x = \frac{1}{3}(Y + 3)^2 - 2 $$

This is now in the standard form:

$$ x = a (Y - k)^2 + h $$

where:

- $a = \frac{1}{3}$
- $k = -3$
- $h = -2$

Identifying the Vertex

From the standard form:

$$ x = a (Y - k)^2 + h $$

the vertex (x, y) is at:

$$ (h, k) $$

Substituting the identified values:

- $h = -2$
- $k = -3$

Vertex: $(-2, -3)$

This point is the parabola's lowest or highest point depending on the opening direction, which we will determine next.

Determining the Focus and Directrix

To find the focus and the directrix, we need to understand the parabola's orientation and the value of p.

Step 1: Recognize the Parabola's Orientation

Since the equation is in the form:

$$ x = a (Y - k)^2 + h $$

and $a = \frac{1}{3} > 0$, the parabola opens to the right.

Step 2: Find the value of p

Recall that in the standard form:

$$ x = a (Y - k)^2 + h $$

the parabola's focus is located at:

$$ (h + p, k) $$

and the directrix is:

$$ x = h - p $$

and the value of a relates to p via:

$$ 4p = \frac{1}{a} $$

Solve for p:

$$ 4p = \frac{1}{a} = \frac{1}{\frac{1}{3}} = 3 $$

$$ p = \frac{3}{4} $$

Note: Since $a = \frac{1}{3}$, then:

$$ p = \frac{1}{4a} \quad \text{or} \quad p = \frac{1}{4a} $$

But the standard relation for parabolas in the form $x = a (Y - k)^2 + h$ is:

$$ 4p = \frac{1}{a} $$

Thus,

$$ p = \frac{1}{4a} = \frac{1}{4 \times \frac{1}{3}} = \frac{1}{\frac{4}{3}} = \frac{3}{4} $$

which confirms the value of p.

Step 3: Find the Focus

The focus, located p units to the right of the vertex:

$$ (x, y) = (h + p, k) $$

Plugging in the values:

$$ (-2 + \frac{3}{4}, -3) $$

Expressed as a decimal or fraction:

$$ -2 + \frac{3}{4} = -2 + 0.75 = -1.25 $$

or as a fraction:

$$ -2 + \frac{3}{4} = -\frac{8}{4} + \frac{3}{4} = -\frac{5}{4} $$

Focus: $\left(-\frac{5}{4}, -3 \right)$

Finding the Directrix

The directrix is a vertical line located p units to the left of the vertex:

$$ x = h - p $$

Calculating:

$$ -2 - \frac{3}{4} = -\frac{8}{4} - \frac{3}{4} = -\frac{11}{4} $$

Expressed as decimal:

$$ -2.75 $$

Directrix: $x = -\frac{11}{4}$ or $x = -2.75$

Summary of Key Features

Feature	Coordinates / Equation
Vertex	$\boxed{(-2, -3)}$
Focus	$\boxed{\left(-\frac{5}{4}, -3 \right)}$
Directrix	$\boxed{x = -\frac{11}{4}}$ (or $x = -2.75$)

Additional Insights and Applications

Understanding how to find the vertex, focus, and directrix is essential not only for graphing parabolas but also for solving real-world problems involving projectile motion, satellite dishes, and reflective properties of parabolic mirrors.

Graphing the Parabola

Given the vertex at $(-2, -3)$, focus at $\left(-\frac{5}{4}, -3 \right)$, and directrix at $x = -\frac{11}{4}$, the parabola opens to the right, with its symmetry axis along the line $y = -3$.

Frequently Asked Questions

How do you find the vertex of the parabola given the equation $y^2 - 6y + 3x - 3 = 0$?

First, rewrite the equation in a standard form by completing the square for y: $y^2 - 6y + 9 - 9 + 3x - 3 = 0$, which simplifies to $(y - 3)^2 = -3x + 12$. Then, express as $(y - 3)^2 = -3(x - 4)$. The vertex is at (4, 3).

What is the focus of the parabola $y^2 - 6y + 3x - 3 = 0$?

From the standard form $(y - 3)^2 = -3(x - 4)$, the parabola opens leftward. The focus is at $(h + p/4, k)$, where p is the coefficient in the form. Here, $p = -3$, so focus is at $(4 + (-3)/4, 3) = (4 - 0.75, 3) = (3.25, 3)$.

How do you determine the directrix of the parabola $y^2 - 6y + 3x - 3 = 0$?

Using the standard form $(y - 3)^2 = -3(x - 4)$, the directrix is a vertical line at $x = h - p/4$. Since $h = 4$ and $p = -3$, the directrix is at $x = 4 - (-3)/4 = 4 + 0.75 = x = 4.75$.

What are the steps to convert the given parabola equation into vertex form?

Isolate y terms and complete the square: $y^2 - 6y + 9 = -3x + 12$. This becomes $(y - 3)^2 = -3(x - 4)$, revealing the vertex at (4, 3).

How can you verify the focus and directrix once you have the vertex form of the parabola?

Identify p from the standard form $(y - k)^2 = 4p(x - h)$. Compute the focus at (h + p, k) and the directrix at x = h - p. For our equation, p = -3/4, so focus at (4 - 0.75, 3) = (3.25, 3) and directrix x = 4 + 0.75 = 4.75.

Why does the parabola $y^2 - 6y + 3x - 3 = 0$ open leftward?

Because in the standard form $(y - 3)^2 = -3(x - 4)$, the coefficient of (x - 4) is negative, indicating the parabola opens to the left.

What is the significance of the focus and directrix in understanding a parabola's shape?

The focus and directrix define the parabola as the set of points equidistant from the focus and the directrix, which helps in graphing and understanding its orientation and geometric properties.

Additional Resources

Find The Vertex, Focus, And Directrix Of The Parabola. Y2 6y 3x 3 = 0 Vertex (x, Y) = Focus (x, Y) =

Understanding the properties of parabolas is fundamental in various fields such as physics, engineering, and mathematics. They serve as the basis for the design of satellite dishes, headlights, and even the paths of projectiles. Today, we delve into the process of analyzing a particular parabola represented by the equation $Y^2 + 6y - 3x + 3 = 0$, with the goal of finding its vertex, focus, and directrix. This exploration not only clarifies the geometric features of this specific parabola but also offers insights into general methods for analyzing quadratic curves.

The Significance of Parabolas in Mathematics and Real Life

Before diving into the technical calculations, it’s worth understanding why parabolas matter. Their unique geometric shape results from a quadratic relationship, which makes them ideal for modeling situations involving constant acceleration, projectile motion, and optics. For instance, satellite dishes are designed as parabolas to focus signals onto a single point—the focus—enhancing communication efficiency.

In the realm of mathematics, analyzing the vertex, focus, and directrix of a parabola provides a comprehensive understanding of its shape, size, and orientation. These features are essential for graphing, transforming, and applying parabolas to practical problems.

Step 1: Recognizing and Rewriting the Equation

The initial step involves examining the given equation:

$Y^2 + 6y - 3x + 3 = 0$

This equation appears to be quadratic in both Y and x, and it resembles the standard form of a parabola, but it’s not immediately clear due to the mixed terms. To analyze it effectively, we need to rewrite it in a form that clearly shows the parabola's orientation and key features.

Step 2: Rearranging to Standard Parabola Form

The goal is to express the equation in a form reminiscent of:

- $(Y - k)^2 = 4p(x - h)$ for a parabola opening to the right or left, or
- $(x - h)^2 = 4p(Y - k)$ for a parabola opening upward or downward.

Given the presence of Y^2 and linear Y terms, completing the square for Y appears to be the most straightforward approach.

Step 2.1: Grouping the Y terms

Rewrite the original equation as:

$Y^2 + 6y = 3x - 3$

Step 2.2: Completing the Square

Complete the square for the Y terms:

- Take half of the coefficient of y (which is 6), divide by 2: $6/2 = 3$
- Square this value: $3^2 = 9$

Add and subtract 9 to the left side to complete the square:

$Y^2 + 6y + 9 - 9 = 3x - 3$

This simplifies to:

$(Y + 3)^2 - 9 = 3x - 3$

Rearranged:

$(Y + 3)^2 = 3x - 3 + 9$

$(Y + 3)^2 = 3x + 6$

Step 3: Expressing in Standard Form

Divide both sides by 3 to isolate x:

$(Y + 3)^2 / 3 = x + 2$

Or equivalently:

$(Y + 3)^2 = 3(x + 2)$

This is now in the form:

$(Y - k)^2 = 4p(x - h)$

where:

- $(Y - k)^2 = 4p(x - h)$

Comparing, we see:

- $k = -3$
- $h = -2$
- $4p = 3$, so $p = 3/4$

Note: Since the squared term involves Y, and the right side involves x, the parabola opens horizontally—either to the right or left depending on the sign of p.

Step 4: Identifying the Vertex

In the standard form $(Y - k)^2 = 4p(x - h)$:

- The vertex is at (h, k).

From our comparison:

- Vertex $(h, k) = (-2, -3)$

This point represents the highest or lowest point on the parabola, and it is the point of symmetry.

Step 5: Determining the Focus and Directrix

The focus and directrix are critical features of a parabola, defining its curvature and orientation.

Focus: Located p units from the vertex along the axis of symmetry, on the side the parabola opens.

Directrix: A line p units from the vertex, on the opposite side of the focus, perpendicular to the axis of symmetry.

Given $(Y - k)^2 = 4p(x - h)$:

- Since $p = 3/4 > 0$, the parabola opens to the right.

Focus Coordinates:

- The focus is at $(h + p, k)$.

Calculating:

- $h + p = -2 + 3/4 = -2 + 0.75 = -1.25$

- $k = -3$

Focus = (-1.25, -3)

Directrix Equation:

- The directrix is a vertical line p units to the left of the vertex:

- $x = h - p = -2 - 3/4 = -2 - 0.75 = -2.75$

Directrix:

- $x = -2.75$

Summary of Key Features

Feature	Coordinates / Equation
Vertex	(-2, -3)
Focus	(-1.25, -3)
Directrix	x = -2.75

Practical Implications and Applications

Understanding these features allows engineers and scientists to manipulate and utilize parabolic shapes effectively. For example, knowing the focus's position helps in designing satellite dishes that focus signals precisely onto the receiver. Similarly, in physics, recognizing the parabola's vertex and focus aids in analyzing projectile trajectories and optical systems.

Additional Insights: The Parabola's Orientation and Shape

Since the parabola opens to the right, its shape is symmetric about the horizontal axis passing through the vertex. The distance between the vertex and focus ($p = 3/4$) indicates the parabola's "width" or "steepness." A larger p would mean a wider parabola, while a smaller p results in a steeper curve.

Final Thoughts

The process of finding the vertex, focus, and directrix of a parabola involves algebraic manipulation, understanding of standard forms, and geometric interpretation. Starting from the given quadratic equation, completing the square reveals the parabola's orientation and key features. Recognizing the standard form $(Y - k)^2 = 4p(x - h)$ provides a straightforward pathway to identifying the parabola's vertex and other properties.

By mastering these techniques, students and professionals can analyze complex quadratic equations with confidence, enabling precise application in science and engineering domains. Whether designing a parabolic reflector or solving physics problems, understanding the core features of parabolas remains an essential mathematical skill.

In summary:

- The vertex of the parabola $Y^2 + 6y - 3x + 3 = 0$ is at (-2, -3).
- The focus is located at (-1.25, -3).
- The directrix is the line $x = -2.75$.

This exploration exemplifies how algebraic techniques combined with geometric insight unlock the deeper properties of quadratic curves, bridging the gap between theoretical mathematics and practical applications.

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