Find The Volume Of The Solid Formed By Rotating The Region Enclosed By Y = E^3x + 1, Y=0, X=0, X=0.1

Find The Volume Of The Solid Formed By Rotating The Region Enclosed By Y = e^{3x} + 1, Y=0, X=0, X=0.1

Introduction

Calculating the volume of a solid of revolution is a fundamental problem in calculus, often encountered in various engineering and scientific applications. The process involves rotating a region bounded by curves around a specified axis, generating a three-dimensional object whose volume can be determined using integral calculus. In this article, we will explore how to find the volume of the solid formed when the region enclosed by the curve $y = e^{3x} + 1$, the line $y = 0$, and the vertical boundaries $x=0$ and $x=0.1$, is revolved about the x-axis.

This problem combines understanding of the exponential function, setting up integrals for volume calculation, and applying the disk method, a common technique when revolving regions around the x-axis. We will walk through the problem step-by-step, from understanding the region to performing the integral calculations, ensuring clarity and thoroughness suitable for students and enthusiasts of calculus.

Understanding the Region and the Boundaries

Graphing and Visualizing the Region

Before delving into calculations, it's essential to understand the region we're revolving:

- The curve $y = e^{3x} + 1$ is an exponential function shifted upward by 1.
- The line $y = 0$ (the x-axis) acts as the lower boundary.
- The vertical lines $x=0$ and $x=0.1$ define the left and right boundaries of the region.

Graphically, for x between 0 and 0.1:

- At $x=0$, $y = e^{0} + 1 = 1 + 1 = 2$.
- At $x=0.1$, $y = e^{0.3} + 1$.

Calculating $e^{0.3}$:

$$
e^{0.3} \approx 1 + 0.3 + \frac{0.3^2}{2} + \frac{0.3^3}{6} \approx 1 + 0.3 + 0.045 + 0.0045 \approx 1.3495
$$

Thus:

$$
y \approx 1.3495 + 1 = 2.3495
$$

The region enclosed is between $y=0$ and $y = e^{3x} + 1$, from $x=0$ to $x=0.1$.

Why is this region significant?

Understanding the shape helps in choosing the appropriate method of volume calculation. Since the region is bounded above by an exponential curve and below by the x-axis, and we are revolved around the x-axis, the resulting solid resembles a "shell" with varying radius and height.

Method for Calculating the Volume

Choosing the Appropriate Method: Disk Method

Since the axis of rotation is the x-axis, and the region is bounded above by $y = e^{3x} + 1$, the disk method is appropriate:

- Each cross-section perpendicular to the x-axis is a disk.
- The radius of each disk at position x is equal to the function value $y = e^{3x} + 1$.

The volume of the entire solid can be obtained by integrating the areas of these disks along x from 0 to 0.1.

Formula for the Disk Method

The volume V is given by:

$$
V = \int_{a}^{b} \pi \left[R(x) \right]^2 dx
$$

where:

- $R(x)$ is the radius of the disk at position x,
- a and b are the limits of integration.

In our case:

- $R(x) = y = e^{3x} + 1$,
- $a = 0$,
- $b = 0.1$.

Thus:

$$
V = \int_{0}^{0.1} \pi \left(e^{3x} + 1 \right)^2 dx
$$

Setting Up the Integral

Expanding the Integrand

To evaluate the integral:

$$
V = \pi \int_{0}^{0.1} (e^{3x} + 1)^2 dx
$$

expand the square:

$$
(e^{3x} + 1)^2 = (e^{3x})^2 + 2 \times e^{3x} \times 1 + 1^2 = e^{6x} + 2e^{3x} + 1
$$

Therefore:

$$
V = \pi \int_{0}^{0.1} \left(e^{6x} + 2e^{3x} + 1 \right) dx
$$

This integral can be separated as:

$$
V = \pi \left(\int_{0}^{0.1} e^{6x} dx + 2 \int_{0}^{0.1} e^{3x} dx + \int_{0}^{0.1} 1 dx \right)
$$

Evaluating the Integral Components

Integral of e^{6x}

$$
\int e^{6x} dx = \frac{1}{6} e^{6x} + C
$$

Evaluated from 0 to 0.1:

$$
\left. \frac{1}{6} e^{6x} \right|_{0}^{0.1} = \frac{1}{6} \left(e^{0.6} - e^{0} \right)
$$

Calculate $e^{0.6}$:

$$
e^{0.6} \approx 1 + 0.6 + \frac{0.6^2}{2} + \frac{0.6^3}{6} \approx 1 + 0.6 + 0.18 + 0.036 = 1.816
$$

Thus:

$$
\frac{1}{6} (1.816 - 1) = \frac{1}{6} \times 0.816 \approx 0.136
$$

Integral of $2e^{3x}$

$$
\int e^{3x} dx = \frac{1}{3} e^{3x} + C
$$

Multiply by 2:

$$
2 \times \frac{1}{3} e^{3x} = \frac{2}{3} e^{3x}
$$

Evaluated from 0 to 0.1:

Calculate $e^{0.3}$:

$$
e^{0.3} \approx 1 + 0.3 + \frac{0.3^2}{2} + \frac{0.3^3}{6} \approx 1 + 0.3 + 0.045 + 0.0045 \approx 1.3495
$$

Thus:

$$
\frac{2}{3} (e^{0.3} - e^{0}) = \frac{2}{3} (1.3495 - 1) = \frac{2}{3} \times 0.3495 \approx 0.233
$$

Integral of the constant 1

$$
\int_{0}^{0.1} 1 dx = 0.1
$$

Calculating the Total Volume

Putting it all together:

$$
V = \pi \left(0.136 + 0.233 + 0.1 \right) = \pi \times 0.469
$$

Finally, multiply by π:

$$
V \approx 3.1416 \times 0.469 \approx 1.473
$$

Final Result and Interpretation

The approximate volume of the solid generated by revolving the specified region about the x-axis is:

\[

\boxed{
V \approx 1.473 \text{ cubic units}
}
\]

This result encapsulates the volume created by the exponential curve \(y = e^{3x} + 1 \) over the interval \([0, 0.1] \), when rotated about the x-axis. The value demonstrates the significant contribution of the exponential growth even over a small interval, and the integral captures the cumulative effect of the changing radius of the disks.

Extensions and Additional Considerations

Alternative Methods

While the disk method is straightforward for this problem, other approaches could include:

- Shell method, especially if the axis of revolution were different.
- Numerical integration techniques for more complex or less manageable functions.

Impact of Changing Boundaries

Altering the interval \([0

Frequently Asked Questions

What is the main goal when finding the volume of the solid formed by rotating the region between y = e^{3x} + 1, y = 0, and x = 0 to x = 0.1?

The main goal is to calculate the volume of the solid generated when the given region is revolved about the x-axis, using integration techniques such as the disk or washer method.

Which method is most appropriate for finding the volume of the solid in this problem?

The disk method is most appropriate because the region is revolved around the x-axis, and the cross-sections are circular disks.

How do you set up the integral to find the volume of the solid?

The volume V is given by the integral $V = \pi \int$ from 0 to 0.1 of $[f(x)]^2\, dx$, where $f(x) = e^{3x} + 1$.

What is the integral expression for the volume in this problem?

$V = \pi \int_0^{0.1} (e^{3x} + 1)^2\, dx$.

How do you evaluate the integral $\int (e^{3x} + 1)^2\, dx$?

Expand the integrand to $e^{6x} + 2e^{3x} + 1$, then integrate term by term: $\int e^{6x}\, dx = (1/6) e^{6x}$, $\int 2e^{3x}\, dx = (2/3) e^{3x}$, and $\int 1\, dx = x$.

What is the value of the definite integral for the volume calculation?

Calculate the antiderivative at $x = 0.1$ and $x = 0$, then subtract to find the definite integral: $V = \pi [(1/6) e^{6x} + (2/3) e^{3x} + x]$ from 0 to 0.1.

What is the approximate numerical value of the volume?

By substituting $x = 0.1$ and $x = 0$ into the antiderivative and multiplying by π, the approximate volume can be computed. For example, $e^{0.6} \approx 1.822$, $e^{0.3} \approx 1.350$, leading to a numerical estimate around 0.219 cubic units.

Why is it important to check the limits of integration in this problem?

Because the region is bounded between $x=0$ and $x=0.1$, ensuring the limits accurately reflect the region's extent is crucial for correct volume calculation.

How does the exponential function affect the volume calculation compared to polynomial functions?

The exponential function introduces exponential growth in the integrand, requiring the use of exponential integration formulas and potentially leading to larger volume values over the interval.

Additional Resources

Volume Calculation

Introduction: Exploring the Fascinating World of Solid of Revolution

When delving into the realm of calculus, one of the most intriguing and visually captivating topics is the concept of finding the volume of a solid formed by rotating a region around a specified axis. This process bridges the gap between algebraic functions and three-dimensional geometry, transforming simple curves into complex, tangible objects. Today, we focus on a specific problem: calculating the volume of the solid obtained by rotating the region enclosed by the curve $y = e^{3x} + 1$, the x-axis ($y=0$), and the vertical lines $x=0$ and $x=0.1$.

This problem not only showcases core techniques in integral calculus but also highlights the power of mathematical visualization. Whether you're a student seeking clarity or an enthusiast eager to deepen your understanding, this in-depth exploration will guide you step-by-step through the process of solving this problem, emphasizing conceptual insights, detailed calculations, and practical considerations.

Understanding the Problem: The Region and Its Boundaries

Before jumping into the calculations, it’s essential to thoroughly understand the geometric region involved.

Region Definition

- Curve: $y = e^{3x} + 1$
- X-axis: $y=0$
- Vertical boundaries: $x=0$ and $x=0.1$

The region in question is bounded on the top by the exponential curve, on the bottom by the x-axis, and on the sides by the lines $x=0$ and $x=0.1$. Visualizing this, imagine a thin strip along the x-axis stretching from 0 to 0.1, with the curve arching above it, creating a "curved strip" that we will rotate around an axis to generate a three-dimensional solid.

Choosing the Axis of Rotation

A crucial aspect of the problem is identifying the axis about which the region is rotated. The problem statement may specify this explicitly or implicitly; in most classical problems of this form, the rotation occurs around the x-axis ($y=0$).

Assumption: The region is rotated about the x-axis, which is the most common scenario unless otherwise specified.

This assumption makes sense because:

- The region is bounded below by $y=0$, the x-axis.
- Rotating around the x-axis will generate a solid of revolution symmetrical around this axis.
- It simplifies the calculation using the disk or washer method.

If the problem intended a different axis, the approach would need to be adjusted accordingly.

Methodology: The Disk Method

The most straightforward technique to find the volume of a solid formed by rotating a region around the x-axis is the disk method. This method involves slicing the solid perpendicular to the axis of rotation, resulting in circular disks whose volumes can be summed via integration.

Why the Disk Method?

- The region is described explicitly as a function $y=f(x)$.
- When rotating around the x-axis, each slice at position x forms a disk with radius $r = y = e^{3x} + 1$.

Formula for Volume Using the Disk Method

The volume V of the solid between $x=a$ and $x=b$ is:

$$
V = \pi \int_{a}^{b} [f(x)]^2\, dx
$$

In this context:

- $a = 0$
- $b = 0.1$
- $f(x) = e^{3x} + 1$

Thus,

$$
V = \pi \int_{0}^{0.1} \left(e^{3x} + 1 \right)^2 dx
$$

Step-by-Step Calculation of the Volume

Now, we proceed to evaluate the integral, which involves expanding the integrand, simplifying, and integrating term by term.

1. Expand the Square of the Function

$$
\left(e^{3x} + 1 \right)^2 = e^{6x} + 2 e^{3x} + 1
$$

This expansion simplifies the integral to:

$$
V = \pi \int_{0}^{0.1} \left(e^{6x} + 2 e^{3x} + 1 \right) dx
$$

---

2. Separate the Integral into Simpler Parts

$$
V = \pi \left(\int_{0}^{0.1} e^{6x} dx + 2 \int_{0}^{0.1} e^{3x} dx + \int_{0}^{0.1} 1 dx \right)
$$

This allows us to evaluate each integral independently.

---

3. Integrate Each Term

a. $\int e^{6x} dx = \frac{1}{6} e^{6x} + C$

b. $\int e^{3x} dx = \frac{1}{3} e^{3x} + C$

c. $\int 1 dx = x + C$

Applying the limits 0 to 0.1:

\[
V = \pi \left[\frac{1}{6} e^{6x} \Big|_{0}^{0.1} + 2 \times \frac{1}{3} e^{3x} \Big|_{0}^{0.1} +

x \Big|_{0}^{0.1} \right]
\]

Simplify each term:

- \(\frac{1}{6} (e^{6 \times 0.1} - e^{0}) = \frac{1}{6} (e^{0.6} - 1)\)
- \(2 \times \frac{1}{3} (e^{3 \times 0.1} - 1) = \frac{2}{3} (e^{0.3} - 1)\)
- \(0.1 - 0 = 0.1\)

Putting it all together:

\[
V = \pi \left[\frac{1}{6} (e^{0.6} - 1) + \frac{2}{3} (e^{0.3} - 1) + 0.1 \right]
\]

Numerical Evaluation

To attain a precise numerical value, we compute the exponentials:

- \(e^{0.6} \approx 1.8221188 \)
- \(e^{0.3} \approx 1.3498588 \)

Plugging these into the expression:

\[
V = \pi \left[\frac{1}{6} (1.8221188 - 1) + \frac{2}{3} (1.3498588 - 1) + 0.1 \right]
\]

Calculate each difference:

- \(1.8221188 - 1 = 0.8221188 \)
- \(1.3498588 - 1 = 0.3498588 \)

Now, multiply:

- \(\frac{1}{6} \times 0.8221188 \approx 0.1370198 \)
- \(\frac{2}{3} \times 0.3498588 \approx 0.2332392 \)

Add all components:

\[
V = \pi \times (0.1370198 + 0.2332392 + 0.1) = \pi \times 0.470259
\]

Finally, multiply by \(\pi \):

\[

V \approx 3.1415926 \times 0.470259 \approx 1.477
\]

Result:

\[
\boxed{
V \approx 1.477 \text{ cubic units}
}
\]

Interpretation and Significance

This volume estimate reveals the scale of the solid generated by rotating the specified region about the x-axis. Notably:

- The exponential growth \(e^{3x} \) causes the radius to increase rapidly, but within the small interval \([0, 0.1] \), the growth remains moderate.
- The resulting volume, approximately 1.477 units^3, reflects the combined effect of exponential expansion and the limited bounds of the region.

Extensions and Variations

While the above approach provides a comprehensive solution, further exploration can include:

- Changing the axis of rotation: For example, rotating around \(y = k \) or \(x = c \) would require different methods, such as shell or washer methods.
- Expanding the interval: Extending beyond \(x=0.1 \) to observe how the volume grows.
- Different functions: Analyzing other exponential or polynomial functions enclosed by similar boundaries.

Conclusion: Mastering the Art of Volume Calculations

Calculating the volume of a solid of revolution is a fundamental skill in calculus, blending algebraic manipulation, geometric visualization, and integral calculus techniques. By carefully defining the

Find The Volume Of The Solid Formed By Rotating The Region Enclosed By Y E 3x 1 Y 0 X 0 X 0 1

Find The Volume Of The Solid Formed By Rotating The Region Enclosed By Y E 3x 1 Y 0 X 0 X 0 1

Related Articles

- Eitan Randomly Selected Volcanoes To Travel To And Study. After Traveling, He Had Seen 131313 Cinder
- Find The Relative Maximum And Minimum Values.f(x,y)=x3+y315xySelect The Correct Choice Below And, If
- Cost Allocation Is Often The Centerpiece Of Conflict That Is Resolved In Court Cases. The Litigation

Back to Home