Find Two Consecutive Even Integers Such That The Smaller Added To Three Times The Larger Gives A Sum

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Understanding how to solve algebraic word problems involving consecutive even integers is a fundamental skill in mathematics that enhances problem-solving abilities. One common type of problem asks you to find two consecutive even integers based on a given relationship between them. In this article, we will explore the process of solving such problems step-by-step, focusing on the specific problem: finding two consecutive even integers such that the smaller added to three times the larger results in a particular sum. We will delve into the methods, formulas, and strategies to approach and solve this problem efficiently, providing clear explanations and practical examples.

Understanding the Problem Statement

Before diving into the solution, it's crucial to understand what the problem is asking. The key components are:

- Two integers: These are the numbers we need to find.
- Consecutive even integers: The numbers are even and follow one after the other, such as 4 and 6, or 10 and 12.
- Relationship: The problem states that when the smaller integer is added to three times the larger, the result is a specific sum (often given in the problem statement).

The problem can be summarized as:

> Find two consecutive even integers such that the smaller added to three times the larger equals a certain sum.

For example, if the sum is 26, the problem becomes: "Find two consecutive even integers such that the smaller added to three times the larger gives 26."

Setting Up the Algebraic Equation

The first step in solving such problems is to translate the word problem into an algebraic equation. Here's a structured approach:

Step 1: Define Variables

- Let x represent the smaller even integer.
- Since the integers are consecutive even integers, the larger will be x + 2 (because even integers differ by 2).

Step 2: Write the Relationship as an Equation

According to the problem:

> Smaller integer + 3 × Larger integer = Sum

Expressed algebraically:

$$ x + 3(x + 2) = \text{Sum} $$

If the sum is given (say, 26), then:

$$ x + 3(x + 2) = 26 $$

Step 3: Simplify and Solve

Distribute and combine like terms:

$$ x + 3x + 6 = 26 $$

$$ 4x + 6 = 26 $$

Subtract 6 from both sides:

$$ 4x = 20 $$

Divide both sides by 4:

$$ x = 5 $$

Since x is 5, which is odd, this indicates the initial assumption needs reconsideration because the problem states the integers are even. Let's verify with an example:

- If x is even, then x + 2 is also even.
- To find the actual integers, choose x as an even number satisfying the equation.

However, in this example, x = 5 is odd, so it suggests a need to check the assumptions or the sum.

Note: If the problem specifies that the integers are even, then the variable x must be even, and the algebraic solution should reflect that.

Adjusting for Even Integers

Because x must be even, and our algebraic solution gives an odd number, it suggests that the sum provided might lead to no solution or requires a different approach.

Alternatively, the problem might specify the sum, and the algebraic process involves verifying the solution.

General Approach to Solving Such Problems

To effectively solve problems involving consecutive even integers and their relationships, follow these key steps:

1. Define variables carefully: Use x for the smaller integer and x + 2 for the larger, ensuring x is even.
2. Translate the problem into an algebraic equation: Based on the relationship described.
3. Simplify and solve the algebraic equation: Use algebraic operations to find x.
4. Verify the solution: Ensure x is even; if not, re-express the problem or check the assumptions.
5. Find the integers: Once x is determined, the two integers are x and x + 2.
6. Check the solution: Substitute back into the original statement to verify correctness.

Practical Example: Solving the Problem Step-by-Step

Let's consider a specific example to illustrate the process thoroughly.

Problem Statement: Find two consecutive even integers such that the smaller added to three times the larger gives 50.

Step 1: Define variables

- Smaller integer: x (even)
- Larger integer: x + 2

Step 2: Write the equation

$$ x + 3(x + 2) = 50 $$

Step 3: Simplify

$$ x + 3x + 6 = 50 $$

$$ 4x + 6 = 50 $$

Step 4: Solve for x

$$ 4x = 44 $$

$$ x = 11 $$

Since $x = 11$, which is odd, but we are looking for even integers, this indicates the initial assumption needs adjustment.

Insight: The algebraic solution suggests x should be even, but the calculation yields an odd number. Therefore, the actual integers that satisfy the condition are $x = 10$ (even), and $x + 2 = 12$.

Let's check:

$$ 10 + 3(12) = 10 + 36 = 46 \neq 50 $$

Next, try $x = 12$:

$$ 12 + 3(14) = 12 + 42 = 54 \neq 50 $$

Try $x = 8$:

$$ 8 + 3(10) = 8 + 30 = 38 \neq 50 $$

Try $x = 14$:

$$ 14 + 3(16) = 14 + 48 = 62 \neq 50 $$

Notice the pattern: the sum increases by 12 when x increases by 2, which indicates the original problem's sum value doesn't match with the integer sequence.

Alternative approach:

Instead of assuming the variable x is any even integer, we can set x as even and solve for x directly using the algebraic equation, ensuring x remains even.

Rearranged:

$$ 4x + 6 = 50 $$
$$ 4x = 44 $$
$$ x = 11 $$

Since x is not even, the solution indicates no even integer solution under these conditions with sum 50.

Conclusion: Sometimes, the problem constraints may lead to no solution, which is an important aspect of algebraic problem-solving.

Key Tips for Solving Consecutive Even Integer Problems

- Always define variables considering the properties of the integers (even or odd).
- Remember that consecutive even integers differ by 2.
- Verify solutions to ensure they satisfy the original conditions and constraints.
- Be prepared for scenarios where no solutions exist given the parameters.

Common Mistakes to Avoid

- Assuming the integers are odd when the problem specifies even integers.
- Forgetting that consecutive even integers differ by 2.
- Making algebraic errors during simplification or solving.
- Overlooking the possibility of no solution due to conflicting constraints.

Conclusion

Finding two consecutive even integers based on a relationship involving their sum, such as the smaller added to three times the larger, requires careful setup of algebraic equations and attention to the properties of even integers. By defining variables accurately, translating the problem into an algebraic expression, simplifying, solving, and verifying the solution, you can confidently tackle similar problems. Remember that understanding the properties of integers and being meticulous in your calculations will lead to correct solutions and deepen your algebraic problem-solving skills. Whether in academic assessments or real-world scenarios, mastering these techniques will enhance your mathematical reasoning and analytical abilities.

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Frequently Asked Questions

What is the mathematical expression for finding two consecutive even integers such that the smaller added to three times the larger gives a sum?

Let the smaller even integer be x, then the larger is x + 2. The equation becomes x + 3(x + 2) = sum, which simplifies to x + 3x + 6 = sum.

How do you set up an equation to solve for two consecutive even integers based on the given condition?

Set the smaller integer as x, the larger as x + 2, then write the equation: x + 3(x + 2) = total sum, and solve for x.

If the sum of the smaller even integer and three times the larger is 20, what are the integers?

Let the smaller be x: x + 3(x + 2) = 20. Simplify: x + 3x + 6 = 20 → 4x + 6 = 20 → 4x = 14 → x = 3. Since integers are even, check for even x; no solution because x=3 is odd. For an even sum, adjust accordingly.

Can the two consecutive even integers be negative? How would that affect the problem?

Yes, the integers can be negative. You set up the same equations with negative x, and solving will yield negative integers if they satisfy the condition.

What is the general approach to find the specific integers once the equation is set up?

Solve the algebraic equation for x, then verify that both x and x + 2 are even integers satisfying the original condition.

How do you verify if the integers found satisfy the original problem statement?

Plug the integers back into the condition: smaller + 3 larger = sum, and check if the equation holds true.

What are common mistakes to avoid when solving for consecutive even integers in this problem?

Avoid assuming the integers are positive without verification, and ensure that the solution for x is even. Also, check that the integers are indeed consecutive evens.

Can this problem be extended to find three consecutive even integers with a similar condition?

Yes, but the setup becomes more complex. You would define the integers as x, x + 2, and x + 4, then write an equation based on the given condition and solve accordingly.

Additional Resources

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Introduction

Mathematics often presents us with intriguing problems that require both logical reasoning and algebraic manipulation to solve. Among these, word problems involving consecutive integers—especially even integers—serve as excellent exercises in translating real-world scenarios into mathematical expressions. The problem at hand, "Find two consecutive even integers such that the smaller added to three times the larger gives a sum," exemplifies this type of challenge.

Such problems are not only pedagogically valuable but also illustrate fundamental algebraic concepts, including variable assignment, equation formation, and solution strategies. This investigative article seeks to thoroughly analyze this problem, explore multiple solution pathways, and contextualize its importance within broader mathematical problem-solving frameworks.

Understanding the Problem

Before delving into the solution process, it is essential to interpret the problem statement accurately. The problem asks us to:

- Find two consecutive even integers.
- The smaller integer, when added to three times the larger integer, results in a specific sum.

Key Elements:

- The integers are even, meaning they are divisible by 2.
- They are consecutive, implying that the larger integer immediately follows the smaller one by 2 units.
- The relationship involves a sum that combines the smaller integer and three times the larger.

Expressed mathematically, the problem can be rephrased as:

Let the smaller even integer be x. Then the larger integer, being the immediate consecutive even number, is $x + 2$.

The sum in question is:

$$
x + 3(x + 2)
$$

The problem, as stated, implies that this sum is a specific value, which should be given or deduced. Since the problem statement appears to be incomplete regarding the exact sum, we will proceed assuming that the goal is to express the sum as an algebraic expression and explore solutions based on this relationship.

Formulating the Algebraic Model

Step 1: Assign Variables

- Smaller integer: x
- Larger integer: $x + 2$

Step 2: Write the Expression

The sum described in the problem:

$$
\text{Sum} = x + 3(x + 2)
$$

Expanding the expression:

$$
x + 3x + 6 = 4x + 6
$$

Without a specified total sum, the problem remains in a general form. To proceed with a concrete solution, we can consider a few scenarios:

- Scenario A: The sum equals a specific value S. For example, if the sum is 30, then:

$$
4x + 6 = 30
$$

- Scenario B: The task is to find the integers based on certain conditions, such as the sum being maximized or minimized under given constraints.

In this article, we will explore both general and specific cases.

Deep Dive: Solving the General Equation

Case 1: Sum equals a specific value S

Suppose the sum is S, an arbitrary constant. Then:

$$
4x + 6 = S
$$

Solving for x:

$$
4x = S - 6
$$
$$
x = \frac{S - 6}{4}
$$

Since x represents an even integer, it must satisfy:

$$
x \equiv 0 \ (\text{mod } 2)
$$

This imposes the condition:

$$
\frac{S - 6}{4} \equiv 0 \ (\text{mod } 2)
$$

which simplifies to:

$$
S - 6 \equiv 0 \ (\text{mod } 8)
$$

because multiplying both sides by 4:

$$
S - 6 \equiv 0 \ (\text{mod } 8)
$$

Therefore, the sum S must be of the form:

$$
S = 8k + 6,\quad k \in \mathbb{Z}
$$

Given this, the value of x becomes:

$$
x = 2k + 1.5

\]

which is not an integer unless \(k \) is such that \(2k + 1.5 \) is integer. But since \(x \) must be an even integer, this suggests that for the original assumption, the sum \(S \) must satisfy certain divisibility conditions to yield an integer \(x \).

Summary:

- For a specific sum \(S \), the solution for the smaller integer is:

\[
x = \frac{S - 6}{4}
\]

- To ensure \(x \) is an even integer, \(S \) must satisfy:

\[
S \equiv 6 \ (\text{mod } 8)
\]

and \(\frac{S - 6}{4} \) must be an even integer, i.e., divisible by 2.

Deep Dive: Specific Examples

To anchor our understanding, consider specific values for \(S \):

Example 1: Sum \(S = 22 \)

Calculate:

\[
x = \frac{22 - 6}{4} = \frac{16}{4} = 4
\]

- \(x = 4 \), which is even.
- Larger integer: \(x + 2 = 6 \).

Verify the sum:

\[
x + 3(x + 2) = 4 + 3 \times 6 = 4 + 18 = 22
\]
which matches the sum \(S = 22 \).

Conclusion: The integers are \(4 \) and \(6 \).

Example 2: Sum \(S = 30 \)

Calculate:

$$
x = \frac{30 - 6}{4} = \frac{24}{4} = 6
$$

- $x = 6$, even.
- Larger integer: 8.

Verify:

$$
6 + 3 \times 8 = 6 + 24 = 30
$$

Solution: integers are 6 and 8.

General Solution and Conditions

From the previous analysis, the general solution involves selecting an appropriate sum S that satisfies the divisibility conditions ensuring both x and $x + 2$ are even integers and satisfy the sum relationship.

Explicitly:

- The smaller integer:

$$
x = \frac{S - 6}{4}
$$

- The larger integer:

$$
x + 2 = \frac{S - 6}{4} + 2
$$

- Valid solutions require:

$$
x \equiv 0 \ (\text{mod } 2)
$$

which leads to the condition:

$$
(S - 6) \equiv 0 \ (\text{mod } 8)
$$

and

$$
x \in \mathbb{Z}
$$

Broader Context: Applications and Educational Significance

This problem exemplifies core algebraic principles such as variable assignment, equation solving, and modular arithmetic. Its educational significance extends to:

- Understanding consecutive integers: Recognizing that consecutive even integers differ by 2.
- Formulating equations: Translating word problems into algebraic expressions.
- Divisibility and modular reasoning: Using modular arithmetic to determine the conditions for integer solutions.

Such problems also foster critical thinking, pattern recognition, and the ability to verify solutions through substitution.

Variations and Extensions

The foundational structure of this problem lends itself to numerous variations, including:

- Finding two consecutive odd integers satisfying similar conditions.
- Extending to more than two integers, such as three consecutive even integers.
- Incorporating additional constraints, such as the sum being less than or greater than a certain value.
- Exploring quadratic relationships when the problem involves squares or higher powers.

These extensions deepen understanding and prepare students for more complex algebraic challenges.

Conclusion

The problem of "Find two consecutive even integers such that the smaller added to three times the larger gives a sum" offers rich insights into algebraic modeling and problem-solving strategies. By assigning variables, translating textual descriptions into equations, and applying modular arithmetic, one can systematically determine the conditions under which solutions exist.

The key takeaways include:

- Recognizing the importance of parity (evenness) in integer problems.
- Understanding how to manipulate algebraic expressions to isolate variables.
- Applying modular reasoning to determine the feasibility of solutions.

Ultimately, such problems serve as excellent pedagogical tools, illustrating the elegance and power of algebra in solving real-world and theoretical puzzles. They exemplify how a simple word problem can unfold into a comprehensive mathematical investigation, enriching both comprehension and appreciation of mathematical problem-solving.

References

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Note: For specific solutions, the sum \(S \) must be provided or assumed. The analysis above demonstrates the process for general and particular cases.

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