# Find Two Numbers Whose Difference Is 42 And Whose Product Is A Minimum. Step 1 If Two Numbers Have A

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When faced with a problem involving two numbers with a specific difference and a goal to minimize or maximize their product, it introduces an interesting challenge in algebra and optimization. In this article, we will explore the process of finding two numbers whose difference is 42 and whose product is minimized. This type of problem is common in various fields such as mathematics, engineering, and economics, where optimization plays a crucial role. We will go through step-by-step methods, including setting up equations, applying calculus, and analyzing the results to achieve an optimal solution.

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## Understanding the Problem: Finding Two Numbers with a Given Difference and Minimum Product

Before diving into complex calculations, it's essential to understand what the problem entails:

- Given Condition: The difference between two numbers is 42.
- Objective: Find the two numbers such that their product is as small as possible.
- Constraints: The difference between the two numbers remains fixed at 42.

This problem can be approached systematically by defining variables, forming algebraic expressions, and then applying calculus to find the minimum of the resulting function.

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## Step 1: Define Variables and Set Up the Equation

The first step in solving the problem involves choosing appropriate variables.

### Choosing Variables

- Let the two numbers be $x$ and $y$.
- Since the difference between the numbers is 42, we can represent this as:
$$
y - x = 42
$$
- To simplify, assume $x$ is one number, then $y = x + 42$.

### Expressing the Product

The goal is to minimize the product $P$ of these two numbers:

$$
P = x \times y
$$

Substitute $y$ with $x + 42$:

$$
P(x) = x \times (x + 42) = x^2 + 42x
$$

Our task reduces to finding the value of $x$ that minimizes $P(x)$.

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## Step 2: Find the Critical Points Using Calculus

To determine the minimum value of the product, calculus offers powerful tools.

### Differentiate the Product Function

- Compute the first derivative of $P(x)$:

$$
P'(x) = 2x + 42
$$

- Set $P'(x) = 0$ to find critical points:

$$
2x + 42 = 0

$$
2x = -42
$$

$$
x = -21
$$

### Find Corresponding $y$

- Recall that $y = x + 42$:

$$
y = -21 + 42 = 21
$$

Thus, the two numbers are $x = -21$ and $y = 21$.

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## Step 3: Verify the Nature of Critical Points

To confirm whether this critical point corresponds to a minimum, examine the second derivative.

### Second Derivative Test

- The second derivative of $P(x)$:

$$
P''(x) = 2
$$

- Since $P''(x) = 2 > 0$, the function $P(x)$ is concave upward at $x = -21$, indicating a minimum point.

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## Step 4: Calculate the Minimum Product

Having identified $x = -21$, compute the corresponding product:

P_{\text{min}} = x \times y = (-21) \times 21 = -441
\]

The minimum product of the two numbers with a difference of 42 is -441.

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## Step 5: Interpret the Results

- The two numbers are -21 and 21.
- Their difference:

\[
21 - (-21) = 42
\]

- Their product:

\[
-21 \times 21 = -441
\]

This result indicates that the minimal product occurs when the two numbers are symmetric around zero, with one being negative and the other positive.

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## Additional Insights and Variations

Understanding this problem allows for exploration of related scenarios:

### 1. When Both Numbers Are Positive

- If both numbers are positive and their difference is 42, then:

\[
x > 0, \quad y = x + 42 > 0
\]

- The product:

\[
P = x(x + 42)
\]

- Since the parabola opens upward, the minimum occurs at:

\[
x = -\frac{b}{2a} = -\frac{42}{2} = -21
\]

- But \( x \) must be positive, so the minimum in the positive domain occurs at the smallest possible \( x \) satisfying the condition, which is at \( x = 0 \).

- At \( x = 0 \), \( y = 42 \), and the product:

\[
P = 0 \times 42 = 0
\]

- Therefore, the minimal positive product with both numbers positive is 0 at \( (0, 42) \).

## 2. Comparing Negative and Positive Solutions

- The previous analysis shows that allowing negative numbers yields a more extreme minimum of -441, while restricting to positive numbers yields a minimal product of 0.

## 3. Real-world Applications

- This type of problem is useful in fields like physics for minimizing energy configurations, economics for profit/loss optimization, and engineering for minimizing costs under constraints.

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# Conclusion: How to Find Two Numbers with a Fixed Difference and a Minimized Product

In summary, solving for two numbers with a fixed difference to minimize their product involves:

- Setting variables and expressing one in terms of the other.
- Formulating the product as a function of a single variable.
- Applying calculus (derivatives) to find critical points.
- Using the second derivative test to confirm minimum points.
- Interpreting the results to identify the specific numbers.

This process is a fundamental application of algebra and calculus in solving optimization problems, demonstrating how mathematical tools can help find solutions that optimize certain criteria under specific constraints.

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## Key Takeaways for Optimization Problems

- Always clearly define variables and constraints.
- Express the target quantity as a function of one variable.
- Use derivatives to locate critical points.
- Confirm the nature of critical points with the second derivative test.
- Consider domain restrictions when interpreting solutions.

By mastering these steps, you can efficiently tackle a wide range of problems involving minimum and maximum values in various fields.

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Meta Description: Discover how to find two numbers with a difference of 42 that produce the minimum product. Learn step-by-step algebraic and calculus methods to solve this optimization problem effectively.

Keywords: find two numbers, minimum product, fixed difference, algebra, calculus, optimization, mathematical problem-solving, minimize product with difference 42

## Frequently Asked Questions

### Find two numbers whose difference is 42 and whose product is minimized. Step 1: If two numbers have a difference of 42, how can we express these numbers in terms of a variable?

Let the two numbers be x and y, with x > y. Since their difference is 42, we can express y as y = x - 42.

### Given the two numbers x and y with y = x - 42, how do we express their product?

The product P of the two numbers is P = x y. Substituting y = x - 42, we get P = x (x - 42) = x^2 - 42x.

## How do we find the value of x that minimizes the product P = x^2 - 42x?

To minimize P, we take the derivative of P with respect to x, set it to zero, and solve for x. So, $dP/dx = 2x - 42 = 0$.

## What is the critical point for the quadratic function P = x^2 - 42x, and how do we find it?

Setting the derivative $2x - 42 = 0$ gives $x = 21$. This is the critical point where P may be minimized.

## What are the corresponding values of the two numbers when x = 21?

Since $y = x - 42$, when $x = 21$, $y = 21 - 42 = -21$. The two numbers are 21 and -21.

## What is the minimum product of the two numbers whose difference is 42?

The minimum product is $P = 21 (-21) = -441$.

## Why is the product minimized when the two numbers are 21 and -21?

Because the quadratic $P = x^2 - 42x$ opens upwards, its vertex at $x = 21$ gives the minimum value of P, corresponding to the pair (21, -21).

## Can the two numbers be both positive or both negative to achieve the minimum product?

No, to minimize the product given a fixed difference of 42, one number must be positive and the other negative, as in 21 and -21, which yields the most negative (minimum) product.

## What is the significance of the sign of the product in this problem?

A negative product indicates the numbers have opposite signs, which is necessary to achieve the minimum product when the difference is fixed at 42.

## Is the pair (21, -21) unique for this problem?

Yes, the pair (21, -21) uniquely minimizes the product for the given difference of 42, as shown by the vertex of the quadratic function.

# Additional Resources

Find Two Numbers Whose Difference Is 42 And Whose Product Is A Minimum: A Comprehensive Analysis

Mathematics often presents us with intriguing problems that combine algebraic reasoning, optimization techniques, and critical thinking. One such classic problem involves finding two numbers that satisfy certain conditions—specifically, having a fixed difference—and then analyzing their product to determine its minimum value. This problem not only sharpens algebraic skills but also deepens understanding of functions, derivatives, and the principles of optimization.

In this detailed review, we will explore the problem statement thoroughly, break down the steps involved, and examine the mathematical concepts underpinning the solution. Our focus will be on understanding "Step 1: If Two Numbers Have A," which introduces initial ideas and foundational assumptions necessary to approach the problem effectively.

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# Understanding the Problem Statement

Before delving into the solution, it’s essential to clarify what the problem entails:

- Given Conditions:
Two numbers, say $x$ and $y$, are such that their difference is 42:
$$
|x - y| = 42
$$

- Objective:
Among all such pairs of numbers, identify the pair for which the product $P = xy$ is minimized.

- Core Challenge:
How do we translate the given condition into a mathematical framework that allows for optimization? And what strategies do we employ to find the minimum product?

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# Step 1: If Two Numbers Have a Difference of 42

The initial step involves understanding and formalizing the relationship between the two numbers based on the difference condition.

## 1.1. Expressing the Relationship Between the Numbers

Since the absolute difference between $x$ and $y$ is 42, the two possible cases are:

- $x - y = 42$
- $y - x = 42$

Without loss of generality, we can assume a specific order to simplify calculations, for example:

$$
x = y + 42
$$

This assumption is valid because the problem is symmetric with respect to the order of the numbers; whether $x$ is larger or smaller, the analysis remains similar.

Note:
Choosing $x = y + 42$ simplifies the algebra without loss of generality. Alternatively, one can assume $y = x + 42$, and similar reasoning applies.

## 1.2. Setting Up Variables and Constraints

Once we assume $x = y + 42$:

- The pair $(x, y)$ is represented as $(y + 42, y)$.

- The only variable is $y$, which can range over all real numbers, but in the context of minimizing the product, it’s often sufficient to consider real numbers unless otherwise specified.

- The problem reduces to analyzing the product as a function of a single variable.

## 1.3. Expressing the Product in Terms of a Single Variable

The product $P = xy$ becomes:

\[

$$
P = (y + 42) \times y
$$

This simplifies to:

$$
P(y) = y^2 + 42y
$$

This quadratic function in $y$ is the foundation for the subsequent optimization process.

## 1.4. Recognizing the Nature of the Function

- The quadratic function $P(y) = y^2 + 42y$ opens upward because the coefficient of $y^2$ is positive.
- Its minimum value occurs at the vertex of the parabola.

Understanding the shape of this quadratic is crucial as it indicates the nature of the extremum:

- Vertex of the parabola:
The vertex gives the minimum point since the parabola opens upward.

- Vertex formula:
For a quadratic $y^2 + 42y$, the vertex $y$-coordinate is at:

$$
y = -\frac{b}{2a} = -\frac{42}{2 \times 1} = -\frac{42}{2} = -21
$$

- Corresponding $x$:
Recall $x = y + 42$, so:

$$
x = -21 + 42 = 21
$$

Result:
The pair $(x, y) = (21, -21)$ (or vice versa depending on the initial assumption) yields the minimum product.

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# Deep Dive into the Optimization Process

Understanding the initial step is crucial, but to truly grasp the problem, one must explore the reasoning behind the optimization process.

## 2.1. Mathematical Foundations of Optimization

- Quadratic functions:
The product $P(y) = y^2 + 42y$ is quadratic, opening upward, which means the vertex represents the minimum point.

- Vertex form:
To better understand the minimum, we can rewrite the quadratic in vertex form:

$$
P(y) = y^2 + 42y = (y + 21)^2 - 441
$$

This is obtained by completing the square:

$$
y^2 + 42y = (y + 21)^2 - 21^2 = (y + 21)^2 - 441
$$

- Interpretation:
The minimum value of $P(y)$ is $-441$, occurring at $y = -21$. The corresponding $x$ is:

$$
x = y + 42 = -21 + 42 = 21
$$

The minimum product is:

$$
P_{\text{min}} = (21) \times (-21) = -441
$$

- Significance:
The negative minimum indicates that the minimal product, in this case, is negative, meaning the two numbers are of opposite signs, which makes sense given their difference.

## 2.2. Alternative Cases and Symmetry

- If we had assumed $y = x + 42$, the analysis would be similar, leading to the same minima but with roles of $x$ and $y$ interchanged.

- The symmetry in the problem suggests that the minimal product is achieved when one number is positive and the other is negative, with their magnitudes related as shown.

## 2.3. Geometric Interpretation

- Viewing the problem geometrically, the set of points $(x, y)$ satisfying $|x - y| = 42$ form two lines:

$$
y = x + 42 \quad \text{and} \quad y = x - 42
$$

- The problem reduces to finding the point on these lines that minimizes the product $xy$.

- The solution involves analyzing the quadratic function along these lines, which geometrically corresponds to minimizing the product over lines at a fixed distance.

## 2.4. Significance of the Sign of the Numbers in the Minimum Product

- The minimum product being negative indicates the two numbers are on opposite sides of zero.

- The magnitude of the product is minimized when the numbers are as large in magnitude as possible but with opposite signs, respecting the fixed difference constraint.

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# Additional Considerations and Broader Perspectives

While the initial step provides a foundation, further exploration reveals interesting insights:

## 3.1. Extending to Absolute Values

- When considering the absolute difference, the problem remains symmetric because:

$$
|x - y| = 42
$$

- The signs of $x$ and $y$ influence the sign of the product $xy$.

- The minimal product, as shown, occurs when one number is positive and the other negative, with magnitudes summing to the minimal point.

## 3.2. Generalization to Other Fixed Differences

- The approach applies broadly to any fixed difference $d$:

$$
P(y) = y^2 + dy
$$

- The vertex occurs at:

$$
y = -\frac{d}{2}
$$

- The minimal product:

$$
P_{\text{min}} = -\frac{d^2}{4}
$$

- This illustrates a general pattern: as the difference increases, the minimal product becomes more negative, with the magnitude proportional to the square of the difference.

## 3.3. Practical Applications

- Optimization problems:
These principles can be applied to real-world problems involving constraints and optimization, such as maximizing profit, minimizing cost, or designing systems with specific parameters.

- Physics and engineering:
Understanding the minimal or maximal values under constraints is fundamental in fields like mechanics, thermodynamics, and electrical engineering.

- Economics and finance:
Similar methods help in portfolio optimization and risk management when

constraints are present.

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## Conclusion: Summarizing the Key Insights

The first step in solving the problem of finding two numbers with a difference of 42 and minimizing their product involves expressing the relationship between the numbers and translating it into a manageable algebraic form. By assuming \( x = y + 42 \), we reduce the two-variable problem to a single-variable quadratic optimization problem.

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