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Introduction

Understanding how to compute the work done when moving a particle in a vector field is fundamental in fields such as physics, engineering, and mathematics. Specifically, in the context of vector calculus, this involves evaluating the line integral of a given vector field along a specified path. This article provides a detailed explanation of how to calculate the work done in moving a particle within a specific force field $\mathbf{F} = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$, from the point $(1,0,0)$ to a designated endpoint. We will explore the concepts of vector fields, line integrals, parameterization of paths, and the step-by-step calculation process.

Understanding the Force Field $\mathbf{F} = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$

What Is a Vector Field?

A vector field assigns a vector to every point in space. In our case, the force field $\mathbf{F}$ assigns the vector:

$$
\mathbf{F}(x, y, z) = y \mathbf{i} + x \mathbf{j} + z \mathbf{k}
$$

to each point (x, y, z) in three-dimensional space.

Components of the Force Field

- X-component: y (multiplied by $\mathbf{i}$)
- Y-component: x (multiplied by $\mathbf{j}$)
- Z-component: z (multiplied by $\mathbf{k}$)

This field is symmetric in the x and y components and linear in z, which influences how the work integral will be evaluated.

The Concept of Work Done in a Force Field

Definition

The work W done in moving a particle along a curve C from point A to point B within a force field $\mathbf{F}$ is given by the line integral:

$$
W = \int_{C} \mathbf{F} \cdot d\mathbf{r}
$$

where:

- $\mathbf{F}$ is the vector field
- $d\mathbf{r}$ is the differential displacement vector along the path C

Significance

Calculating this integral measures how much work the force field does on the particle as it moves along C. The path C can be any curve connecting the initial and final points, but the integral's value depends on the path unless the field is conservative.

Step-by-Step Calculation

1. Define the Path

Since the problem states moving from $(1, 0, 0)$ to an unspecified endpoint, for completeness, we will consider a specific endpoint; let's assume we're moving to $(0, 1, 1)$. If a different endpoint is specified, the same method applies.

Note: If the endpoint differs, simply update the calculations accordingly.

2. Parameterize the Path

Choosing a simple path simplifies calculations. Common choices are:

- Straight line path
- Piecewise linear path
- Curved paths

Here, we'll use a straight-line path from $(1, 0, 0)$ to $(0, 1, 1)$.

Parameterization:

Let t vary from 0 to 1:

$$
\mathbf{r}(t) = \mathbf{r}_0 + t(\mathbf{r}_1 - \mathbf{r}_0)
$$

where:

- $\mathbf{r}_0 = (1, 0, 0)$
- $\mathbf{r}_1 = (0, 1, 1)$

Thus,

$$
\mathbf{r}(t) = (1 - t, t, t)
$$

for $t \in [0, 1]$.

3. Compute $d\mathbf{r}$

Differentiating $\mathbf{r}(t)$:

$$
\frac{d\mathbf{r}}{dt} = \left(-1, 1, 1\right)
$$

and

$$
d\mathbf{r} = \frac{d\mathbf{r}}{dt} dt = (-1, 1, 1) dt
$$

4. Express $\mathbf{F}$ Along the Path

Substitute $x(t), y(t), z(t)$:

$$
x(t) = 1 - t
$$
$$
y(t) = t
$$
$$
z(t) = t
$$

Then,

$$
\mathbf{F}(\mathbf{r}(t)) = y(t)\mathbf{i} + x(t)\mathbf{j} + z(t)\mathbf{k} = t \mathbf{i} + (1 - t) \mathbf{j} + t \mathbf{k}
$$

5. Compute the Dot Product $\mathbf{F}(\mathbf{r}(t)) \cdot d\mathbf{r}$

$$
\mathbf{F}(\mathbf{r}(t)) \cdot d\mathbf{r} = [t, (1 - t), t] \cdot [-1, 1, 1] dt
$$

Calculating:

$$
t \times (-1) + (1 - t) \times 1 + t \times 1 = -t + 1 - t + t = (-t - t + t) + 1 = (-t) + 1
$$

Simplify:

$$
(-t) + 1
$$

Thus, the integrand simplifies to:

$$
(1 - t) dt
$$

6. Set Up the Integral

The work done:

$$
W = \int_{t=0}^{1} (1 - t) dt
$$

7. Evaluate the Integral

$$
W = \int_{0}^{1} (1 - t) dt = \left[t - \frac{t^2}{2} \right]_0^1 = \left(1 - \frac{1}{2} \right) - (0 - 0) = \frac{1}{2}
$$

Final Result: Work Done

The work done in moving the particle along the straight path from $(1,0,0)$ to $(0,1,1)$ within the force field $\mathbf{F} = y \mathbf{i} + x \mathbf{j} + z \mathbf{k}$ is $\boxed{\frac{1}{2}}$ units of work.

Additional Considerations

1. Different Paths

The calculation above assumes a straight line. If the path differs, re-parameterize accordingly. For complex paths, the integral may require more advanced techniques, but the procedure remains similar.

2. Path Independence and Conservative Fields

In some cases, the work integral is path-independent if the force field is conservative. To check whether $\mathbf{F}$ is conservative, verify if it can be expressed as the gradient of a scalar potential function ϕ:

$$
\mathbf{F} = \nabla \phi
$$

If so, the work depends only on the endpoints, simplifying calculations.

3. Verifying Conservativeness

Calculate curl of $\mathbf{F}$:

$$
\nabla \times \mathbf{F} = \left| \begin{matrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial_x & \partial_y & \partial_z \\ y & x & z \end{matrix} \right|
$$

Compute:

$$
\nabla \times \mathbf{F} = \left(\frac{\partial z}{\partial y} - \frac{\partial x}{\partial z} \right) \mathbf{i} - \left(\frac{\partial z}{\partial x} - \frac{\partial y}{\partial z} \right) \mathbf{j} + \left(\frac{\partial x}{\partial y} - \frac{\partial y}{\partial x} \right) \mathbf{k}
$$

Since all partial derivatives are zero:

$$
\nabla \times \mathbf{F} = (0 - 0) \mathbf{i} - (0 - 0) \mathbf{j} + (0 - 0) \mathbf{k} = \mathbf{0}
$$

The curl is zero everywhere, indicating $\mathbf{F}$ is conservative, and the line integral depends only on endpoints.

Conclusion

Calculating the work done in a force field involves understanding the vector field's components, parameterizing the path, and performing the line integral of $\mathbf{F} \cdot d\mathbf{r}$. For the specific force field $\mathbf{F} = y \mathbf{i} + x \mathbf{j} + z \mathbf{k}$, moving a particle along a straight-line path from $(1,0,0)$ to $(0,1,1)$, the work done

Frequently Asked Questions

What is the force field F = Yi + Xj + Zk in vector notation?

The force field is given by the vector function F(x, y, z) = y i + x j + z k, where i, j, k are the unit vectors in the x, y, and z directions respectively.

How do you interpret the work done by a force field in a line integral?

The work done by a force field in moving a particle along a curve is calculated as the line integral of the force field along that curve, expressed as W = ∫ C F · dr.

What are the initial and final points of the particle's movement in this problem?

The particle moves from the point (1, 0, 0) to an unspecified final point; typically, the problem would specify the endpoint to compute the work.

How do you set up the line integral for this force field moving from (1, 0, 0) to a given point?

You parameterize the path from the initial to the final point, substitute into F, compute dr, and evaluate the line integral W = ∫ F · dr along that parameterized path.

What is a common parameterization for a straight-line path from (1, 0, 0) to (x2, y2, z2)?

A common parameterization is r(t) = (1, 0, 0) + t[(x2 - 1), y2, z2], with t in [0, 1], which linearly interpolates between the two points.

How do you compute the differential dr in the parameterization?

Differential dr is calculated as the derivative of r(t) with respect to t times dt: dr/dt = (dx/dt, dy/dt, dz/dt), so dr = (dx/dt, dy/dt, dz/dt) dt.

How do you evaluate the line integral once the parameterization and dr are established?

Substitute the parameterized coordinates into F, compute the dot product with dr, and integrate with respect to t over the interval [0, 1].

What is the significance of the direction of the path in calculating work done?

Since the work depends on the path, different paths between the same points can yield different results unless the force field is conservative; in that case, the work is path-independent.

Is the force field F = Yi + Xj + Zk conservative, and how does that affect the calculation?

To determine if F is conservative, check if its curl is zero. If it is conservative, the work done depends only on the endpoints, and it can be computed using a potential function, simplifying the calculation.

Additional Resources

Force Field Calculation: Work Done in Moving a Particle from (1, 0, 0)

When delving into the fascinating world of vector calculus and physics, understanding how force fields influence particle motion is fundamental. Specifically, calculating the work done by a force field in moving a particle along a path provides critical insights into energy transfer, potential functions, and the nature of the field itself. Today, we explore this concept through a detailed examination of the force field F = Yi + Xj + Zk and the process of calculating the work done in moving a particle from the point (1, 0, 0) to an arbitrary endpoint.

Understanding the Force Field F = Yi + Xj + Zk

Before we can compute work, it's essential to thoroughly understand the force field in question.

Nature and Components of the Force Field

The given force field:

$$
\mathbf{F}(X, Y, Z) = Y \mathbf{i} + X \mathbf{j} + Z \mathbf{k}
$$

can be broken down into its component functions:

- X-component: $F_x = Y$
- Y-component: $F_y = X$

- Z-component: $F_z = Z$

This structure indicates that the force at any point in space depends linearly on the coordinates, with a symmetry between the x and y components. It is reminiscent of certain physical phenomena where forces depend on positional relationships, such as in magnetic or gravitational fields, albeit in a simplified, mathematical form.

Visualizing the Force Field

Visualizing this vector field helps understand its behavior:

- At points where $Y = 0$, the force component along the x-axis is zero.
- Along lines where $X = 0$, the y-component is zero.
- The z-component is directly proportional to Z itself, indicating a kind of 'spring-like' or self-referential behavior along the Z-axis.

This field exhibits symmetry across the plane $X=Y$, and its divergence and curl can be computed to analyze whether it is conservative or not (a key factor in work calculation).

Path of the Particle and the Work Calculation

The core problem involves moving a particle from the point $(1, 0, 0)$ to an endpoint, which we will define as (X_2, Y_2, Z_2). For simplicity, and to keep the problem fully specified, let's assume the endpoint is $(0, 1, 0)$, a common choice that allows us to explore the field's behavior over a known path.

Why Path Matters: Work and Line Integrals

In vector calculus, the work W done by a force $\mathbf{F}$ in moving a particle along a path C from point A to point B is given by the line integral:

$$
W = \int_{C} \mathbf{F} \cdot d\mathbf{r}
$$

where:

- $d\mathbf{r}$ is the differential displacement vector along the path,
- The integral sums the force component in the direction of motion at each infinitesimal segment.

Key point: If the force field $\mathbf{F}$ is conservative—that is, if it can be expressed as the gradient of a scalar potential function—the work done is path-independent, depending only on the endpoints. Otherwise, the path matters.

Analyzing the Force Field for Conservativeness

To determine whether the work calculation simplifies, we investigate if $\mathbf{F}$ is conservative.

Computing the Curl of $\mathbf{F}$

The curl of a vector field $\mathbf{F} = P \mathbf{i} + Q \mathbf{j} + R \mathbf{k}$ is:

$$
\nabla \times \mathbf{F} =
\left(\frac{\partial R}{\partial Y} - \frac{\partial Q}{\partial Z} \right) \mathbf{i}
+ \left(\frac{\partial P}{\partial Z} - \frac{\partial R}{\partial X} \right) \mathbf{j}
+ \left(\frac{\partial Q}{\partial X} - \frac{\partial P}{\partial Y} \right) \mathbf{k}
$$

For our $\mathbf{F}$:

- $P = Y$
- $Q = X$
- $R = Z$

Calculations:

$$
\begin{aligned}
\nabla \times \mathbf{F} &=
\left(\frac{\partial Z}{\partial Y} - \frac{\partial X}{\partial Z} \right) \mathbf{i} +
\left(\frac{\partial Y}{\partial Z} - \frac{\partial Z}{\partial X} \right) \mathbf{j} +
\left(\frac{\partial X}{\partial Y} - \frac{\partial Y}{\partial X} \right) \mathbf{k} \\
&= (0 - 0) \mathbf{i} + (0 - 0) \mathbf{j} + (1 - 1) \mathbf{k} \\
&= \mathbf{0}
\end{aligned}
$$

Since the curl is zero everywhere, $\mathbf{F}$ is a conservative vector field.

Implications of Conservativeness

Given that $\nabla \times \mathbf{F} = \mathbf{0}$ and assuming the domain is simply connected (which it is in $\mathbb{R}^3$), we conclude:

- $\mathbf{F}$ is conservative.

- There exists a potential function $\phi (X, Y, Z)$ such that $\nabla \phi = \mathbf{F}$.

This simplifies work calculation significantly because the work in moving from point A to B depends only on the endpoints:

$$
W = \phi(B) - \phi(A)
$$

Finding the Potential Function ϕ

Since $\nabla \phi = \mathbf{F}$:

$$
\frac{\partial \phi}{\partial X} = F_x = Y
$$
$$
\frac{\partial \phi}{\partial Y} = F_y = X
$$
$$
\frac{\partial \phi}{\partial Z} = F_z = Z
$$

Let's find ϕ:

1. Integrate $F_x = Y$ with respect to X:

$$
\phi(X, Y, Z) = XY + g(Y, Z)
$$

2. Differentiate ϕ with respect to Y:

$$
\frac{\partial \phi}{\partial Y} = X + \frac{\partial g}{\partial Y} = X
$$

which implies:

$$
\frac{\partial g}{\partial Y} = 0
$$

Thus, $g(Y, Z)$ does not depend on Y:

$$
g(Y, Z) = g(Z)

\]

3. Differentiate \(\phi\) with respect to \(Z\):

\[
\frac{\partial \phi}{\partial Z} = \frac{\partial}{\partial Z} (XY + g(Z)) = g'(Z)
\]

Given that:

\[
\frac{\partial \phi}{\partial Z} = Z
\]

we have:

\[
g'(Z) = Z \implies g(Z) = \frac{Z^2}{2} + C
\]

Choosing the constant of integration \(C = 0\) for simplicity, the potential function is:

\[
\boxed{
\phi(X, Y, Z) = XY + \frac{Z^2}{2}
}
\]

Calculating the Work Done

Since the field is conservative, the work in moving from \((1, 0, 0)\) to \((0, 1, 0)\) is:

\[
W = \phi(0, 1, 0) - \phi(1, 0, 0)
\]

Compute each:

- \(\phi(0, 1, 0) = (0)(1) + \frac{0^2}{2} = 0\)
- \(\phi(1, 0, 0) = (1)(0) + \frac{0^2}{2} = 0\)

Therefore:

\[
W = 0 - 0 = 0
\]

Result: The work done in moving the particle along any path from \((1, 0, 0)\) to \((0, 1, 0)\) in this force field is zero.

Interpreting the Result and Practical Insights

This zero work outcome aligns with the properties of conservative fields:

- The force field does no net work in moving the particle between these two points.
- The potential function \(\phi = XY + Z^2/2\) provides a scalar measure of the energy landscape, with local minima and maxima corresponding to equilibrium points.

Practical implications:

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