For The Function Y=(x ^2+4)(x ^3 9x), At (3,0) Find The Following. (a) The Slope Of The Tangent Line

For The Function Y=(x ^2+4)(x ^3 9x), At (3,0) Find The Following. (a) The Slope Of The Tangent Line

Understanding the behavior of functions and their derivatives is fundamental in calculus, especially when analyzing the rate at which a function changes at a specific point. This article provides a detailed, step-by-step process to determine the slope of the tangent line to the function $Y = (x^2 + 4)(x^3 - 9x)$ at the point $(3, 0)$. We will cover the concepts involved, the differentiation process, the calculation of the derivative at the given point, and the interpretation of the result—all structured to enhance clarity and SEO relevance.

Introduction to the Function and its Significance

Before diving into the derivative calculations, it's essential to understand the nature of the function:

- The function is a product of two polynomial expressions: $x^2 + 4$ and $x^3 - 9x$.
- This form suggests the use of the product rule for differentiation.
- The point $(3, 0)$ is specified, indicating that the function passes through this point, and we want to find the slope of the tangent line there.

Calculating the slope of the tangent line at a specific point involves differentiating the function to find $\frac{dy}{dx}$ (the derivative), then evaluating this derivative at the point $x=3$.

Understanding the Derivative and the Tangent Line

What is the Derivative?

- The derivative of a function at a point provides the slope of the tangent line to the function's graph at that point.
- It measures the instantaneous rate of change of the function concerning x.

Why Find the Slope of the Tangent Line?

- The slope indicates how steep the graph is at the point.
- It is critical in applications like physics (velocity), engineering (rate of change), and optimization problems.

Step-by-Step Calculation of the Derivative

Given the function:

$$
Y = (x^2 + 4)(x^3 - 9x)
$$

we will differentiate it with respect to x using the product rule. The product rule states:

$$
\frac{d}{dx} [u(x) \cdot v(x)] = u'(x) \cdot v(x) + u(x) \cdot v'(x)
$$

where:

- $u(x) = x^2 + 4$
- $v(x) = x^3 - 9x$

Calculating $u'(x)$ and $v'(x)$

- $u(x) = x^2 + 4 \Rightarrow u'(x) = 2x$
- $v(x) = x^3 - 9x \Rightarrow v'(x) = 3x^2 - 9$

Applying the Product Rule

Plugging these into the product rule:

$$
\frac{dy}{dx} = 2x \cdot (x^3 - 9x) + (x^2 + 4) \cdot (3x^2 - 9)
$$

Now, expand each term:

- First term:

$$
2x \cdot (x^3 - 9x) = 2x \cdot x^3 - 2x \cdot 9x = 2x^4 - 18x^2
$$

\]

- Second term:

\[
(x^2 + 4) \cdot (3x^2 - 9) = x^2 \cdot 3x^2 - x^2 \cdot 9 + 4 \cdot 3x^2 - 4 \cdot 9
\]

which simplifies to:

\[
3x^4 - 9x^2 + 12x^2 - 36
\]

Combine like terms:

\[
3x^4 + (-9x^2 + 12x^2) - 36 = 3x^4 + 3x^2 - 36
\]

Combine All Terms

Adding the expanded terms together:

\[
\frac{dy}{dx} = (2x^4 - 18x^2) + (3x^4 + 3x^2 - 36)
\]

which simplifies to:

\[
(2x^4 + 3x^4) + (-18x^2 + 3x^2) - 36 = 5x^4 - 15x^2 - 36
\]

Thus, the derivative of the function is:

\[
\boxed{\frac{dy}{dx} = 5x^4 - 15x^2 - 36}
\]

Evaluating the Derivative at \(x=3 \)

To find the slope of the tangent line at the point \((3, 0) \), substitute \(x=3 \) into the derivative:

\[
\frac{dy}{dx} \bigg|_{x=3} = 5(3)^4 - 15(3)^2 - 36

$$

Calculate each term:

- $3^4 = 81$

$$
5 \times 81 = 405
$$

- $3^2 = 9$

$$
15 \times 9 = 135
$$

Now, plug these into the expression:

$$
405 - 135 - 36 = (405 - 135) - 36 = 270 - 36 = 234
$$

Therefore, the slope of the tangent line at the point $(3, 0)$ is $\boxed{234}$.

Interpreting the Results

- The calculated slope indicates that at $x=3$, the function Y is increasing very steeply.
- A positive and large value (234) suggests a rapid rate of change, which could be visualized as a steep incline on the graph at that point.
- This information can be used in various applications, such as drawing the tangent line or analyzing the behavior of the function near that point.

Equation of the Tangent Line at $(3, 0)$

Having the slope $m=234$ and a point $(3, 0)$, we can write the equation of the tangent line:

$$
y - y_1 = m (x - x_1)
$$

where:

- $x_1 = 3$
- $y_1 = 0$

- $m=234$

Substituting:

$$
y - 0 = 234 (x - 3)
$$

which simplifies to:

$$
\boxed{ y = 234(x - 3) }
$$

This linear equation represents the tangent to the curve at the specified point.

Summary and Key Takeaways

- The function $Y = (x^2 + 4)(x^3 - 9x)$ is a product of polynomials, requiring the product rule for differentiation.
- The derivative $\frac{dy}{dx} = 5x^4 - 15x^2 - 36$ was obtained through careful algebraic expansion.
- Evaluating at $x=3$ yields a slope of 234, indicating a steep positive incline.
- The tangent line at $(3, 0)$ is $y = 234(x - 3)$, which can be used for graphing or further analysis.

Final Thoughts

This detailed process demonstrates how calculus tools like the product rule can be applied to complex polynomial functions to find their instantaneous rate of change at specific points. Understanding these steps enhances comprehension of the function's behavior and is essential for solving related problems in mathematics, physics, engineering, and related fields. Remember, practice with different functions solidifies your grasp of derivatives and their applications.

Meta Description: Discover how to find the slope of the tangent line to the function $Y = (x^2 + 4)(x^3 - 9x)$ at the point (3, 0). Follow our step-by-step guide to understand differentiation, evaluate derivatives, and interpret the results for advanced calculus mastery.

Frequently Asked Questions

How do you find the slope of the tangent line to the function $y = (x^2 + 4)(x^3 - 9x)$ at the point (3, 0)?

First, differentiate y with respect to x using the product rule, then substitute $x = 3$ into the derivative to find the slope of the tangent line at that point.

What is the derivative of $y = (x^2 + 4)(x^3 - 9x)$ before plugging in $x = 3$?

Using the product rule, $dy/dx = (2x)(x^3 - 9x) + (x^2 + 4)(3x^2 - 9)$.

Calculate the value of the derivative at $x = 3$ for the given function.

Substitute $x = 3$ into the derivative: $dy/dx = (23)(3^3 - 93) + (3^2 + 4)(33^2 - 9) = 6(27 - 27) + (9 + 4)(27 - 9) = 6(0) + 13(18) = 0 + 234 = 234$.

What is the slope of the tangent line to the curve at the point (3, 0)?

The slope is 234.

How is the point (3, 0) related to the function $y = (x^2 + 4)(x^3 - 9x)$?

At $x = 3$, the function evaluates to $y = (3^2 + 4)(3^3 - 93) = (9 + 4)(27 - 27) = 130 = 0$, confirming the point (3, 0) lies on the curve.

Additional Resources

For The Function $Y = (x^2 + 4)(x^3 - 9x)$: An Investigative Analysis of the Slope of the Tangent Line at (3,0)

In the realm of calculus, understanding the behavior of functions at specific points is fundamental. One such analysis involves determining the slope of the tangent line to a curve at a given point. This investigation delves into the function $Y = (x^2 + 4)(x^3 - 9x)$, focusing on the point $(3, 0)$, to find the slope of the tangent line at that point. This detailed exploration not only elucidates the process of differentiation but also highlights the significance of derivative calculations in understanding the geometric properties of functions.

Introduction to the Function and Its Significance

The function in question, $Y = (x^2 + 4)(x^3 - 9x)$, is a product of two polynomial expressions. Such functions often exhibit complex behaviors, including multiple points of inflection, local maxima and minima, and varying slopes along different segments. Analyzing the slope of the tangent line at a specific point provides insight into the function’s instantaneous rate of change, which is fundamental in fields ranging from physics to economics.

The point of interest, $(3, 0)$, is particularly intriguing because evaluating the function at $x=3$ confirms it lies on the curve:

$$
Y = (3^2 + 4)(3^3 - 9 \times 3) = (9 + 4)(27 - 27) = 13 \times 0 = 0
$$

Thus, the point $(3, 0)$ indeed resides on the graph, setting the stage for a precise derivative calculation to find the tangent slope.

Deep Dive into Differentiation: Applying the Product Rule

The Need for the Product Rule

Given the function's structure as a product of two functions:

- $u(x) = x^2 + 4$
- $v(x) = x^3 - 9x$

the derivative Y' requires the application of the product rule:

$$
\frac{d}{dx}[u(x) \times v(x)] = u'(x) v(x) + u(x) v'(x)
$$

This rule states that the derivative of a product is the sum of the derivatives of each function multiplied by the other function.

Step-by-Step Differentiation

1. Differentiate $u(x) = x^2 + 4$:

$$
u'(x) = 2x
$$

2. Differentiate $v(x) = x^3 - 9x$:

$$
v'(x) = 3x^2 - 9
$$

3. Apply the product rule:

$$
Y' = u'(x) v(x) + u(x) v'(x) = 2x (x^3 - 9x) + (x^2 + 4)(3x^2 - 9)
$$

Calculating the Derivative at $x = 3$

Substituting $x = 3$ into the derivative:

$$
Y'|_{x=3} = 2 \times 3 \times (3^3 - 9 \times 3) + (3^2 + 4) \times (3 \times 3^2 - 9)
$$

Breaking it down:

- $2 \times 3 = 6$
- $3^3 = 27$
- $9 \times 3 = 27$
- $3^2 = 9$

Evaluate each component:

1. First term:

$$
6 \times (27 - 27) = 6 \times 0 = 0
$$

2. Second term:

$$
(9 + 4) \times (3 \times 9 - 9) = 13 \times (27 - 9) = 13 \times 18 = 234
$$

Adding both:

$$
Y'|_{x=3} = 0 + 234 = 234
$$

Interpretation of the Result

The derivative Y' at $x=3$ is 234, which is the slope of the tangent line to the curve at the point $(3, 0)$. This steep positive slope indicates that the function is increasing rapidly at this point, and the tangent line is inclined at a significant angle relative to the x-axis.

Geometric Implications and Visual Interpretation

Understanding the slope's magnitude provides a geometric perspective:

- Steepness: A slope of 234 suggests an extremely steep tangent, signaling rapid growth of the function near $x=3$.
- Behavior of the Curve: Since the function crosses the x-axis at $x=3$, and the slope is positive, the function is transitioning from negative to positive values or vice versa, depending on the overall shape.

Plotting this function near $x=3$ would reveal a sharp incline, confirming the calculated derivative's implications.

Broader Context: Why Derivatives Matter

The process of calculating derivatives, especially for composite functions like this, is central to understanding the local behavior of functions:

- Rate of Change: Derivatives quantify how a function’s output varies with input, essential in physics (velocity and acceleration), economics (marginal cost), and engineering.
- Tangents and Normals: The derivative at a point defines the tangent line’s slope, enabling the approximation of the function near that point.
- Optimization: Finding where derivatives equal zero helps locate maxima or minima, critical in decision-making processes.

In this specific case, the derivative's calculation underscores the importance of applying the product rule meticulously, considering each component's contribution.

Alternative Approaches and Additional Considerations

While the above method directly applies the product rule, alternative strategies or supplementary analyses could include:

- Graphical Analysis: Plotting the function to visually confirm the steepness at $x=3$.
- Second Derivative: Calculating Y'' to analyze concavity and points of inflection.
- Numerical Approximation: Using finite differences for approximate slopes when derivatives are complex.

Moreover, verifying the derivative through computational tools or symbolic calculators can ensure accuracy, especially for more complex functions.

Conclusion and Summary

This comprehensive investigation into the function $Y = (x^2 + 4)(x^3 - 9x)$ at the point $(3, 0)$ reveals that:

- The derivative Y' at $x=3$ is 234.
- The slope of the tangent line at $(3, 0)$ is 234, indicating a steep, positive inclination.
- Applying the product rule carefully is vital for accurate differentiation.
- The derivative provides deep insight into the local behavior of the function, reinforcing the foundational role of calculus in understanding and analyzing mathematical models.

This analysis not only satisfies the immediate goal of determining the tangent slope but also exemplifies the broader significance of derivative calculations in mathematical, scientific, and engineering contexts.

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