For Which Of The Potential Energies In Fig. 4.33 Could Psi(x) Be An Energy Eigenfunction? Sketch Any

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Understanding the conditions under which a wave function $\Psi(x)$ can serve as an energy eigenfunction is fundamental in quantum mechanics. In the context of the potential energies depicted in Fig. 4.33, analyzing whether a given $\Psi(x)$ corresponds to an eigenstate provides insight into the nature of quantum states, bound and scattering states, and the influence of potential profiles on the solutions of the Schrödinger equation. This article offers a comprehensive discussion on which potential energy configurations from Fig. 4.33 could support $\Psi(x)$ as an eigenfunction, along with illustrative sketches and detailed explanations.

Understanding the Concept of Energy Eigenfunctions in Quantum Mechanics

What Is an Energy Eigenfunction?

An energy eigenfunction $\Psi(x)$ is a solution to the time-independent Schrödinger equation:

$$
\hat{H}\Psi(x) = E\Psi(x)
$$

where $\hat{H}$ is the Hamiltonian operator, typically expressed as:

$$
\hat{H} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x)
$$

and E is the corresponding energy eigenvalue. For $\Psi(x)$ to be an eigenfunction:
- It must satisfy boundary conditions relevant to the potential.
- It must be normalizable (integrable with finite probability).
- It must be an eigenfunction of the Hamiltonian with a definite energy E.

The physical significance is that the system’s state has a definite energy, and such states form the basis for understanding quantum systems.

Relationship to Potential Energy Profiles

The shape and nature of the potential $V(x)$ critically determine the form of $\Psi(x)$:

- Bound states occur in potential wells, leading to discrete energy levels and normalizable $\Psi(x)$.
- Scattering states occur in free or repulsive potentials, with $\Psi(x)$ typically non-normalizable but physically meaningful in a scattering context.
- Potential barriers can support resonant states or tunneling phenomena, influencing the form of $\Psi(x)$.

Analyzing the Potentials in Fig. 4.33

While the actual figure is not provided here, typical potential energy diagrams in quantum mechanics include:

- Infinite potential well (box)
- Finite potential well
- Potential barrier
- Step potential
- Harmonic oscillator potential
- Double well potential

Each potential has characteristic solutions and implications for whether $\Psi(x)$ can be an eigenfunction.

Potential 1: Infinite Square Well

- Description: $V(x) = 0$ for $0 < x < a$, infinite elsewhere.
- Eigenfunctions: $\Psi_n(x) = \sqrt{\frac{2}{a}} \sin \left(\frac{n\pi x}{a} \right)$, $n=1,2,3,\ldots$.
- Conditions for $\Psi(x)$: Must vanish at the boundaries and be normalizable within the well.
- Likelihood: If $\Psi(x)$ matches a sine function within the well and zero outside, it can be an eigenfunction.

Potential 2: Finite Square Well

- Description: $V(x) = -V_0$ (a negative constant) within a finite region, zero outside.
- Eigenfunctions: Bound states are localized within the well, decaying exponentially outside.
- Conditions: $\Psi(x)$ must be continuous, finite, and decay outside the well.
- Likelihood: $\Psi(x)$ that matches decaying exponentials outside and sinusoidal within can be eigenfunctions.

Potential 3: Potential Barrier

- Description: $V(x)$ is higher in a region, acting as a barrier.
- Eigenfunctions: Scattering states with oscillatory behavior outside, possibly exponential decay within.
- Conditions: $\Psi(x)$ can represent incident, reflected, and transmitted waves, but must satisfy boundary conditions at the barrier edges.
- Likelihood: $\Psi(x)$ resembling plane waves or superpositions can be eigenfunctions for scattering states.

Potential 4: Step Potential

- Description: $V(x)$ changes abruptly at a point, e.g., from zero to a finite value.
- Eigenfunctions: Combinations of incident, reflected, and transmitted waves.
- Conditions: $\Psi(x)$ must match boundary conditions at the discontinuity.
- Likelihood: $\Psi(x)$ that are plane waves on either side can be eigenfunctions.

Potential 5: Harmonic Oscillator

- Description: $V(x) = \frac{1}{2}m\omega^2 x^2$.
- Eigenfunctions: Hermite functions multiplied by a Gaussian.
- Conditions: $\Psi(x)$ must decay at infinity and be normalizable.
- Likelihood: Hermite polynomial solutions times Gaussian are eigenfunctions.

Potential 6: Double Well or More Complex Potentials

- Description: Multiple minima, possibly supporting tunneling states.
- Eigenfunctions: Symmetric and antisymmetric combinations, localized in each well.
- Conditions: Must satisfy boundary conditions and be continuous.
- Likelihood: Well-structured wave functions matching these criteria can serve as eigenfunctions.

Determining Which Potentials Support $\Psi(x)$ as an Eigenfunction

To determine whether $\Psi(x)$ can be an eigenfunction for a particular potential $V(x)$, consider the following criteria:

1. Compatibility with the Schrödinger Equation

- $\Psi(x)$ must satisfy:

$$
-\frac{\hbar^2}{2m} \frac{d^2 \Psi}{dx^2} + V(x) \Psi = E \Psi
$$

\]

- Verify if the form of \(\Psi(x)\) fits this differential equation for the given \(V(x)\).

2. Boundary Conditions and Normalization

- \(\Psi(x)\) should vanish or be finite at boundaries.
- Must be normalizable: \(\int |\Psi(x)|^2 dx < \infty\).

3. Continuity and Differentiability

- \(\Psi(x)\) and its derivative should be continuous across potential discontinuities unless boundary conditions specify otherwise.

4. Physical Consistency

- The form of \(\Psi(x)\) should make physical sense given the potential's nature (e.g., decay in classically forbidden regions).

5. Matching with Known Eigenfunctions

- Comparing \(\Psi(x)\) to known solutions for standard potentials helps determine if it can be an eigenfunction.

Sketching the Wave Function \(\Psi(x)\) for Different Potentials

Visual representation aids understanding of how \(\Psi(x)\) behaves in various potential landscapes.

Infinite Potential Well

- \(\Psi(x)\): Sine wave confined within the well, zero outside.
- Sketch: Sinusoidal within \(0 < x < a\), zeros at boundaries.

Finite Potential Well

- \(\Psi(x)\): Sinusoidally oscillating inside the well, exponential decay outside.
- Sketch: Oscillations within the well; tails tapering off outside.

Potential Barrier

- $\Psi(x)$: Oscillatory incident and transmitted waves, exponential decay inside the barrier.
- Sketch: Plane wave approaching the barrier, partial reflection, and transmission with exponential decay inside.

Harmonic Oscillator

- $\Psi(x)$: Gaussian envelope multiplied by Hermite polynomials.
- Sketch: Symmetric bell-shaped curves with nodes at specific positions.

Step Potential

- $\Psi(x)$: Plane wave on one side, possibly reflected or transmitted wave on the other.
- Sketch: Continuous wavefunctions with different wavelengths on each side, matching at the interface.

Conclusion: Which Potential Profiles in Fig. 4.33 Support $\Psi(x)$ as an Eigenfunction?

Based on the analysis outlined, the potential energies in Fig. 4.33 that can support $\Psi(x)$ as an eigenfunction are those where $\Psi(x)$ satisfies the Schrödinger equation, boundary conditions, and normalization criteria.

Key Points:

- Potential Well (Infinite or Finite): If $\Psi(x)$ resembles a bound state wave function (oscillatory within the well, decaying outside), it can be an eigenfunction.
- Harmonic Oscillator Potential: $\Psi(x)$ matching Hermite-Gaussian forms can be eigenfunctions.
- Step or Barrier Potentials: $\Psi(x)$ resembling plane waves or exponential decay solutions can serve as scattering eigenstates.
- Complex or Double Wells: $\Psi(x)$ localized in a well or symmetric/antisymmetric combinations can be eigenfunctions.

Sketches of such wave functions illustrate their behavior across the potential landscape

Frequently Asked Questions

Which potential energies in Fig. 4.33 allow the wavefunction

Psi(x) to be an energy eigenfunction?

Potential energies where Psi(x) satisfies the time-independent Schrödinger equation as an eigenfunction—typically regions with well-defined, stationary potentials—allow Psi(x) to be an energy eigenfunction. For example, bound states in finite or infinite potential wells or constant potential regions typically support eigenfunctions.

How does the shape of Psi(x) relate to the potential energy in Fig. 4.33 for it to be an eigenfunction?

If Psi(x) corresponds to a stationary state, it must satisfy the Schrödinger equation with the given potential, meaning its shape should match the form of solutions (e.g., sinusoidal in free regions, exponential decay in classically forbidden regions). Regions where Psi(x) is sinusoidal or exponential, consistent with the potential, indicate that Psi(x) could be an eigenfunction.

Can Psi(x) be an eigenfunction in regions where the potential energy is discontinuous in Fig. 4.33? Why or why not?

Yes, Psi(x) can be an eigenfunction in regions with discontinuous potential energies, provided it satisfies boundary conditions at the discontinuities (continuity of Psi(x) and its derivative). Such piecewise solutions are common in quantum systems with step potentials or finite barriers.

Is it possible for Psi(x) to be an eigenfunction in a potential energy region that is not constant? How would its form look like?

Yes, Psi(x) can be an eigenfunction in non-constant potentials, but its form will generally be more complex, often involving special functions or numerical solutions. The key requirement is that Psi(x) satisfies the Schrödinger equation with the given potential and boundary conditions.

What features of Fig. 4.33 would help determine whether Psi(x) is an eigenfunction for a particular potential?

Features such as the shape of Psi(x) (oscillatory, exponential decay), boundary conditions, and how it matches the potential energy profile help determine if it is an eigenfunction. Specifically, stationary states typically have well-behaved, normalized wavefunctions that satisfy the boundary conditions imposed by the potential.

Additional Resources

For Which Of The Potential Energies In Fig. 4.33 Could Psi(x) Be An Energy Eigenfunction? Sketch Any — this question invites a deep exploration of quantum mechanical wavefunctions in various potential energy landscapes. It’s a fundamental inquiry into how the form of a wavefunction, denoted by Psi(x), relates to the underlying potential energy function V(x). Understanding this relationship is crucial for solving the Schrödinger equation and interpreting physical systems, from bound states in atoms to particles in potential wells.

In this article, we will analyze the potential energies depicted in Fig. 4.33, examining which of these could support Psi(x) as an energy eigenfunction. We will explore the characteristics of wavefunctions that correspond to different potential profiles, how boundary conditions influence possible energy eigenstates, and provide sketches illustrating these concepts. This comprehensive guide aims to clarify the conditions under which a given wavefunction can be an eigenfunction of the Hamiltonian for various potentials.

Understanding the Relationship Between Psi(x) and V(x)

Before diving into specific potentials, it's essential to recall that in quantum mechanics, the wavefunction Psi(x) of a particle satisfies the time-independent Schrödinger equation:

$$
-\frac{\hbar^2}{2m} \frac{d^2 \Psi(x)}{dx^2} + V(x) \Psi(x) = E \Psi(x)
$$

where:
- $\hbar$ is the reduced Planck constant,
- m is the particle's mass,
- $V(x)$ is the potential energy function,
- E is the energy eigenvalue associated with $\Psi(x)$.

For $\Psi(x)$ to be an eigenfunction, it must satisfy this differential equation with some real eigenvalue E. The form of $\Psi(x)$ and the nature of $V(x)$ are intimately connected: specific potential shapes support particular wavefunctions and energy spectra.

General Criteria for Psi(x) to be an Energy Eigenfunction

Key properties of eigenfunctions:

- Bound states: The wavefunction is normalizable; it tends to zero as $x \to \pm \infty$.
- Continuity and differentiability: $\Psi(x)$ must be continuous and have a continuous first derivative, except possibly at points where the potential is infinite.
- Matching boundary conditions: For confined systems, $\Psi(x)$ must vanish at the boundaries or decay exponentially outside the potential well.
- Nodal structure: The number of nodes in $\Psi(x)$ corresponds to the principal quantum number for bound states.

Analyzing the Potentials in Fig. 4.33

Suppose Fig. 4.33 depicts several potential energy functions $V(x)$. These could include:

1. Infinite Square Well (Box)
2. Finite Square Well
3. Harmonic Oscillator (Parabolic Potential)

4. Step Potential
5. Potential Barrier
6. Potential Well with Finite or Infinite Walls
7. Other composite potentials (e.g., double wells)

Our goal is to determine, for each potential, whether the provided $\Psi(x)$ could be an eigenfunction.

Step-by-Step Analysis

1. Infinite Square Well

Potential Profile:
- $V(x) = 0$ for $0 < x < a$,
- $V(x) = \infty$ elsewhere.

Wavefunction Characteristics:
- $\Psi(x)$ must be zero at the walls $x=0$ and $x=a$.
- Inside, it takes the form of sine functions: $\Psi_n(x) = \sqrt{\frac{2}{a}} \sin \left(\frac{n \pi x}{a} \right)$.

Could $\Psi(x)$ be an eigenfunction?
- If $\Psi(x)$ is zero outside the well and sinusoidal inside, it could be an energy eigenfunction.
- If $\Psi(x)$ does not vanish at the boundaries or is non-zero outside, it cannot be an eigenfunction of the infinite well.

Sketch:
- A sine wave confined strictly within $0 < x < a$, zero at boundaries.

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2. Finite Square Well

Potential Profile:
- $V(x) = 0$ inside a region $[-a/2, a/2]$,
- $V(x) = V_0$ outside, with V_0 finite.

Wavefunction Characteristics:
- Inside, $\Psi(x)$ resembles sinusoidal or exponential decay depending on the energy.
- Outside, $\Psi(x)$ decays exponentially if bound.

Could $\Psi(x)$ be an eigenfunction?
- Yes, if $\Psi(x)$ matches the form of solutions for bound states: sinusoidal inside and exponential decay outside, with continuity conditions satisfied.

Sketch:
- Oscillatory within the well, exponential tail outside.

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3. Harmonic Oscillator

Potential Profile:
- $V(x) = \frac{1}{2} m \omega^2 x^2$, a parabola.

Wavefunction Characteristics:
- The eigenfunctions are Hermite polynomials multiplied by a Gaussian: $\Psi_n(x) \propto H_n(\sqrt{\alpha} x) e^{-\alpha x^2/2}$.

Could $\Psi(x)$ be an eigenfunction?
- Yes, if $\Psi(x)$ resembles a Gaussian multiplied by a polynomial with the proper nodal structure.

Sketch:
- Bell-shaped, possibly with nodes depending on the energy level.

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4. Step Potential and Barriers

Potential Profile:
- Sudden changes in $V(x)$ at certain points.

Wavefunction Characteristics:
- $\Psi(x)$ may be oscillatory in regions where $E > V(x)$ and exponential where $E < V(x)$.

Could $\Psi(x)$ be an eigenfunction?
- Yes, if it satisfies boundary conditions and the matching conditions at discontinuities.

Sketch:
- Oscillations in allowed regions, decay in forbidden regions.

Conditions Where $\Psi(x)$ Cannot Be an Eigenfunction

- If $\Psi(x)$ does not satisfy boundary conditions (e.g., does not vanish at infinite walls or boundaries).
- If $\Psi(x)$ is not normalizable (e.g., does not decay sufficiently at infinity).
- If $\Psi(x)$ has discontinuities or non-physical behavior at potential discontinuities.
- If the shape of $\Psi(x)$ does not match the expected form of solutions for the potential in question.

Sketching $\Psi(x)$ for Potential Types

Infinite Well:
- Draw a sine wave confined between two vertical lines, zero at the boundaries, with nodes inside.

Finite Well:
- Similar to the infinite well but with exponential tails outside, wavefunction peaks inside.

Harmonic Oscillator:
- Bell-shaped curve with polynomial modulation, with nodes at higher energies.

Step Potential:
- Oscillatory in the allowed region, exponentially decaying in forbidden regions, with a sharp change at the step.

Summary Table

Potential Type	\(\Psi(x)\) form support	Key features for eigenfunction	Boundary conditions	Typical sketch
Infinite Square Well	Yes	Sinusoidal, zero at boundaries	Zero at edges	Sinusoid confined between walls
Finite Square Well	Yes	Sinusoidal + exponential tail	Continuity at boundaries	Oscillations with decay outside
Harmonic Oscillator	Yes	Gaussian modulated by Hermite polynomials	Decay at infinity	Bell-shaped with nodes
Step Potential	Yes	Oscillatory + decay in forbidden regions	Match at interface	Sinusoid + exponential tail
Potential Barrier	No	Not support bound states in barrier	N/A	Wavefunction decays or transmits

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Final Remarks

The question "For which of the potential energies in Fig. 4.33 could \(\Psi(x)\) be an energy eigenfunction?" hinges on matching the features of \(\Psi(x)\) with the characteristics of solutions to the Schrödinger equation for each potential profile.

- If \(\Psi(x)\) aligns with the expected mathematical form—satisfies boundary conditions, is normalizable, and matches the physical constraints—it can be an eigenfunction.
- If it does not, then it is not an eigenfunction, although it might be a superposition of eigenstates or a non-stationary state.

In practice, sketching \(\Psi(x)\) in relation to the potential helps visualize these relationships. For example, a sinusoidal wave confined within a well suggests an eigenstate of that well, whereas a wavefunction that extends infinitely without decay might not correspond to a bound state.

In conclusion, by examining the shape and boundary behavior of \(\Psi(x)\) alongside the potential profiles, one can determine which potentials in Fig. 4.33 support \(\Psi(x)\) as an energy eigenfunction. This process exemplifies the core principles of

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Be An Energy Eigenfunction Sketch Any

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