

Force $\mathbf{F} = (-6.0 \text{ N}) \mathbf{i} + (3.7 \text{ N}) \mathbf{j}$ Acts On A Particle With Position Vector $\mathbf{R} = (2.7 \text{ m}) \mathbf{i} + (4.4 \text{ m}) \mathbf{j}$. What

Force $\mathbf{F} = (-6.0 \text{ N}) \mathbf{i} + (3.7 \text{ N}) \mathbf{j}$ Acts On A Particle With Position Vector $\mathbf{R} = (2.7 \text{ m}) \mathbf{i} + (4.4 \text{ m}) \mathbf{j}$. What

Understanding the interaction between a force and a particle's position vector is fundamental in physics, especially in classical mechanics. The given force vector $\mathbf{F} = -6.0\mathbf{i} + 3.7\mathbf{j}$ Newtons and the position vector $\mathbf{R} = 2.7\mathbf{i} + 4.4\mathbf{j}$ meters provide a basis for analyzing the particle's behavior, the work done by the force, and the physical significance of these vectors.

This article delves into the mathematical and physical interpretation of these vectors, explores their relationships, and discusses relevant concepts such as vector dot products, work calculation, and the significance of the force's direction relative to the particle's position.

Understanding the Vectors: Force and Position

Vector Representation in Physics

In physics, vectors are mathematical quantities characterized by both magnitude and direction. They are essential for describing quantities like force, velocity, acceleration, and position in space.

- Force Vector $\mathbf{F}$: Represents the magnitude and direction of the force acting on an object.
- Position Vector $\mathbf{R}$: Represents the location of the particle relative to a reference point, usually the origin.

Both vectors are expressed in component form, typically along the Cartesian axes, denoted as $\mathbf{i}$ (x-axis) and $\mathbf{j}$ (y-axis).

Analyzing the Given Vectors

Force Vector $\mathbf{F}$

$$\mathbf{F} = -6.0\mathbf{i} + 3.7\mathbf{j}$$

- Magnitude of $\mathbf{F}$:

The magnitude $|\mathbf{F}|$ can be calculated using the Pythagorean theorem:

$$|\mathbf{F}| = \sqrt{(-6.0)^2 + (3.7)^2} = \sqrt{36 + 13.69} = \sqrt{49.69} \approx 7.05 \text{ N}$$

- Direction of $\mathbf{F}$:

The angle θ_F relative to the positive x-axis:

$$\theta_F = \arctan\left(\frac{3.7}{-6.0}\right) \approx \arctan(-0.6167) \approx -31.7^\circ$$

Since the x-component is negative and the y-component is positive, the force vector points toward the second quadrant, at approximately $(180^\circ - 31.7^\circ = 148.3^\circ)$.

Position Vector $\mathbf{R}$

$$\mathbf{R} = 2.7\mathbf{i} + 4.4\mathbf{j}$$

- Magnitude of $\mathbf{R}$:

$$|\mathbf{R}| = \sqrt{(2.7)^2 + (4.4)^2} = \sqrt{7.29 + 19.36} = \sqrt{26.65} \approx 5.16 \text{ m}$$

- Direction of $\mathbf{R}$:

$$\theta_R = \arctan\left(\frac{4.4}{2.7}\right) \approx \arctan(1.629) \approx 58.0^\circ$$

This indicates the position vector points roughly at (58°) above the positive x-axis.

Physical Interpretation of the Vectors

The Significance of Force and Position

- The force vector's components suggest it acts predominantly in the negative x-direction and positive y-direction.
- The position vector indicates the particle's location is in the first quadrant, somewhat above the x-axis.

Understanding these vectors' orientations helps infer how the force influences the particle's motion, the work done, and the potential energy changes.

Calculating the Work Done by the Force

Work as a Dot Product

The work (W) done by a force $(\mathbf{F})$ when moving a particle from the origin to position $(\mathbf{R})$ is given by the dot product:

$$W = \mathbf{F} \cdot \mathbf{R}$$

This scalar quantity measures how much of the force acts in the direction of displacement, indicating the energy transferred.

Performing the Dot Product Calculation

Given:

$$\mathbf{F} = -6.0\mathbf{i} + 3.7\mathbf{j}$$
$$\mathbf{R} = 2.7\mathbf{i} + 4.4\mathbf{j}$$

Calculating:

$$\begin{aligned} & \backslash[\\ W &= (-6.0)(2.7) + (3.7)(4.4) = -16.2 + 16.28 = 0.08 \backslash, \text{\texttt{J}} \\ & \backslash] \end{aligned}$$

Interpretation:

- The work done by the force in moving the particle from the origin to the position vector is approximately 0.08 Joules, which is very small.
- The positive value suggests a slight component of the force acts in the same direction as the displacement, contributing marginally to the particle's energy.

Implications and Further Analysis

Directionality and Force-Displacement Relationship

- The nearly zero work indicates that the force has a minimal component in the direction of the particle's position vector.
- The force's direction, being in the second quadrant, forms an angle with the displacement vector, which is at about 58 degrees in the first quadrant.

Potential for Further Calculations

- Work done over different paths: Since work depends on the path in the presence of non-conservative forces, analyzing different trajectories could be insightful.
- Particle dynamics: Using Newton's second law, $\mathbf{F} = m \mathbf{a}$, one could determine acceleration, velocity, and motion over time.
- Energy considerations: The work-energy theorem relates the work done to the change in kinetic energy of the particle.

Conclusion

The vectors provided for force and position offer a comprehensive view of the particle's environment and interactions. Calculating magnitudes and directions helps understand how the force acts relative to the particle's location, and the dot product reveals the small amount of work involved in moving the particle from the origin to its position.

Understanding these fundamental vector operations and their physical meanings is crucial in classical mechanics, enabling scientists and engineers to analyze systems, predict motion, and design mechanisms effectively. Whether in analyzing simple particle motion or complex engineering systems, the principles illustrated by these vectors serve as foundational tools for deeper insights into the physical world.

Frequently Asked Questions

What is the magnitude of the force F acting on the particle?

The magnitude of force F is calculated as $\sqrt{(-6.0 \text{ N})^2 + (3.7 \text{ N})^2} \approx 7.03 \text{ N}$.

What is the direction (angle) of the force F relative to the positive x-axis?

The angle $\theta = \arctangent(3.7 / -6.0) \approx -31.8^\circ$, which indicates the force points left and slightly upward.

Is the force F acting in the same direction as the position vector R ?

No, the force vector F points in a different direction than the position vector R ; F points left and up, while R points right and up.

What is the dot product of the force F and the position vector R ?

The dot product is $(-6.0 \text{ N})(2.7 \text{ M}) + (3.7 \text{ N})(4.4 \text{ M}) = -16.2 + 16.28 = 0.08 \text{ N}\cdot\text{m}$, nearly zero, indicating almost orthogonal vectors.

What is the physical significance of the dot product between F and R in this context?

The dot product represents the work done or energy transfer if F acts along R ; a near-zero value suggests minimal work in the direction of R .

How would you compute the torque τ about the origin due to force F acting at position R ?

Torque $\tau = R \times F = (2.7 \text{ M})(3.7 \text{ N}) - (4.4 \text{ M})(-6.0 \text{ N})$ in k-component form, resulting in $\tau_z = (2.7)(3.7) - (4.4)(-6.0) \approx 10.0 + 26.4 = 36.4 \text{ N}\cdot\text{m}$.

Additional Resources

Force $\mathbf{F} = (-6.0 \text{ N}) \mathbf{i} + (3.7 \text{ N}) \mathbf{j}$ Acts On A Particle With Position Vector $\mathbf{R} = (2.7 \text{ M}) \mathbf{i} + (4.4 \text{ M}) \mathbf{j}$ – this scenario presents a classic problem in vector calculus and physics, involving the analysis of forces acting on particles within a coordinate system. Understanding how to interpret and manipulate such vector quantities is fundamental in mechanics, electromagnetism, and related fields. In this comprehensive guide, we'll break down the problem step by step, explore the relevant concepts, and demonstrate how to analyze and compute the various parameters involved.

Introduction to Vector Quantities in Physics

In physics, many quantities such as force, velocity, acceleration, and position are represented as vectors. Vectors encapsulate both magnitude and direction, making them essential for describing physical phenomena in three-dimensional space.

Why Use Vectors?

- Directional Representation: Vectors specify the exact direction of a quantity, not just its size.
- Component Analysis: They allow for decomposition into components, simplifying calculations.
- Mathematical Operations: Vectors can be added, subtracted, and dot or cross-multiplied to analyze physical systems.

Breakdown of the Given Data

Let's analyze the problem statement:

- Force vector, $\mathbf{F}$: $\mathbf{F} = -6.0 \text{ N} \mathbf{i} + 3.7 \text{ N} \mathbf{j}$
- Position vector, $\mathbf{R}$: $\mathbf{R} = 2.7 \text{ M} \mathbf{i} + 4.4 \text{ M} \mathbf{j}$

Here, the vectors are expressed in component form using unit vectors $\mathbf{i}$ (x-direction) and $\mathbf{j}$ (y-direction).

Understanding the Physical Context

The problem involves a force acting on a particle located at a certain position. Several questions naturally arise:

- Is the force derived from a potential or related to a field?

- What is the relationship between this force and the position vector?
- Are we asked to find a component of the force, the work done, or the potential energy?

In many cases, such problems relate to calculating work, understanding the force's direction relative to the position vector, or determining the force's effect on the particle's motion.

Step 1: Visualizing the Vectors

Plotting the vectors on a coordinate plane can help visualize their directions and magnitudes:

- Position vector $\mathbf{R}$: Starts at the origin, extends to the point (2.7 m, 4.4 m).
- Force vector $\mathbf{F}$: Points in the negative x-direction (since the x-component is -6.0 N) and positive y-direction (3.7 N).

This visualization aids in understanding the relative directions and potential interactions.

Step 2: Calculating the Magnitude of the Force

The magnitude of the force vector $\mathbf{F}$ is vital:

$$|\mathbf{F}| = \sqrt{(-6.0)^2 + (3.7)^2} \text{ N}$$

Calculating:

$$|\mathbf{F}| = \sqrt{36.0 + 13.69} = \sqrt{49.69} \approx 7.05 \text{ N}$$

This gives us an overall measure of the force's strength.

Step 3: Computing the Magnitude of the Position Vector

Similarly, the magnitude of $\mathbf{R}$:

$$|\mathbf{R}| = \sqrt{(2.7)^2 + (4.4)^2} \text{ m}$$

Calculating:

$$|\mathbf{R}| = \sqrt{7.29 + 19.36} = \sqrt{26.65} \approx 5.16, \text{ m}$$

Step 4: Determining the Angle Between Force and Position Vectors

Finding the angle θ between $\mathbf{F}$ and $\mathbf{R}$:

$$\cos \theta = \frac{\mathbf{F} \cdot \mathbf{R}}{|\mathbf{F}| |\mathbf{R}|}$$

Dot product calculation:

$$\mathbf{F} \cdot \mathbf{R} = (-6.0)(2.7) + (3.7)(4.4) = -16.2 + 16.28 = 0.08, \text{ N}\cdot\text{m}$$

Calculating $\cos \theta$:

$$\cos \theta = \frac{0.08}{7.05 \times 5.16} \approx \frac{0.08}{36.42} \approx 0.0022$$

Angle θ :

$$\theta = \arccos(0.0022) \approx 89.9^\circ$$

This indicates the force is nearly perpendicular to the position vector.

Step 5: Interpreting the Results

Force and Position Relationship:

Given the near 90° angle, the force acts almost perpendicular to the position vector. This is typical in scenarios involving magnetic forces or certain field interactions where the force is perpendicular to displacement, implying no work is done in this configuration.

Work Done Calculation:

If asked to compute work done by the force when moving the particle from the origin to position $\mathbf{R}$:

$$W = \mathbf{F} \cdot \mathbf{R}$$

Using the dot product:

$$W = 0.08 \text{ N}\cdot\text{m}$$

Since the work is very close to zero, the force does negligible work in moving the particle along this path.

Additional Considerations

1. Vector Projection:

Sometimes, it's necessary to find the component of the force along the displacement:

$$F_{\parallel} = |\mathbf{F}| \cos \theta \approx 7.05 \times 0.0022 \approx 0.0155 \text{ N}$$

This tiny component indicates minimal influence along the displacement direction.

2. Force Components:

Breaking down the force into components:

$$\begin{aligned} F_x &= -6.0 \text{ N} \\ F_y &= 3.7 \text{ N} \end{aligned}$$

Similarly, the position components:

$$\begin{aligned} R_x &= 2.7 \text{ m} \\ R_y &= 4.4 \text{ m} \end{aligned}$$

Practical Applications of Such Analysis

Understanding forces in vector form is crucial in many physical systems. Let's explore some practical contexts where similar analysis applies:

a) Magnetic Forces on Moving Charges

Magnetic force $\mathbf{F} = q \mathbf{v} \times \mathbf{B}$ is always perpendicular to velocity, leading to circular or helical motion. Analyzing the force's direction relative to displacement helps in understanding particle trajectories.

b) Electric Fields and Potential Energy

In electrostatics, force vectors derived from electric fields influence the potential energy stored in the system, which can be calculated from the work done by forces along paths.

c) Mechanical Systems and Work-Energy Theorem

Calculating work done by forces during motion helps determine changes in kinetic energy, vital in designing mechanical systems.

Summary and Key Takeaways

- Vector analysis provides a powerful framework for understanding physical phenomena involving direction and magnitude.
- Calculating dot products helps determine the angle between vectors and the work done.
- In the given problem, the force acts nearly perpendicular to the position vector, implying minimal work is done in displacing the particle.
- Recognizing the physical interpretation of vector components and their relationships enhances comprehension of complex systems.

Final Thoughts

Mastering vector operations and their physical interpretations is fundamental for physicists, engineers, and anyone engaged in analyzing systems with directional quantities. As demonstrated, breaking down complex problems into manageable steps—visualization, calculation, interpretation—provides clarity and insight, enabling precise and meaningful conclusions about the physical world.

Remember: Whether you're working on particle dynamics, electromagnetic forces, or mechanical systems, a solid grasp of vector calculus is your most valuable tool in unlocking the secrets of the universe.

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