Given F Of X Is Equal To The Quantity X Plus 3 End Quantity Over The Quantity X Squared Plus 2 Times

Given F Of X Is Equal To The Quantity X Plus 3 End Quantity Over The Quantity X Squared Plus 2 Times is a mathematical expression that often appears in calculus and algebra. Understanding this function, its properties, and how to manipulate it is essential for students and professionals working in mathematics, engineering, economics, and related fields. This article aims to explore this function in detail, covering its domain, range, derivatives, integrals, and applications, all while optimizing for SEO to ensure clarity and accessibility.

Understanding the Function: Given F Of X Is Equal To The Quantity X Plus 3 End Quantity Over The Quantity X Squared Plus 2 Times

Breaking Down the Function

The given function can be written more explicitly as:

$$F(x) = (x + 3) / (x^2 + 2)$$

This is a rational function, where the numerator is a linear expression (x + 3) and the denominator is a quadratic expression ($x^2 + 2$). Rational functions are crucial in calculus because they often involve asymptotic behavior, limits, and derivatives that reveal the function's increasing and decreasing intervals.

Key Characteristics of the Function

- Domain: The set of all real numbers except where the denominator is zero.
- Range: All real values that F(x) can take based on the domain.
- Asymptotes: Vertical asymptotes where the denominator approaches zero, and possibly horizontal or oblique asymptotes based on the degree of numerator and denominator.

Determining the Domain and Range of the Function

Domain of F(x)

Since the function involves a denominator, it is undefined where the denominator equals zero:

$$x^2 + 2 = 0$$

Because $x^2 + 2 = 0$ implies $x^2 = -2$, which has no real solutions, the denominator is never zero for real x. Therefore, the domain of F(x) is all real numbers:

- **Domain:** ℝ (all real numbers)

This is a significant property because the function is defined everywhere on the real number line, simplifying many calculus operations.

Range of F(x)

To find the range, analyze the possible output values of the function. Since the numerator and denominator are both polynomial expressions, the function can take various real values. To determine the range, consider the limits as x approaches infinity and negative infinity:

- As $x \to \pm\infty$, the dominant terms are x in numerator and x^2 in denominator, so:

$$F(x) \approx x / x^2 = 1 / x \to 0$$

Thus, the horizontal asymptote is $y = 0$.

To find if F(x) can actually equal 0, set $F(x) = 0$:

$$(x + 3) / (x^2 + 2) = 0$$

which implies:

$$x + 3 = 0 \to x = -3$$

Since $x = -3$ is in the domain, the function can take the value 0 at $x = -3$.

Therefore, the range includes 0, but what about other values? To analyze further, consider solving for x in terms of y:

$$F(x) = y \to (x + 3) = y(x^2 + 2)$$

which simplifies to:

$$x + 3 = yx^2 + 2y$$

Rearranged as a quadratic in x:

$$yx^2 - x + (2y - 3) = 0$$

For real solutions, the discriminant must be non-negative:

$$\Delta = (-1)^2 - 4 y (2y - 3) \geq 0$$

Calculating:

$$\Delta = 1 - 4y(2y - 3) = 1 - 8y^2 + 12y$$

Set this ≥ 0:

$$1 - 8y^2 + 12y \geq 0$$

This quadratic inequality in y can be solved to find the range of y values. Solving:

$1 - 8y^2 + 12y \geq 0$
$\Rightarrow -8y^2 + 12y + 1 \geq 0$

Multiply through by -1 (reversing inequality):

$8y^2 - 12y - 1 \leq 0$

Now, solve the quadratic:

$8y^2 - 12y - 1 = 0$

Discriminant:

$D = (-12)^2 - 4 \, 8 \, (-1) = 144 + 32 = 176$

Square root of D:

$\sqrt{176} \approx 13.266$

Solutions for y:

$$y = [12 \pm \sqrt{176}] / (2 \, 8) = [12 \pm 13.266] / 16$$

Calculate:

- y_1:

```
y = (12 + 13.266) / 16 ≈ 25.266 / 16 ≈ 1.579
```

- y_2:

```
y = (12 - 13.266) / 16 ≈ -1.266 / 16 ≈ -0.079
```

Since the quadratic in y is ≤ 0 between these roots, the range of F(x) is approximately:

- **Range:** $y \in [-0.079, 1.579]$

Thus, the function outputs all values between approximately -0.079 and 1.579.

Calculus of the Function: Derivatives and Critical Points

Finding the Derivative of F(x)

The derivative of a rational function can be computed using the quotient rule:

$$\text{If } F(x) = u(x) / v(x), \text{ then } F'(x) = (u'v - uv') / v^2$$

Here,

- $u(x) = x + 3 \Rightarrow u'(x) = 1$
- $v(x) = x^2 + 2 \Rightarrow v'(x) = 2x$

Applying the quotient rule:

$$F'(x) = [1 \ (x^2 + 2) - (x + 3) \ 2x] / (x^2 + 2)^2$$

Simplify numerator:

- First term: $x^2 + 2$
- Second term: $2x(x + 3) = 2x^2 + 6x$

Thus,

$$F'(x) = [x^2 + 2 - (2x^2 + 6x)] / (x^2 + 2)^2 = (-x^2 - 6x + 2) / (x^2 + 2)^2$$

Critical points occur where numerator = 0:

$$-x^2 - 6x + 2 = 0$$

Multiply both sides by -1:

$$x^2 + 6x - 2 = 0$$

Use quadratic formula:

$$x = [-6 \pm \sqrt{(36 - 4 \ 1 \ (-2))}] / 2 = [-6 \pm \sqrt{(36 + 8)}] / 2 = [-6 \pm \sqrt{44}] / 2$$

Simplify $\sqrt{44}$:

$\sqrt{44} \approx 6.633$

Calculate:

- $x \approx [-6 + 6.633] / 2 \approx 0.633 / 2 \approx 0.317$
- $x \approx [-6 - 6.633] / 2 \approx -12.633 / 2 \approx -6.317$

These are the critical points where the function may have local maxima or minima.

Interpreting the Derivative

- For $x < -6.317$, $F'(x) > 0$ (function increasing)
- Between -6.317 and 0.317, $F'(x) < 0$ (function decreasing)
- For $x > 0.317$, $F'(x) > 0$ (function increasing)

This behavior indicates a local maximum at $x \approx -6.317$ and a local minimum at $x \approx 0.317$.

Integrating the Function: Area Under the Curve

Calculating the Indefinite Integral

The indefinite integral of $F(x)$:

$$\int F(x)\,dx = \int (x + 3) / (x^2 + 2)\,dx$$

To evaluate this integral, consider substitution methods:

- Split the integral:

Frequently Asked Questions

What is the function F of X given as (X + 3) over (X squared + 2)?

The function F of X is defined as $F(X) = (X + 3) / (X^2 + 2)$.

How can I find the domain of the function F(X) = (X + 3) / (X^2 + 2)?

Since the denominator $X^2 + 2$ is always positive (never zero for real X), the domain is all real numbers, $\mathbb{R}$.

What is the derivative of $F(X) = (X + 3) / (X^2 + 2)$?

Using the quotient rule, $F'(X) = [(1)(X^2 + 2) - (X + 3)(2X)] / (X^2 + 2)^2$, which simplifies to $(X^2 + 2 - 2X^2 - 6X) / (X^2 + 2)^2 = (-X^2 - 6X + 2) / (X^2 + 2)^2$.

How do I find the critical points of F(X)?

Set the derivative F'(X) to zero: $(-X^2 - 6X + 2) / (X^2 + 2)^2 = 0$. This occurs when the numerator is zero: $-X^2 - 6X + 2 = 0$, which simplifies to $X^2 + 6X - 2 = 0$. Solve this quadratic to find critical points.

What is the behavior of F(X) as X approaches infinity?

As X approaches infinity, the dominant terms are X in the numerator and X^2 in the denominator, so F(X) approaches 0 from the positive side.

Is the function F(X) continuous and differentiable everywhere?

Yes. Since the denominator $X^2 + 2$ is never zero, F(X) is continuous and differentiable for all real numbers.

How can I find the local maxima or minima of F(X)?

Find the critical points by setting the derivative to zero and determine their nature using the second derivative test or the first derivative sign change analysis.

What is the value of F(X) at X = 0?

$F(0) = (0 + 3) / (0^2 + 2) = 3 / 2 = 1.5$.

Can I simplify F(X) further?

No, the function is already in its simplest form as a rational expression. No common factors can be canceled.

Additional Resources

Given F of X Is Equal To The Quantity X Plus 3 End Quantity Over The Quantity X Squared Plus 2 Times

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Investigative Analysis of the Function \(F(x) = \frac{x + 3}{x^2 + 2} \)

Introduction

Mathematics, especially calculus and algebra, forms the backbone of many scientific and engineering disciplines. Among the fundamental concepts are functions—rules that assign exactly one output to each input. The function in focus, $F(x) = \frac{x + 3}{x^2 + 2}$, presents a compelling case study for analysis due to its rational form, domain considerations, and potential applications.

This detailed review aims to explore the properties, behavior, and significance of this function through a comprehensive investigative lens. Such an exploration not only enhances understanding but also demonstrates the function's relevance in real-world contexts and advanced mathematical applications.

Dissecting the Function: An Overview

The Basic Structure

The function $F(x) = \frac{x + 3}{x^2 + 2}$ is a rational function, characterized by a polynomial numerator and denominator. Its form suggests certain inherent features:

- Numerator: $x + 3$, a linear polynomial.
- Denominator: $x^2 + 2$, a quadratic polynomial with no real roots.

Domain Analysis

The domain of $F(x)$ is all real numbers where the denominator is non-zero:

$$x^2 + 2 \neq 0$$

Since $x^2 \geq 0$ for all real x, and adding 2 ensures:

$$x^2 + 2 \geq 2 > 0$$

Therefore, the denominator never zeroes out, and the domain is:

$$\boxed{
\text{Domain} = (-\infty, \infty)
}$$

This is a significant property, indicating the function is defined everywhere on the real line.

Deep Dive: Behavior and Characteristics

1. End Behavior and Horizontal Asymptotes

Understanding the limits of $F(x)$ as $x \to \pm \infty$ reveals how the function behaves at the extremities of its domain.

- As $x \to \infty$:

$$
F(x) \approx \frac{x}{x^2} = \frac{1}{x} \to 0
$$

- As $x \to -\infty$:

$$
F(x) \approx \frac{x}{x^2} = \frac{1}{x} \to 0
$$

Conclusion: The function approaches zero from both sides, indicating a horizontal asymptote at:

$$
\boxed{
y = 0
}
$$

2. Symmetry and Parity

Examining whether $F(x)$ is even, odd, or neither:

- Check $F(-x)$:

$$
F(-x) = \frac{-x + 3}{(-x)^2 + 2} = \frac{-x + 3}{x^2 + 2}
$$

- Compare to $F(x)$:

$$
F(x) = \frac{x + 3}{x^2 + 2}
$$

- Is $F(-x) = F(x)$? No, unless $x = 0$, so the function is not even.

- Is $F(-x) = -F(x)$?

$$
- F(x) = - \frac{x + 3}{x^2 + 2} = \frac{-x - 3}{x^2 + 2}
$$

which is not equal to $F(-x)$. Therefore, the function is neither even nor odd.

3. Critical Points and Extrema

To find the local maxima or minima, compute the derivative $F'(x)$:

$$
F(x) = \frac{x + 3}{x^2 + 2}
$$

Using the quotient rule:

$$
F'(x) = \frac{(1)(x^2 + 2) - (x + 3)(2x)}{(x^2 + 2)^2}
$$

Simplify numerator:

$$
x^2 + 2 - 2x(x + 3) = x^2 + 2 - 2x^2 - 6x = -x^2 - 6x + 2
$$

Thus:

$$
F'(x) = \frac{-x^2 - 6x + 2}{(x^2 + 2)^2}
$$

Critical points occur where numerator zero:

$$
-x^2 - 6x + 2 = 0
$$

Multiply through by -1:

$$
x^2 + 6x - 2 = 0
$$

Apply quadratic formula:

$$
x = \frac{-6 \pm \sqrt{36 - 4 \times 1 \times (-2)}}{2} = \frac{-6 \pm \sqrt{36 + 8}}{2} = \frac{-6 \pm \sqrt{44}}{2}
$$

Simplify:

$$
x = \frac{-6 \pm 2 \sqrt{11}}{2} = -3 \pm \sqrt{11}
$$

Critical points at:

$$
x = -3 + \sqrt{11} \quad \text{and} \quad x = -3 - \sqrt{11}
$$

\]

In-Depth: Nature of Critical Points

To classify these critical points, evaluate the second derivative or analyze the first derivative's sign changes.

1. Sign of $F'(x)$:

- For x less than $-3 - \sqrt{11}$, numerator $-x^2 - 6x + 2$ is positive or negative?

- For x between the critical points and beyond, similar analysis applies.

2. Numerical Approximation:

$$
\sqrt{11} \approx 3.317
$$

- First critical point:

$$
x \approx -3 + 3.317 \approx 0.317
$$

- Second critical point:

$$
x \approx -3 - 3.317 \approx -6.317
$$

Interpretation:

- At $x \approx -6.317$, the function may have a local maximum or minimum.

- At $x \approx 0.317$, the function may have the opposite extremum.

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4. Function Values at Critical Points

Calculate $F(x)$ at these points:

- At $x \approx -6.317$:

$$
F(-6.317) \approx \frac{-6.317 + 3}{(-6.317)^2 + 2} = \frac{-3.317}{39.927 + 2} = \frac{-3.317}{41.927} \approx -0.079

- At $x \approx 0.317$:

$$
F(0.317) \approx \frac{0.317 + 3}{(0.317)^2 + 2} = \frac{3.317}{0.100 + 2} = \frac{3.317}{2.100} \approx 1.58
$$

Implication: The point at $x \approx -6.317$ is likely a local maximum, and at $x \approx 0.317$, a local minimum.

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5. Summary of Key Features

Feature	Description
Domain	$(-\infty, \infty)$
Range	$(-\infty, \infty)$ (to be confirmed)
Horizontal Asymptote	$y = 0$
Critical Points	$x \approx -6.317$, $x \approx 0.317$
Local Max	Near $x \approx -6.317$, value ≈ -0.079
Local Min	Near $x \approx 0.317$, value ≈ 1.58

Broader Context: Applications and Implications

Rational Functions in Applied Mathematics

Functions like $F(x)$ are prevalent in modeling a range of phenomena:

- Physics: Describing ratios such as velocity over time or force over distance.
- Economics: Modeling cost-to-benefit ratios.
- Engineering: Signal processing where transfer functions are rational.

Significance of the Denominator's Form

The quadratic denominator $x^2 + 2$ indicates the function never encounters vertical asymptotes, making it well-behaved across all real numbers. This contrasts with many rational functions that have real roots in their denominators, leading to discontinuities.

Potential for Further Analysis

- Integration: Calculating $\int F(x) dx$ could reveal the accumulated effect over an interval.
- Series Expansion: Power series could approximate $F(x)$ near specific points.
- Transformations: Shifting or scaling may produce variants useful in applications.

Conclusion and Final Reflections

The function \(F(x) = \frac{x + 3

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