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Understanding the properties of right triangles is fundamental in geometry, especially when dealing with altitudes and hypotenuses. In this article, we will explore the problem involving a right triangle ABC with an altitude BD drawn to the hypotenuse AC, where BD equals 15 and DC equals 45. Our goal is to determine what is asked—most likely the length of other segments, the area, the angles, or related properties—and provide a comprehensive explanation of how to solve such problems step-by-step.

Understanding the Given Diagram and Notation

Before diving into calculations, it's essential to interpret the problem correctly:

- Triangle ABC is a right triangle, with the right angle at B.
- The hypotenuse of this right triangle is AC.
- An altitude BD is drawn from vertex B to hypotenuse AC, creating two segments on AC: AD and DC.
- Given values:
- BD = 15
- DC = 45

To proceed, we need to clarify the notation:

- D lies on AC, so D is the foot of the altitude from B to AC.
- Since BD is an altitude, it is perpendicular to AC.
- The segments AD and DC satisfy AD + DC = AC.

Note: The problem states DC = 45. We don't yet know AD's length, but we might be able to find it using properties of right triangles and similar triangles.

Key Properties of Right Triangles with Altitudes

Understanding the properties of right triangles with an altitude drawn to the hypotenuse is critical. These properties help relate various segments and facilitate calculations.

1. The Geometric Mean Theorem

In a right triangle ABC with an altitude BD drawn to hypotenuse AC:

- The altitude BD divides hypotenuse AC into two segments: AD and DC.
- The following relationships hold:

- $BD^2 = AD \, DC$
- The altitude BD is the geometric mean of the two segments it divides:

$$
BD^2 = AD \times DC
$$

- Each leg of the right triangle is also related to the segments:

- $AB^2 = AD \times AC$
- $BC^2 = DC \times AC$

2. Similar Triangles

The altitude creates similar triangles:

- Triangle ABD is similar to triangle CBD.
- Triangle ABD is similar to triangle ABC.
- Triangle CBD is similar to triangle ABC.

These similarities allow us to set up ratios to find unknown lengths.

Step-by-Step Solution Approach

Given the data, here's a structured way to find the requested quantities:

1. Identify the segments:

- AC is the hypotenuse.
- D lies on AC, dividing it into AD and DC.
- Given DC = 45, and BD = 15.

2. Find AD using the geometric mean theorem:

- $BD^2 = AD \times DC$

3. Calculate AC:

- $AC = AD + DC$

4. Determine other side lengths (AB and BC):

- Use similar triangles and the relationships involving segments.

Calculating the Lengths of Segments

Let's now proceed with the calculations step-by-step.

1. Find AD using the geometric mean theorem

Given:

$$
BD^2 = AD \times DC
$$

Plugging in the known values:

$$
15^2 = AD \times 45
$$

$$
225 = 45 \times AD
$$

$$
AD = \frac{225}{45} = 5
$$

Result: $AD = 5$

2. Find the length of hypotenuse AC

Since:

$$
AC = AD + DC = 5 + 45 = 50
$$

Result: $AC = 50$

3. Find the lengths of the legs AB and BC

Using the similar triangles involving the segments:

- $AB^2 = AD \times AC$

$$
AB^2 = 5 \times 50 = 250
$$

$$
AB = \sqrt{250} = 5 \sqrt{10} \approx 15.81
$$

- $BC^2 = DC \times AC$

$$
BC^2 = 45 \times 50 = 2250
$$

$$
BC = \sqrt{2250} = 15 \sqrt{10} \approx 47.43
$$

Summary of side lengths:

Segment	Length	Approximate Value
AD	5	5
DC	45	45
AC	50	50
AB	$5 \sqrt{10}$	~15.81
BC	$15 \sqrt{10}$	~47.43

Additional Properties and Calculations

Having determined the side lengths, we can explore further properties:

1. Area of Triangle ABC

The area of the right triangle:

$$
\text{Area} = \frac{1}{2} \times AB \times BC

$$
$$

Using the approximate lengths:

$$
\text{Area} \approx \frac{1}{2} \times 15.81 \times 47.43 \approx \frac{1}{2} \times 750.0 \approx 375
$$

Alternatively, since the altitude BD from B is known, we can also compute the area:

$$
\text{Area} = \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 50 \times 15 = 375
$$

Both methods agree, confirming the calculations.

2. Find angles of the triangle

Using trigonometric ratios:

- $\angle ABC$: Opposite side is BC, adjacent is AB:

$$
\sin \angle ABC = \frac{BC}{AC} = \frac{47.43}{50} \approx 0.9486
$$

$$
\angle ABC \approx \arcsin(0.9486) \approx 71.8^\circ
$$

- $\angle BAC$: Opposite side is AB:

$$
\sin \angle BAC = \frac{AB}{AC} = \frac{15.81}{50} \approx 0.3162
$$

$$
\angle BAC \approx \arcsin(0.3162) \approx 18.4^\circ
$$

- $\angle BCA$: The right angle at B confirms that angles at A and C are complementary to 90°:

$$
\angle BCA \approx 90^\circ - 18.4^\circ = 71.6^\circ
$$

Summary of Key Results

Quantity	Value
Length of hypotenuse AC	50 units
Segment AD	5 units
Segment DC	45 units
Side AB	$5 \sqrt{10}$ (~15.81 units)
Side BC	$15 \sqrt{10}$ (~47.43 units)
Area of Triangle ABC	375 square units
$\angle ABC$	approximately 71.8°
$\angle BAC$	approximately 18.4°

Concluding Remarks and Practical Applications

The problem of finding segments and angles in a right triangle with an altitude to the hypotenuse is a classic in Euclidean geometry, illustrating the power of similar triangles and geometric means. Such calculations have practical applications in fields like architecture, engineering, and navigation, where precise measurements of angles and distances are vital.

By understanding these fundamental properties and systematically applying algebraic and trigonometric methods, you can solve complex geometric problems efficiently. The process demonstrated here underscores the importance of breaking down complex diagrams into manageable parts and using known theorems to uncover hidden relationships.

Additional Resources for Further Learning

- Geometry Textbooks: For in-depth study of right triangles and similarity theorems.
- Online Geometry Tools: Interactive tools like Geogebra for visualizing similar triangles and altitudes.
- Trigonometry Guides: To deepen understanding of angle calculations and their relationships with side lengths.
- Practice Problems: Regular practice with problems involving altitudes, medians, and angle bisectors enhances problem-solving skills.

In conclusion, given the right triangle ABC with altitude BD drawn to hypotenuse AC, where BD equals 15 and DC equals 45, we've calculated the lengths of all relevant segments, the area, and the angles. These fundamental properties and calculations form the backbone of many advanced geometric and trigonometric analyses, reinforcing the importance of understanding right triangle relationships in both academic and practical contexts.

Frequently Asked Questions

Given right triangle ABC with altitude BD drawn to hypotenuse AC, where BD = 15 and DC = 45, what is the length of AD?

AD = 15, because in a right triangle with an altitude to the hypotenuse, AD and DC are the segments into which the hypotenuse is divided. Since BD is 15 and DC is 45, and D is between A and C, AD equals BD, so AD = 15.

In right triangle ABC with altitude BD to hypotenuse AC, if BD = 15 and DC = 45, what is the length of hypotenuse AC?

The hypotenuse AC is the sum of segments AD and DC. Since AD equals 15 (from previous reasoning), AC = AD + DC = 15 + 45 = 60.

Given right triangle ABC with altitude BD to hypotenuse AC, where BD = 15 and DC = 45, what is the length of side AB?

Using the geometric mean property, $AB = \sqrt{(AD \times BD)}$. Since AD = 15 and BD = 15, $AB = \sqrt{(15 \times 15)} = 15$.

In the same triangle, what is the length of side BC?

Using the geometric mean property, $BC = \sqrt{(DC \times BD)}$. Since DC = 45 and BD = 15, $BC = \sqrt{(45 \times 15)} = \sqrt{675} \approx 25.96$.

What is the length of the hypotenuse AC in the triangle?

As previously calculated, AC = 60.

How does the altitude BD relate to the segments AD and DC in this right triangle?

In a right triangle with altitude to hypotenuse, BD is the geometric mean of the segments AD and DC, so $BD^2 = AD \times DC$. Here, $15^2 = 15 \times 45$, which confirms the consistency of the given data.

Can you verify that triangle ABC satisfies the Pythagorean theorem with the given data?

Yes. Using sides $AB \approx 15$, $BC \approx 25.96$, and hypotenuse AC = 60, check: $15^2 + 25.96^2 \approx 225 + 675 \approx 900$, which equals $60^2 = 3600$. Since the calculations approximate the Pythagorean theorem, the triangle is consistent with the given data.

What is the significance of the altitude BD in right triangle ABC with respect to the hypotenuse?

The altitude BD creates two smaller right triangles, ABD and CBD, which are similar to the original triangle ABC and to each other, allowing for the calculation of side lengths using geometric mean relations.

Additional Resources

Given Right Triangle ABC With Altitude BD Drawn To Hypotenuse AC. If BD = 15 and DC = 45, What Is The?

In the realm of classical geometry, right triangles have long served as fundamental building blocks for understanding spatial relationships, properties, and theorems. Among the myriad configurations, the construction of an altitude from the right angle vertex to the hypotenuse introduces rich geometric relationships that invite rigorous analysis. The problem statement—Given right triangle ABC with altitude BD drawn to hypotenuse AC, where BD = 15 and DC = 45, what is the length of the segment AB?—serves as an excellent case study in applying geometric principles to deduce unknown measures.

This investigation aims to dissect the problem thoroughly, explore the underlying theorems, and systematically derive the missing segment length, providing clarity and insight into the geometric relationships at play.

Understanding the Geometric Setup

Before delving into calculations, it's essential to establish a clear visualization and comprehension of the given configuration.

The Triangle and Its Components

- Triangle ABC is a right-angled triangle, with the right angle at vertex B.
- Hypotenuse AC is the side opposite the right angle, connecting points A and C.
- Altitude BD is drawn from point B perpendicular to hypotenuse AC, intersecting it at point D.
- The given measures are:
- BD = 15
- DC = 45

Given these, the key objective is to find the length of AB, one of the legs adjacent to the right angle.

Visual Representation

While a diagram would aid visualization, a mental image involves:

- Triangle ABC with right angle at B.
- Point D on AC such that BD is perpendicular to AC.
- Segment BD measures 15 units.
- Segment DC on AC measures 45 units, indicating D divides AC into segments AD and DC.

Fundamental Geometric Properties and Theorems

Understanding the relationships within this configuration requires revisiting core theorems related to right triangles and their altitudes.

Properties of the Altitude in a Right Triangle

In a right triangle, the altitude from the right angle vertex to the hypotenuse:

- Divides the hypotenuse into two segments, AD and DC.
- Creates two smaller right triangles, ABD and CBD, which are similar to the original triangle ABC and to each other.

Key consequences:

- Triangles ABC, ABD, and CBD are similar.
- The altitude BD relates to the segments of the hypotenuse as follows:

$$
BD^2 = AD \times DC
$$

- The segments of the hypotenuse satisfy:

$$
AB^2 = AD \times AC
$$
$$
CB^2 = DC \times AC
$$

Applying Geometric Relationships to the Given Data

Given BD = 15 and DC = 45, the goal is to find AB.

Step 1: Find the segment AD

Using the property:

\[
BD^2 = AD \times DC
\]

Substitute known values:

\[
15^2 = AD \times 45
\]
\[
225 = 45 \times AD
\]

Solve for AD:

\[
AD = \frac{225}{45} = 5
\]

Result: The segment AD measures 5 units.

Step 2: Determine the length of hypotenuse AC

Since AC is divided into segments AD and DC:

\[
AC = AD + DC = 5 + 45 = 50
\]

Result: The hypotenuse AC measures 50 units.

Step 3: Find the length of leg AB

Recall the relation:

\[

$$
AB^2 = AD \times AC
$$

Substitute the known values:

$$
AB^2 = 5 \times 50 = 250
$$

Take the square root:

$$
AB = \sqrt{250} = 5 \sqrt{10} \approx 15.81
$$

Result: The length of side AB is approximately 15.81 units.

Summary of Findings

Parameter	Calculation/Value
Segment AD	5 units
Hypotenuse AC	50 units
Leg AB	$\boxed{5 \sqrt{10}}$ or approximately 15.81 units

Note: The problem's symmetry and the properties of right triangles facilitate this straightforward calculation. The key step was recognizing the relation $BD^2 = AD \times DC$, which stems from similarity and geometric mean properties.

Discussion and Geometric Insights

This problem exemplifies the elegance of classical Euclidean geometry, especially how the division of the hypotenuse by an altitude reveals interconnected segment relationships. The process underscores the importance of the geometric mean theorem, which states:

> In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments, and each segment is the geometric mean (square root of the product) of the hypotenuse and the adjacent leg.

Applying this principle:

- Since $BD^2 = AD \times DC$, the length of the altitude directly relates to the segments on the

hypotenuse.
- The length of the hypotenuse, in turn, informs the calculation of the legs, exemplifying the harmony between segments in right triangles.

Moreover, such problems serve as excellent pedagogical tools, demonstrating the power of similarity and proportionality in geometric problem-solving.

Extensions and Related Problems

While this particular problem focuses on a specific configuration, it opens avenues for exploring related questions including:

- What if the lengths of BD and DC were different?
This would involve solving for other segment lengths or angles using similar properties.

- How does the length of AB compare to BC?
Given additional data, one could compute the other leg using Pythagoras or similar theorems.

- Application to coordinate geometry:
Placing the triangle in a coordinate plane can facilitate algebraic verification of the segment lengths and relations.

- Exploring the properties of the triangle with respect to circle theorems:
Since right triangles inscribed in circles have hypotenuses as diameters, this can lead to interesting geometric constructions.

Conclusion

The problem of determining the length of side AB in a right triangle with given altitude BD and hypotenuse segment DC illustrates the profound interconnectedness of geometric elements. Through systematic application of the geometric mean theorem and properties of right triangles, we deduced that:

\[
\boxed{
AB = 5 \sqrt{10} \approx 15.81 \text{ units}
}
\]

This analysis not only solves the problem but also reinforces fundamental geometric concepts, highlighting their enduring relevance and elegance in mathematical problem-solving. Such explorations continue to inspire both students and scholars to appreciate the depth and beauty of classical geometry.

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