Given The Two Points, C(3, 2, 1) And D(r = 5, = 20, = 70), Find: (a) The Spherical Coordinates Of C;

Given The Two Points, C(3, 2, 1) And D(r = 5, = 20, = 70), Find: (a) The Spherical Coordinates Of C; is a common problem in coordinate geometry, especially in the context of converting Cartesian coordinates to spherical coordinates. Understanding how to perform this conversion is essential for students and professionals working in fields like physics, engineering, and computer graphics. This article provides a comprehensive guide to finding the spherical coordinates of point C, given its Cartesian coordinates, and discusses the related concepts necessary to understand the process thoroughly.

Understanding Coordinate Systems: Cartesian and Spherical Coordinates

Before diving into the calculation, it is vital to understand the differences between Cartesian and spherical coordinate systems.

Cartesian Coordinates

- Represent a point in 3D space with three values: (x, y, z).
- These coordinates specify a point's position relative to mutually perpendicular axes, traditionally labeled X, Y, and Z.
- For example, point C is given as (3, 2, 1), where:
- x = 3
- y = 2
- z = 1

Spherical Coordinates

- Represent a point in space with three parameters: (r, θ, φ).
- These parameters have specific geometric meanings:
- r: The radial distance from the origin to the point.
- θ (theta): The polar angle, measured from the positive z-axis downward.
- φ (phi): The azimuthal angle, measured from the positive x-axis in the x-y plane.
- The relationships among these variables are crucial for converting between coordinate systems.

Conversion Formulas from Cartesian to Spherical

Coordinates

To convert a point from Cartesian to spherical coordinates, we use the following formulas:

Radial Distance (r)

- $r = \sqrt{x^2 + y^2 + z^2}$

Polar Angle (θ)

- $\theta = \arccos \left(\frac{z}{r} \right)$

Azimuthal Angle (φ)

- $\varphi = \arctan2(y, x)$

Here, the function $\arctan2(y, x)$ computes the angle considering the signs of both y and x to determine the correct quadrant.

Calculating the Spherical Coordinates of Point C(3, 2, 1)

Let's now apply these formulas step-by-step to point C(3, 2, 1).

Step 1: Calculate the Radial Distance (r)

Using the formula:
$$r = \sqrt{x^2 + y^2 + z^2}$$
substitute the values:
$$r = \sqrt{3^2 + 2^2 + 1^2}$$
$$r = \sqrt{9 + 4 + 1}$$
$$r = \sqrt{14}$$
$$r \approx 3.7417$$

Step 2: Calculate the Polar Angle (θ)

Using:
$$\theta = \arccos \left(\frac{z}{r} \right)$$
substitute the values:
$$\theta = \arccos \left(\frac{1}{3.7417} \right)$$
$$\theta = \arccos (0.2673)$$

Using a calculator or computational tool:

$$ \theta \approx 74.5^\circ $$
or in radians:
$$ \theta \approx 1.3 \text{ radians} $$

Step 3: Calculate the Azimuthal Angle (φ)

Using:
$$ \varphi = \arctan2(y, x) $$
substitute:
$$ \varphi = \arctan2(2, 3) $$

Calculating:
$$ \varphi \approx 33.7^\circ $$
or in radians:
$$ \varphi \approx 0.588 \text{ radians} $$

Final Spherical Coordinates of Point C

- Radial distance (r): approximately 3.7417 units
- Polar angle (θ): approximately 74.5 degrees or 1.3 radians
- Azimuthal angle (φ): approximately 33.7 degrees or 0.588 radians

Expressed in the standard spherical coordinate notation:
$$ (r, \theta, \varphi) \approx (3.7417, 74.5^\circ, 33.7^\circ) $$

Understanding the Role of Point D and Its Coordinates

While the primary focus is on finding the spherical coordinates of point C, the given point D(r=5, =20, =70) can serve as an example of how spherical coordinates are represented and how they relate to Cartesian coordinates.

- The notation indicates:
- $r = 5$
- $\theta = 20^\circ$
- $\varphi = 70^\circ$

This can be converted back into Cartesian coordinates if needed, or used to understand the spatial relationship between points C and D.

Applications of Spherical Coordinates

Understanding how to convert between coordinate systems is vital in many scientific and engineering fields, including:

- **Physics:** Describing the position of particles in three-dimensional space, especially in quantum mechanics and electromagnetism.
- **Engineering:** Designing systems involving spherical geometries, such as antennas or spherical tanks.
- **Computer Graphics:** Rendering 3D objects and environments, where spherical coordinates facilitate camera positioning and lighting calculations.
- **Astronomy:** Calculating the position of celestial objects relative to Earth.

Understanding the conversion process enhances spatial reasoning and problem-solving skills across these disciplines.

Summary and Key Takeaways

- The Cartesian coordinates of point C(3, 2, 1) can be converted to spherical coordinates using the formulas:
- $r = \sqrt{x^2 + y^2 + z^2}$
- $θ = \arccos \left(\frac{z}{r} \right)$
- $φ = \arctan2(y, x)$
- For point C:
- $r \approx 3.7417$
- $θ \approx 74.5^\circ$
- $φ \approx 33.7^\circ$
- These coordinates provide a different perspective on the location of point C, useful in various technical applications.

Mastering coordinate transformations is essential for advanced spatial analysis and problem-solving in multiple scientific fields.

Further Resources for Learning

- Textbooks on coordinate systems and vector calculus
- Online tutorials on 3D coordinate transformations
- Interactive tools for visualizing Cartesian and spherical coordinates
- Mathematical software like MATLAB, WolframAlpha, or GeoGebra for computations and visualizations

By practicing these conversions and understanding their geometric interpretations, students and professionals can develop a deeper intuition for three-dimensional space and enhance their analytical capabilities in complex spatial problems.

Frequently Asked Questions

What are the steps to convert Cartesian coordinates to spherical coordinates for point C(3, 2, 1)?

To convert Cartesian coordinates (x, y, z) to spherical coordinates (r, θ, φ), you calculate $r = \sqrt{x^2 + y^2 + z^2}$, $\theta = \arccos(z / r)$, and $\varphi = \arctan(y / x)$. For C(3, 2, 1), compute $r = \sqrt{9 + 4 + 1} = \sqrt{14}$, then $\theta = \arccos(1 / \sqrt{14})$, and $\varphi = \arctan(2 / 3)$.

What is the value of the radial distance r for point C(3, 2, 1)?

The radial distance r is $\sqrt{3^2 + 2^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$, approximately 3.74 units.

How do you calculate the polar angle θ for point C in spherical coordinates?

θ is calculated as $\arccos(z / r)$. For C(3, 2, 1), $\theta = \arccos(1 / \sqrt{14})$, which is approximately 74.5 degrees.

How is the azimuthal angle φ determined for point C in spherical coordinates?

φ is determined by $\arctan(y / x)$. For C(3, 2, 1), $\varphi = \arctan(2 / 3)$, approximately 33.7 degrees.

What are the spherical coordinates of point C(3, 2, 1)?

The spherical coordinates are approximately ($r = \sqrt{14} \approx 3.74$, $\theta \approx 74.5°$, $\varphi \approx 33.7°$).

Given point D(5, 20, 70), how would you convert it to spherical coordinates?

Calculate $r = \sqrt{5^2 + 20^2 + 70^2} = \sqrt{25 + 400 + 4900} = \sqrt{5325} \approx 73.0$. Then, $\theta = \arccos(70 / 73)$, and $\varphi = \arctan(20 / 5)$.

Why is converting Cartesian coordinates to spherical coordinates useful?

It simplifies calculations involving distances, angles, and physical problems with spherical symmetry, such as in physics and engineering.

What is the significance of the angles θ and φ in spherical coordinates?

θ (polar angle) measures the angle from the positive z-axis, while φ (azimuthal angle) measures the angle from the positive x-axis in the xy-plane, defining the point's direction in space.

How can understanding the relationship between points C and D help in 3D spatial analysis?

Analyzing their spherical coordinates allows for easier calculation of distances, angles, and spatial relationships, which is essential in fields like navigation, astronomy, and robotics.

Additional Resources

Understanding the Conversion from Cartesian to Spherical Coordinates: A Deep Dive into Point C(3, 2, 1) and Point D(r=5, θ=20°, φ=70°)

In the realm of three-dimensional coordinate systems, understanding how points are represented and converted between different coordinate systems is fundamental to fields ranging from mathematics and physics to engineering and computer graphics. The problem at hand involves two specific points: C with Cartesian coordinates (3, 2, 1) and D given in what appears to be spherical coordinates with parameters (r=5, θ=20°, φ=70°). Our goal is to determine the spherical coordinates of point C, given its Cartesian coordinates, and to analyze the structure and significance of these coordinate transformations.

This article presents an in-depth exploration of the concepts, methodologies, and calculations involved in converting Cartesian coordinates to spherical coordinates, the significance of these coordinate systems, and the implications of the given data. It is structured to guide the reader through the foundational principles, detailed calculations, and broader applications, providing clarity and comprehensive understanding.

Understanding Coordinate Systems: Cartesian vs. Spherical

Cartesian Coordinates: The Standard Framework

Cartesian coordinates are perhaps the most familiar way to represent points in three-dimensional space. Each point is described by three numerical values (x, y, z), corresponding to its position along mutually perpendicular axes—x-axis, y-axis, and z-axis. This system is intuitive and widely used because it directly relates to physical space, making measurements straightforward.

For point C(3, 2, 1), the Cartesian coordinates specify:
- x = 3
- y = 2
- z = 1

Spherical Coordinates: An Alternative Perspective

Spherical coordinates, on the other hand, describe a point in space based on its distance from the origin and two angles. This system is particularly useful in applications involving radial symmetry, such as electromagnetism, astronomy, and geophysics.

A point in spherical coordinates is typically represented as (r, θ, φ), where:
- r: the radial distance from the origin to the point
- θ (theta): the polar angle, measured from the positive z-axis ($0° \leq \theta \leq 180°$)
- φ (phi): the azimuthal angle, measured in the x-y plane from the positive x-axis ($0° \leq \varphi < 360°$)

Note: Different conventions exist, but the above is standard in physics and engineering.

Converting Cartesian Coordinates to Spherical Coordinates

The core of the problem involves the transformation from Cartesian (x, y, z) to spherical (r, θ, φ). The conversion formulas stem from basic geometric relationships:

1. Radial distance (r):

$$
r = \sqrt{x^2 + y^2 + z^2}
$$

2. Polar angle (θ):

$$
\theta = \arccos \left(\frac{z}{r} \right)
$$

This measures the angle between the positive z-axis and the line segment connecting the origin to the point.

3. Azimuthal angle (φ):

$$
\phi = \arctan \left(\frac{y}{x} \right)
$$

Adjustments are needed based on the quadrant in which the point lies, to ensure φ is within the correct range.

Step-by-Step Calculation for Point C(3, 2, 1)

Let's now process the Cartesian point C(3, 2, 1) and find its corresponding spherical coordinates.

Calculating the Radial Distance (r)

$$
r = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14} \approx 3.7417
$$

Thus, the radial distance r is approximately 3.74 units.

Determining the Polar Angle (θ)

$$
\theta = \arccos \left(\frac{z}{r} \right) = \arccos \left(\frac{1}{3.7417} \right)
$$

Calculate the ratio:

$$
\frac{1}{3.7417} \approx 0.2673
$$

Now, find the arccosine in degrees:

$$
\theta = \arccos(0.2673) \approx 74.4^\circ
$$

This angle indicates the inclination of point C from the positive z-axis.

Calculating the Azimuthal Angle (φ)

$$
\phi = \arctan \left(\frac{y}{x} \right) = \arctan \left(\frac{2}{3} \right)
$$

Calculating:

$$
\phi = \arctan(0.6667) \approx 33.7^\circ
$$

Since x and y are both positive, φ is in the first quadrant, which is consistent.

Summary for Point C:
- Radial distance, $r \approx 3.74$
- Polar angle, $\theta \approx 74.4^\circ$
- Azimuthal angle, $\phi \approx 33.7^\circ$

Expressed in spherical coordinates:

$$
\boxed{
\text{C in spherical coordinates} \approx (r=3.74, \theta=74.4^\circ, \phi=33.7^\circ)
}
$$

\]

Analyzing Point D Given in Spherical Coordinates

The second point, D, is given as ($r=5$, $\theta=20°$, $\varphi=70°$). This provides a reference to understand the spatial positioning and to compare with point C.

Interpreting Point D:

- Radial distance: 5 units from the origin.
- Polar angle: 20°, indicating it is relatively close to the positive z-axis.
- Azimuthal angle: 70°, placing it in the first quadrant of the x-y plane.

This point is closer to the z-axis than point C, which has a larger θ (~74.4°), meaning C is more inclined relative to the z-axis.

Implications and Broader Context

Significance of Coordinate Systems in Applications

Understanding the conversion between coordinate systems is critical in various scientific and engineering fields:

- Physics: Electromagnetic fields, quantum mechanics, and classical mechanics often utilize spherical coordinates for problems involving symmetric potentials or fields.
- Astronomy: Celestial objects are frequently described in spherical coordinates relative to Earth or the Sun.
- Engineering: Antenna design, radar systems, and 3D modeling often rely on spherical coordinates for spatial analysis.

Advantages of Spherical Coordinates

- Simplify the description of points and regions with spherical symmetry.
- Facilitate solving partial differential equations like Laplace's and Helmholtz's equations.
- Allow easier interpretation of angles and distances in three-dimensional space.

Limitations and Challenges

- Conversion formulas can be complex near singularities (e.g., at the origin or along the axes).
- Ambiguities in angle measurement require careful handling of quadrants and ranges.

- Computational considerations arise when implementing these conversions in algorithms.

Further Considerations and Extensions

Determining the Spherical Coordinates of Point D

Suppose the problem requires verifying the spherical coordinates of point D or converting between coordinate systems. Given D's parameters ($r=5$, $\theta=20°$, $\varphi=70°$), converting back to Cartesian coordinates involves:

$$
x = r \sin \theta \cos \phi
$$
$$
y = r \sin \theta \sin \phi
$$
$$
z = r \cos \theta
$$

Calculations:
- Convert angles to radians for computational accuracy:

$$
\theta = 20^\circ = \frac{\pi}{9} \approx 0.3491\, \text{rad}
$$

$$
\phi = 70^\circ = \frac{7\pi}{18} \approx 1.2217\, \text{rad}
$$

- Compute Cartesian coordinates:

$$
x = 5 \times \sin(20^\circ) \times \cos(70^\circ) \approx 5 \times 0.3420 \times 0.3420 \approx 0.585
$$

$$
y = 5 \times 0.3420 \times \sin(70^\circ) \approx 5 \times 0.3420 \times 0.9397 \approx 1.610
$$

$$
z = 5 \times \cos(20^\circ) \approx 5 \times 0.9397 \approx 4.698
$$

Thus, point D in Cartesian coordinates is approximately (0.585, 1.61, 4.70).

Comparative Analysis of Points C and D

Analyzing the spatial relationship between points C and D reveals:

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Given The Two Points C 3 2 1 And D R 5 20 70 Find A The Spherical Coordinates Of C

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