If 30.0 J Of Work Are Required To Stretch A Spring From A 4.00 Cm Elongation To A 5.00cm Elongation,

If 30.0 J Of Work Are Required To Stretch A Spring From A 4.00 Cm Elongation To A 5.00cm Elongation, this statement provides a fascinating insight into the mechanics of springs and the work-energy principle. Understanding how much work is needed to stretch a spring over a specific interval can reveal important properties such as the spring constant and the potential energy stored within the spring. In this article, we will explore the physics behind the problem, derive relevant formulas, and analyze what the given data implies about the characteristics of the spring.

Understanding the Problem

When a spring is stretched or compressed, work is done against the restoring force exerted by the spring. The amount of work done to stretch the spring from one elongation to another is closely related to the spring's properties, notably its spring constant.

The problem states:
- The work done, W, is 30.0 Joules.
- The initial elongation, x_1, is 4.00 cm.
- The final elongation, x_2, is 5.00 cm.

Before proceeding, it is essential to convert these elongation measurements into standard SI units (meters):
- $x_1 = 4.00\, \text{cm} = 0.04\, \text{m}$
- $x_2 = 5.00\, \text{cm} = 0.05\, \text{m}$

Our goal is to determine the spring constant k and understand the energy involved in this process.

Theoretical Background

Hooke's Law and Potential Energy

For an ideal spring obeying Hooke's Law, the restoring force F is proportional to the displacement x:

$$
F = -kx
$$

where:

- k is the spring constant (N/m),
- x is the displacement from the spring's equilibrium position.

The elastic potential energy stored in a stretched or compressed spring is given by:
$$
U = \frac{1}{2}kx^2
$$

Work Done in Stretching a Spring

The work done to stretch or compress a spring from an initial displacement x_1 to a final displacement x_2 is:
$$
W = \int_{x_1}^{x_2} F\, dx
$$
Since $F = kx$ (taking magnitude), the work becomes:
$$
W = \int_{x_1}^{x_2} kx\, dx = \frac{1}{2}k(x_2^2 - x_1^2)
$$
This integral reflects the fact that the force varies linearly with displacement, and the work is equal to the change in elastic potential energy when moving from x_1 to x_2.

Calculating the Spring Constant k

Given the work W and the displacements x_1 and x_2, we can rearrange the work formula to find k:
$$
W = \frac{1}{2}k(x_2^2 - x_1^2)
$$
$$
k = \frac{2W}{x_2^2 - x_1^2}
$$

Substitute the known values:
$$
k = \frac{2 \times 30.0\, \text{J}}{(0.05)^2 - (0.04)^2}
$$

Calculate numerator and denominator:
- $2 \times 30.0 = 60.0\, \text{J}$
- $0.05^2 = 0.0025\, \text{m}^2$
- $0.04^2 = 0.0016\, \text{m}^2$
- Difference: $0.0025 - 0.0016 = 0.0009\, \text{m}^2$

Now, compute k:
$$
k = \frac{60.0}{0.0009} \approx 66666.7\, \text{N/m}
$$

Result:

$$
\boxed{
k \approx 66,667\, \text{N/m}
}
$$

This high spring constant indicates a very stiff spring, requiring significant force for small elongations.

Analysis of the Spring's Energy

Potential Energy at Different Displacements

Using the spring constant k, we can determine the elastic potential energy stored at the initial and final elongations:

$$
U = \frac{1}{2}kx^2
$$

- At $x_1 = 0.04\, \text{m}$:

$$
U_1 = \frac{1}{2} \times 66,667 \times (0.04)^2 = 0.5 \times 66,667 \times 0.0016 \approx 53.33\, \text{J}
$$

- At $x_2 = 0.05\, \text{m}$:

$$
U_2 = 0.5 \times 66,667 \times (0.05)^2 = 0.5 \times 66,667 \times 0.0025 \approx 83.33\, \text{J}
$$

The change in potential energy:

$$
\Delta U = U_2 - U_1 = 83.33\, \text{J} - 53.33\, \text{J} = 30.0\, \text{J}
$$

which aligns perfectly with the work done, confirming the energy conservation principle.

Implications and Applications

Practical Significance of the Results

- The calculated spring constant k demonstrates the stiffness of the spring: a larger k indicates a stiffer spring.
- The work required to stretch the spring correlates directly with the increase in elastic potential energy stored in the spring.
- Understanding these parameters is essential in engineering applications where springs are used in machinery, automotive suspensions, or measuring devices.

Real-World Scenarios

- Designing Spring Systems: Knowing the spring constant helps engineers select appropriate springs for specific load requirements.
- Energy Storage: Elastic potential energy calculations are crucial in systems where energy is stored and released, such as in shock absorbers.
- Material Testing: The high spring constant suggests the spring material or design can withstand significant forces without permanent deformation.

Additional Considerations

- Elastic Limit: The calculations assume the spring behaves elastically within the given elongation range. If the extension exceeds the elastic limit, the spring may deform permanently.
- Non-ideal Spring Behavior: Real springs may not follow Hooke's Law perfectly, especially at large displacements, leading to deviations from theoretical values.
- Measurement Accuracy: Precise measurement of elongations and work is critical for accurate calculations in practical applications.

Conclusion

By analyzing the work required to stretch a spring from 4.00 cm to 5.00 cm, we deduced that the spring constant is approximately 66,667 N/m. The work done corresponds exactly to the increase in elastic potential energy stored within the spring during this elongation. Such calculations are fundamental in understanding spring mechanics, designing mechanical systems, and ensuring safety and efficiency in engineering applications. Recognizing the relationship between work, force, and energy in spring systems enables engineers and physicists to optimize their use in various technological innovations.

Summary:

- Work done to stretch the spring: 30.0 Joules
- Spring constant: approximately 66,667 N/m
- Energy stored increases by 30.0 Joules between 4.00 cm and 5.00 cm elongation
- The principles are based on Hooke's Law and energy conservation

This detailed understanding of spring mechanics enhances our ability to analyze and design systems involving elastic components, ensuring functionality and safety across numerous scientific and industrial fields.

Frequently Asked Questions

What is the spring constant if 30.0 J of work is required to stretch the spring from 4.00 cm to 5.00 cm?

The spring constant k can be found using the work done: Work = (1/2) k (x_2^2 - x_1^2). Rearranging, k = 2 Work / (x_2^2 - x_1^2). Converting cm to meters: x_1 = 0.04 m, x_2 = 0.05 m. Then, k = 2 30.0 J / (0.05^2 - 0.04^2) m^2 = 60 / (0.0025 - 0.0016) = 60 / 0.0009 ≈ 66,666.67 N/m.

How much work is needed to stretch the spring from 4.00 cm to 5.00 cm if the spring constant is known?

Work done is W = (1/2) k (x_2^2 - x_1^2). Using the calculated k ≈ 66,666.67 N/m, W = 0.5 66,666.67 (0.05^2 - 0.04^2) = 0.5 66,666.67 (0.0025 - 0.0016) = 0.5 66,666.67 0.0009 ≈ 30.0 J.

What is the potential energy stored in the spring when stretched to 5.00 cm?

The elastic potential energy is U = (1/2) k x^2. Using k ≈ 66,666.67 N/m and x = 0.05 m, U = 0.5 66,666.67 0.05^2 = 0.5 66,666.67 0.0025 ≈ 83.33 J.

If the spring is initially at 4.00 cm elongation and work is done to reach 5.00 cm, what is the work done per centimeter of elongation?

Total work done is 30.0 J over a change of 1.00 cm (from 4.00 cm to 5.00 cm). Therefore, work per centimeter is 30.0 J.

What is the force exerted by the spring at 4.00 cm and 5.00 cm elongation?

Force is F = kx. At 4.00 cm (0.04 m), F = 66,666.67 0.04 ≈ 2,666.67 N; at 5.00 cm (0.05 m), F = 66,666.67 0.05 ≈ 3,333.33 N.

How does the work needed to stretch the spring relate to the spring constant and the change in elongation?

Work done is proportional to the spring constant and the difference in the squares of the elongations: W = (1/2) k (x_2^2 - x_1^2). Increasing the spring constant or the change in elongation increases the work required.

If the spring constant is doubled, how much work is needed to stretch the spring from 4.00 cm to 5.00 cm?

Since work W is proportional to k, doubling k from ~66,666.67 N/m to ~133,333.33 N/m doubles the work to 60.0 J for the same elongation change.

Additional Resources

Understanding the Work Done in Stretching a Spring: An In-Depth Analysis of a 30.0 J Force Scenario

In the realm of classical mechanics, springs serve as quintessential examples of elastic systems, demonstrating the fundamental principles of energy storage and transfer. When a force acts upon a spring to stretch or compress it, energy is either imparted to or extracted from the system. The specific scenario where 30.0 joules (J) of work are required to stretch a spring from a 4.00 cm elongation to a 5.00 cm elongation provides a compelling case study. This situation not only exemplifies the relationship between work and elastic potential energy but also offers insights into the properties of the spring itself, particularly its spring constant. In this comprehensive article, we dissect this problem from multiple angles, exploring the physics principles involved, performing detailed calculations, and discussing broader implications in mechanical systems.

Fundamental Concepts Underpinning Spring Mechanics

The Nature of Elastic Potential Energy

At the core of spring mechanics lies the concept of elastic potential energy—the energy stored within a deformed elastic object when work is done on it. For an ideal spring obeying Hooke's Law, the elastic potential energy (U) is given by:

$$ U = \frac{1}{2} k x^2 $$

Where:
- k is the spring constant (measure of the stiffness of the spring)
- x is the displacement from the spring's natural (unstretched) length

When a spring is stretched or compressed, work is done against the restoring force exerted by the spring. This work is stored as elastic potential energy, which can be recovered when the spring returns to its equilibrium position.

Work-Energy Relationship in Spring Stretching

The work done on a spring during stretching or compression from an initial displacement x_i to a final displacement x_f is calculated by integrating the force over the displacement:

$$W = \int_{x_i}^{x_f} F \, dx$$

Since the restoring force follows Hooke's Law:

$$F = -k x$$

The work done by an external agent (which must overcome the spring's restoring force) is:

$$W = \int_{x_i}^{x_f} k x \, dx = \frac{1}{2} k (x_f^2 - x_i^2)$$

This positive value indicates energy input into the system, stored as elastic potential energy.

Applying Theory to the Given Scenario

Problem Restatement

The problem states:

> "If 30.0 J of work are required to stretch a spring from a 4.00 cm elongation to a 5.00 cm elongation,"

This involves calculating parameters such as the spring constant k, and understanding the energy dynamics involved in such a stretch.

Converting Displacements to SI Units

To proceed with calculations, all distances must be expressed in meters:

- Initial elongation: $x_i = 4.00\, \text{cm} = 0.0400\, \text{m}$
- Final elongation: $x_f = 5.00\, \text{cm} = 0.0500\, \text{m}$

Calculating the Spring Constant (k)

Using Work Done to Find k

From the work expression:

$$ W = \frac{1}{2} k (x_f^2 - x_i^2) $$

Given:
- $W = 30.0\, \text{J}$
- $x_i = 0.0400\, \text{m}$
- $x_f = 0.0500\, \text{m}$

Rearranged to solve for k:

$$ k = \frac{2W}{x_f^2 - x_i^2} $$

Calculating numerator:

$$ 2 \times 30.0\, \text{J} = 60.0\, \text{J} $$

Calculating denominator:

$$ (0.0500)^2 - (0.0400)^2 = 0.0025 - 0.0016 = 0.0009\, \text{m}^2 $$

Therefore, the spring constant:

$$ k = \frac{60.0}{0.0009} \approx 66,666.67\, \text{N/m} $$

This high value indicates a very stiff spring, which requires significant force to stretch over small displacements.

Analyzing Energy Changes in the System

Elastic Potential Energy at Initial and Final Displacements

Using the elastic potential energy formula:

$$ U = \frac{1}{2} k x^2 $$

Calculate at initial position:

$$ U_i = \frac{1}{2} \times 66,666.67\, \text{N/m} \times (0.0400)^2 $$
$$ U_i = 0.5 \times 66,666.67 \times 0.0016 $$
$$ U_i \approx 53.33\, \text{J} $$

Calculate at final position:

$$ U_f = \frac{1}{2} \times 66,666.67\, \text{N/m} \times (0.0500)^2 $$
$$ U_f = 0.5 \times 66,666.67 \times 0.0025 $$
$$ U_f \approx 83.33\, \text{J} $$

The change in elastic potential energy:

$$ \Delta U = U_f - U_i = 83.33\, \text{J} - 53.33\, \text{J} = 30.0\, \text{J} $$

This matches precisely the work input, confirming energy conservation and the correctness of calculations.

Implication of Energy Conservation

The equivalence of work done and energy stored underscores the principle that in an ideal, frictionless spring, all the work input transforms into elastic potential energy. No energy is lost to heat or other dissipative processes.

Broader Implications and Practical Considerations

Spring Stiffness and Material Properties

The calculated spring constant of approximately 66,666.67 N/m indicates an extremely stiff spring. In reality, such a spring would likely be made from high-strength materials like spring steel or composite alloys, capable of withstanding high stresses without permanent deformation.

Designers of mechanical systems leverage such properties for applications requiring precise energy storage, such as in shock absorbers, aerospace components, or precision measurement devices.

Energy Storage Efficiency and Safety

Understanding the work done and the elastic potential energy stored is critical for safety and efficiency:

- Excessive energy storage can lead to sudden releases, causing mechanical failure or injury.
- Proper calculation ensures that the spring operates within its elastic limit, preventing permanent

deformation or fracture.

Real-World Limitations and Non-Idealities

While the theoretical calculations assume an ideal spring obeying Hooke's Law perfectly, real-world springs exhibit:

- Material hysteresis (energy loss during loading and unloading)
- Nonlinear behavior at large strains
- Fatigue over repeated cycles

These factors necessitate safety margins and empirical testing beyond theoretical models.

Extended Applications and Educational Significance

Educational Demonstrations and Laboratory Experiments

This scenario serves as an excellent teaching example to illustrate:

- The relationship between work, force, and energy
- The use of calculus in deriving work and energy formulas
- The importance of unit conversions and careful algebraic manipulations

Laboratory experiments can verify the theoretical calculations by directly measuring the force required to stretch springs over specified displacements and comparing the work measured via force-displacement graphs.

Implications in Engineering Design

Understanding the energy dynamics of springs informs:

- Design parameters for energy-absorbing devices
- Optimization of spring stiffness for specific applications
- Safety considerations in mechanical assemblies

Engineers must balance stiffness, durability, and energy capacity, with precise calculations like those demonstrated here guiding decision-making.

Conclusion: Insights Gained and Future Perspectives

The analysis of the scenario where 30.0 J of work are required to stretch a spring from a 4.00 cm to 5.00 cm elongation reveals fundamental principles of elastic energy storage, the relationship between work and potential energy, and the importance of the spring constant. The calculations demonstrate that a spring with a high stiffness (approximately 66,666.67 N/m) can store exactly the input energy when stretched over a small displacement, aligning theoretical predictions with physical intuition.

This case underscores the necessity of precise calculations in mechanical design, safety considerations when deploying elastic systems, and the educational value of such problems in illustrating core physics principles. Future investigations could explore non-ideal behaviors, energy losses, and dynamic stretching scenarios, enriching our understanding of real-world elastic systems.

In sum, the interplay between work, energy, and spring properties exemplifies the elegance of classical mechanics and its practical applications across engineering, physics, and technology.

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