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Understanding the Problem: Exploring the Nature of the Exponential Function

When confronted with a problem involving an exponential function, the key is to understand the fundamental properties of exponential functions and how they relate to given data points. The problem states that F(x)f(x) is an exponential function with specific values at certain points:

- F(-2.5) = 1
- F(2.5) = 1
- F(3.5) = 15

Our goal is to find a certain quantity related to this function—most likely the general form of F(x) or some specific value.

Step 1: Clarify the Nature of the Function

Before solving for any unknowns, let's analyze what it means for F(x)f(x) to be an exponential function.

Key properties of exponential functions:

- They can be written in the form:
$$F(x)f(x) = A \cdot r^{x}$$
where:
- A is a constant coefficient,
- r is the base of the exponential,
- x is the variable.

Given the data points, the function exhibits specific behavior at different x-values, which can help us determine A, r, and any other constants.

Step 2: Analyze the Given Data Points

Let's examine the points:

- $F(-2.5) = 1$
- $F(2.5) = 1$
- $F(3.5) = 15$

Notice that the function has the same value (1) at both -2.5 and 2.5. This symmetry suggests the exponential function might be centered or symmetric around a certain point.

Implication:

- Since $F(-2.5) = F(2.5)$, the function might involve even powers or symmetric properties, or perhaps the function's form includes a negative exponent that cancels out at these points.

Step 3: Establish Equations from the Data Points

Assuming the exponential function has the form:

$$
F(x)f(x) = A \cdot r^{x}
$$

then:

- At $x = -2.5$:

$$
A \cdot r^{-2.5} = 1
$$

- At $x = 2.5$:

$$
A \cdot r^{2.5} = 1
$$

- At $x = 3.5$:

$$
A \cdot r^{3.5} = 15
$$

Step 4: Solve for the Parameters A and r

Using the first two equations:

$$
A \cdot r^{-2.5} = 1 \quad \text{(Equation 1)}
$$
$$
A \cdot r^{2.5} = 1 \quad \text{(Equation 2)}
$$

Dividing Equation 2 by Equation 1:

$$
\frac{A \cdot r^{2.5}}{A \cdot r^{-2.5}} = \frac{1}{1}
$$

Simplifies to:

$$
r^{2.5 - (-2.5)} = 1
$$
$$
r^{5} = 1
$$

Therefore,

$$
r^{5} = 1
$$

which implies:

$$
r = 1^{1/5} = 1
$$

But if $r = 1$, then from Equation 1:

$$
A \cdot 1^{-2.5} = A = 1
$$

Similarly, from Equation 2:

$$
A \cdot 1^{2.5} = A = 1
$$

Now, check the third point:

$$
F(3.5) = A \cdot r^{3.5} = 1 \cdot 1^{3.5} = 1
$$

But the given value at $x=3.5$ is 15, which contradicts this.

Conclusion: The assumption of a simple form $A \cdot r^{x}$ may not suffice; perhaps the function involves additional parameters or a different form.

Step 5: Consider a More General Form — Including a Vertical Shift

Given the symmetry and the behavior, a more appropriate model may be:

$$
F(x)f(x) = C \cdot r^{x}
$$

or

$$
F(x)f(x) = a \cdot b^{x}
$$

where a and b are constants.

Let's revisit the data:

- $F(-2.5) = 1 \Rightarrow a \cdot b^{-2.5} = 1$
- $F(2.5) = 1 \Rightarrow a \cdot b^{2.5} = 1$

Dividing the two:

$$
\frac{a \cdot b^{2.5}}{a \cdot b^{-2.5}} = 1
$$
$$
b^{2.5 - (-2.5)} = 1
$$

\]

$$
b^{5} = 1
$$

Again, $b^5=1 \Rightarrow b=1$.

As before, this leads to inconsistency with the third point.

Step 6: Recognize the Need for a Different Functional Form

The symmetry suggests that the function could be of the form:

$$
F(x)f(x) = k \cdot r^{x}
$$

but possibly with a multiplicative factor that accounts for the jump at $x=3.5$.

Alternatively, perhaps the function is a piecewise exponential or involves a shift:

$$
F(x)f(x) = A \cdot r^{x} \quad \text{or} \quad F(x)f(x) = A \cdot r^{x} + D
$$

Step 7: Using Logarithms to Find r

Given the previous inconsistency, perhaps the function is of the form:

$$
F(x)f(x) = A \cdot r^{x}
$$

and the data indicates that:

$$
A \cdot r^{-2.5} = 1
$$

$$
A \cdot r^{2.5} = 1
$$

which implies:

$$
A = r^{2.5} \quad \text{and} \quad A = r^{-2.5}
$$

Combining:

$$
r^{2.5} = r^{-2.5}
$$

which implies:

$$
r^{2.5} = r^{-2.5} \Rightarrow r^{2.5 + 2.5} = r^{5} = 1
$$

Thus,

$$
r^{5} = 1
$$

and

$$
r = 1^{1/5} = 1
$$

again leading to the same inconsistency.

Step 8: Re-express the Data in Terms of Ratios

Given the repeated pattern, perhaps the function is symmetric around $x=0$, with the same value at symmetric points.

Since:

$$
F(-2.5) = F(2.5) = 1
$$

and

$$
F(3.5) = 15

\]

consider the possibility that the function is of the form:

\[
F(x)f(x) = M \cdot r^{|x|}
\]

which is even in \(x\).

Using \(x=2.5\):

\[
F(2.5) = M \cdot r^{2.5} = 1
\]
and at \(x=3.5\):

\[
F(3.5) = M \cdot r^{3.5} = 15
\]

Dividing the second by the first:

\[
\frac{M \cdot r^{3.5}}{M \cdot r^{2.5}} = \frac{15}{1} \Rightarrow r^{1} = 15
\]

Thus:

\[
r = 15
\]

From the first:

\[
M \cdot (15)^{2.5} = 1
\]

Calculate \(15^{2.5}\):

\[
15^{2.5} = 15^{2} \times 15^{0.5} = 225 \times \sqrt{15} \approx 225 \times 3.873 = 872.7
\]

Therefore,

\[
M = \frac{1}{872.7} \approx 0.001147
\]

The function:

$$
F(x)f(x) = 0.001147 \times 15^{|x|}
$$

Frequently Asked Questions

Given that F(x)f(x) is an exponential function with F(-2.5) = 1 and F(2.5) = 1, and F(3.5) = 15, how can we determine the general form of F(x)?

Since F(x)f(x) is exponential and F(-2.5) = 1, F(2.5) = 1, and F(3.5) = 15, we can assume $F(x) = a b^x$. Using the given points, we set up equations: $a b^{-2.5} = 1$, $a b^{2.5} = 1$, and $a b^{3.5} = 15$. Solving these will help find the values for a and b.

Why do the values F(-2.5) = 1 and F(2.5) = 1 suggest symmetry in the exponential function?

Because F(-2.5) = F(2.5) = 1, it indicates the function might be symmetric about the y-axis or centered at x = 0, implying that the base b of the exponential could be related to reciprocal powers, or that the function exhibits even symmetry.

How can we find the base b of the exponential function using the given data points?

From the points F(-2.5) = 1 and F(2.5) = 1, and assuming $F(x) = a b^x$, we get $a b^{-2.5} = 1$ and $a b^{2.5} = 1$. Dividing these equations gives $b^{5} = 1$, so b = 1 or b = -1. Since exponential functions typically have positive bases, b = 1, which suggests the function is constant, but since F(3.5)=15, $b \neq 1$. Therefore, more analysis is needed.

Given that F(3.5) = 15 and the previous points, how do we determine the coefficient a in the exponential function?

Using $F(3.5) = a b^{3.5} = 15$, once we find the value of b, we can substitute back to solve for a. For example, if b is known, then $a = 15 / b^{3.5}$.

What is the significance of these points in determining the exact exponential function F(x)?

The points provide specific values that allow us to set up equations to solve for the parameters of the exponential function. By using these points, we can determine the base b and the coefficient a, thus fully defining F(x).

Additional Resources

If F(x)f(x) Is An Exponential Function Where F(-2.5)=1, f(2.5)=1, and F(3.5)=15, then Find The

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Introduction

Mathematical analysis often involves deciphering the nature of complex functions based on given conditions. When confronted with the problem: "If $F(x)f(x)$ is an exponential function where $F(-2.5) = 1$, $f(2.5) = 1$, and $F(3.5) = 15$, then find the functions $F(x)$ and $f(x)$," we are invited into a deep exploration of functional relationships. This problem embodies the intriguing interplay between functions and their products, with the added twist that the product itself exhibits exponential behavior.

This investigation aims to rigorously analyze the problem, identify the underlying mathematical principles, and systematically derive the functions $F(x)$ and $f(x)$. The approach combines algebraic manipulations, properties of exponential functions, and logical deductions to arrive at a comprehensive solution.

Understanding the Problem and Initial Observations

Restatement of the Problem

- The product $F(x)f(x)$ is an exponential function, say $E(x) = F(x)f(x)$.
- Known values:
- $F(-2.5) = 1$
- $f(2.5) = 1$
- $F(3.5) = 15$

Key Questions

- What is the form of $F(x)$ and $f(x)$?
- How can the given data points help determine these functions?
- How does the exponential nature of $E(x)$ influence the structure of $F(x)$ and $f(x)$?

Initial Hypotheses

- Since $E(x) = F(x)f(x)$ is exponential, it can be written as $E(x) = A \cdot r^x$, where A and r are constants.
- The values provided suggest that at specific points, the product takes particular values, which could help determine A and r.

Formulating the Relationship: The Exponential Product

Assumption: $E(x) = F(x)f(x)$ is an exponential function

Let:

$$
E(x) = F(x)f(x) = A \cdot r^x
$$

where:

- A is the initial value constant,
- r is the common ratio base.

Using the Given Data Points

The known functional values are at different points:

1. $F(-2.5) = 1$
2. $f(2.5) = 1$
3. $F(3.5) = 15$

Since the product $E(x)$ is exponential, we can relate the known points to $E(x)$:

$$
E(-2.5) = F(-2.5)f(-2.5) = 1 \cdot f(-2.5) = \text{unknown}
$$

$$
E(2.5) = F(2.5)f(2.5) = F(2.5) \cdot 1 = F(2.5)
$$

$$
E(3.5) = F(3.5)f(3.5) = 15 \cdot f(3.5)
$$

But only the values of $F(-2.5)$, $f(2.5)$, and $F(3.5)$ are directly given, not the values of $f(-2.5)$, $F(2.5)$, or $f(3.5)$. This suggests that perhaps the functions F and f are related in a way that allows the reconstruction of their forms.

Exploring Possible Functional Forms

Hypothesis: $F(x)$ and $f(x)$ are exponential functions themselves

Suppose:

$$
F(x) = P \cdot a^x
$$
$$
f(x) = Q \cdot b^x
$$

where P, Q, a, b are constants to be determined.

In this case, their product becomes:

$$
E(x) = F(x)f(x) = P Q \cdot (a b)^x
$$

which is exponential with base $a b$.

Matching with the known data points:

- At $x = 2.5$,

$$
E(2.5) = F(2.5)f(2.5) = P Q \cdot (a b)^{2.5}
$$

- At $x = 3.5$,

$$
E(3.5) = F(3.5)f(3.5) = P Q \cdot (a b)^{3.5}
$$

- At $x = -2.5$,

$$
E(-2.5) = P Q \cdot (a b)^{-2.5}
$$

But from the given data:

- $F(-2.5) = 1$, so

$$
F(-2.5) = P a^{-2.5} = 1 \Rightarrow P = a^{2.5}
$$

- $f(2.5) = 1$, so

$$
f(2.5) = Q b^{2.5} = 1 \Rightarrow Q = b^{-2.5}
$$

- $F(3.5) = 15$, so

$$
F(3.5) = P a^{3.5} = a^{2.5} \cdot a^{3.5} = a^{6.0} = 15
$$

Thus,

$$
a^{6} = 15 \Rightarrow a = 15^{1/6}
$$

Similarly, from $f(2.5) = 1$:

$$
Q = b^{-2.5}
$$

Now, considering the product $E(x) = P Q \cdot (a b)^x$:

- At $x=2.5$:

$$
E(2.5) = P Q \cdot (a b)^{2.5}
$$

Calculate $P Q$:

$$
P Q = a^{2.5} \cdot b^{-2.5}
$$

Also,

$$
a b = a \times b
$$

Thus,

$$
E(2.5) = a^{2.5} \cdot b^{-2.5} \cdot (a b)^{2.5} = a^{2.5} b^{-2.5} \cdot a^{2.5} b^{2.5} = a^{2.5 + 2.5} \cdot b^{-2.5 + 2.5} = a^{5} \cdot b^{0} = a^{5}
$$

Recall that at $x=2.5$, $E(2.5) = F(2.5)f(2.5)$. Since $f(2.5) = 1$, then:

$$
E(2.5) = F(2.5) = \text{unknown}
$$

Similarly, at $x=3.5$:

$$
E(3.5) = P Q \cdot (a b)^{3.5}
$$

which simplifies to:

$$
E(3.5) = a^{2.5} b^{-2.5} \cdot (a b)^{3.5} = a^{2.5} b^{-2.5} \cdot a^{3.5} b^{3.5} = a^{6.0} \cdot b^{1.0}
$$

But from the previous step, $a^{6} = 15$, so:

$$
E(3.5) = 15 \cdot b
$$

From the known data:

$$
F(3.5)f(3.5) = E(3.5) = 15 \cdot f(3.5)
$$

But $F(3.5) = 15$, so:

$$
E(3.5) = 15 \cdot f(3.5) = 15 \cdot Q b^{3.5}
$$

Recall that $Q = b^{-2.5}$, so:

$$
E(3.5) = 15 \cdot b^{-2.5} \cdot b^{3.5} = 15 \cdot b^{1.0} = 15b
$$

which matches the earlier calculation.

Now, considering the previous expression:

$$
E(2.5) = a^{5}
$$

But we also know that:

$$
E(2.5) = F(2.5)f(2.5) = F(2.5) \times 1 = F(2.5)
$$

If F X F X Is An Exponential Function Where F 2 5 1f 2 5 1 And F 3 5 15f 3 5 15 Then Find The

If F X F X Is An Exponential Function Where F 2 5 1f 2 5 1 And F 3 5 15f 3 5 15 Then Find The

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