If N=28, X(xbar)=30, And S=16, Construct A Confidence Interval At A 90% Confidence Level. Assume The

Understanding the Basics of Confidence Intervals

If N=28, X̄=30, And S=16, Construct A Confidence Interval At A 90% Confidence Level. Assume The statement introduces a common statistical problem faced by researchers, students, and data analysts alike. Confidence intervals are essential tools in inferential statistics, allowing us to estimate an unknown population parameter (such as the mean) based on sample data. In this article, we will explore how to construct a confidence interval with the given data, explaining each step in detail, and providing insights into the underlying concepts.

What Is a Confidence Interval?

Before diving into calculations, it's crucial to understand what a confidence interval (CI) represents. A confidence interval is a range of values derived from sample data that is believed, with a certain level of confidence, to contain the true population parameter.

Key points about confidence intervals:

- They provide an estimated range rather than a single point estimate.
- The confidence level (e.g., 90%) indicates the proportion of such intervals that would contain the true parameter if the sampling process is repeated numerous times.
- They are widely used in hypothesis testing, quality control, and decision-making processes.

The Given Data and Its Significance

Let's analyze the data provided:

- Sample size (N): 28
- Sample mean (X̄): 30
- Sample standard deviation (S): 16
- Confidence level: 90%

Understanding these values:

- The sample size (28) suggests a moderate amount of data.
- The sample mean (30) is our point estimate for the population mean.
- The sample standard deviation (16) measures the variability within the sample.
- The confidence level indicates that we want to be 90% confident that the interval contains the true

population mean.

Choosing the Correct Statistical Method

Since the sample size is less than 30, and assuming the population distribution is approximately normal or the sample size is sufficiently large to invoke the Central Limit Theorem, we should use the t-distribution for constructing the confidence interval.

Why use the t-distribution?

- When the population standard deviation (σ) is unknown, and the sample size is small, the t-distribution provides a more accurate interval.
- For larger sample sizes (typically $N \geq 30$), the z-distribution could be used, but here, the conservative approach is to use the t-distribution.

Summary:

Parameter	Value
Sample size (N)	28
Degrees of freedom (df)	N - 1 = 27
Sample mean ($\bar{X}$)	30
Sample standard deviation (S)	16
Confidence level	90%

Calculating the Confidence Interval

The formula for a confidence interval for the mean when using the t-distribution is:

$$ \text{CI} = \bar{X} \pm t_{\alpha/2, df} \times \frac{S}{\sqrt{N}} $$

Where:

- $\bar{X}$ is the sample mean
- $t_{\alpha/2, df}$ is the t-value corresponding to the desired confidence level and degrees of freedom
- S is the sample standard deviation
- N is the sample size

Step 1: Find the t-value

For a 90% confidence level, the significance level α is:

$$ \alpha = 1 - 0.90 = 0.10 $$

Thus, $\alpha/2 = 0.05$.

Using a t-distribution table or calculator for \(df = 27\) and \(\alpha/2 = 0.05\), the critical t-value is approximately:

\[
t_{0.05, 27} \approx 1.703
\]

Step 2: Calculate the standard error (SE)

\[
SE = \frac{S}{\sqrt{N}} = \frac{16}{\sqrt{28}} \approx \frac{16}{5.2915} \approx 3.022
\]

Step 3: Compute the margin of error (ME)

\[
ME = t_{\alpha/2, df} \times SE = 1.703 \times 3.022 \approx 5.146
\]

Step 4: Construct the confidence interval

\[
\text{Lower limit} = \bar{X} - ME = 30 - 5.146 \approx 24.854
\]

\[
\text{Upper limit} = \bar{X} + ME = 30 + 5.146 \approx 35.146
\]

Final Confidence Interval:

\[
\boxed{
(24.85, 35.15)
}
\]

This means we are 90% confident that the true population mean lies between approximately 24.85 and 35.15 based on the sample data.

Interpreting the Confidence Interval

Understanding the meaning of this interval is vital:

- 90% confidence implies that if we were to take many samples and compute a confidence interval for each, approximately 90% of those intervals would contain the true population mean.
- The interval provides a plausible range for the population mean based on the sample data.
- The wider the interval, the more uncertainty exists; in this case, variability in the data (S=16) contributes to the width.

Factors Affecting Confidence Interval Width

Several factors influence the size of the confidence interval:

- Sample size (N): Larger N reduces the standard error, leading to a narrower interval.
- Sample variability (S): Higher variability increases the interval width.
- Confidence level: Higher confidence levels (e.g., 95%) produce wider intervals.
- Population distribution: Assumptions about normality impact the choice of method and the accuracy of the interval.

Practical Applications of Confidence Intervals

Confidence intervals are widely used across various fields:

- Healthcare: Estimating average blood pressure in a population.
- Manufacturing: Determining the average strength of a material.
- Market research: Estimating customer satisfaction scores.
- Environmental science: Assessing average pollutant levels.

By understanding how to construct and interpret confidence intervals, professionals can make informed decisions based on data.

Summary and Final Thoughts

Constructing a confidence interval involves understanding your data, choosing the appropriate distribution, and applying the correct formulas. In the case where $N=28$, $\bar{X}=30$, and $S=16$, at a 90% confidence level, the calculated interval is approximately from 24.85 to 35.15. This range provides a plausible estimate for the population mean, allowing decision-makers to assess the underlying data with a quantifiable level of confidence.

By mastering these steps, you can confidently analyze data and communicate findings effectively, ensuring your statistical insights are both accurate and meaningful.

Additional Tips for Accurate Confidence Interval Construction

- Always verify the assumptions underlying the use of t-distribution.
- Use reliable statistical software or tables for critical t-values.
- Ensure data quality and proper sampling techniques to enhance interval reliability.
- Remember that confidence intervals are estimates; they do not guarantee the true parameter lies within the interval in every case.

Conclusion

Confidence intervals are fundamental tools for statistical inference, enabling us to make educated guesses about population parameters based on sample data. By carefully following the steps—selecting the right distribution, calculating standard error, and applying the correct formulas—you can construct meaningful confidence intervals that inform research, policy, and business decisions. The example of constructing a 90% confidence interval for the mean with N=28, $\bar{X}$=30, and S=16 demonstrates how these principles come together in practice, providing valuable insights from data analysis.

Frequently Asked Questions

What is the sample size (N) given in the problem?

The sample size N is 28.

What is the sample mean ($\bar{X}$) provided?

The sample mean $\bar{X}$ is 30.

What is the sample standard deviation (S) provided?

The sample standard deviation S is 16.

What confidence level is specified for constructing the confidence interval?

The confidence level specified is 90%.

Which distribution should be used to construct the confidence interval in this case?

Since the population standard deviation is unknown and the sample size is less than 30, the t-distribution should be used.

How do you calculate the standard error (SE) in this scenario?

Standard error SE = S / sqrt(N) = 16 / sqrt(28).

What is the critical t-value for a 90% confidence interval with degrees of freedom df = N - 1?

The critical t-value for df=27 at 90% confidence level is approximately 1.703 (value may vary slightly depending on t-distribution table).

How do you compute the margin of error (ME) for the confidence interval?

Margin of error ME = t SE.

What are the steps to construct the 90% confidence interval?

Calculate SE = 16 / sqrt(28), find t-value for df=27 at 90%, compute ME = t SE, then the confidence interval is ($\bar{X}$ - ME, $\bar{X}$ + ME).

What is the final 90% confidence interval based on the given data?

First, SE ≈ 16 / 5.2915 ≈ 3.022; t ≈ 1.703; ME ≈ 1.703 3.022 ≈ 5.144. Therefore, the interval is (30 - 5.144, 30 + 5.144) ≈ (24.856, 35.144).

Additional Resources

Constructing a Confidence Interval When N=28, $\bar{X}$=30, and S=16 at a 90% Confidence Level

Understanding how to construct a confidence interval is a fundamental skill in statistics, especially when analyzing sample data to estimate an unknown population parameter. In this guide, we will walk through the process step-by-step for a scenario where the sample size (N) is 28, the sample mean ($\bar{X}$) is 30, and the sample standard deviation (S) is 16, aiming to build a 90% confidence interval for the population mean.

Introduction to Confidence Intervals

A confidence interval provides a range of plausible values for an unknown population parameter—most commonly, the population mean (μ). When we collect a sample, the sample mean ($\bar{X}$) serves as a point estimate, but due to sampling variability, we need a way to express the uncertainty inherent in this estimate. The confidence interval does exactly that: it indicates the range within which the true population mean is expected to lie with a specified level of confidence (e.g., 90%).

Key Concepts and Assumptions

Before constructing the confidence interval, it’s essential to understand the foundational concepts and assumptions involved:

- Sample Size (N): 28
- Sample Mean ($\bar{X}$): 30
- Sample Standard Deviation (S): 16
- Confidence Level: 90%

- Population Distribution: Assumed to be approximately normal or the sample size large enough for the Central Limit Theorem to apply.

Note: Since the sample size is less than 30, we typically use the t-distribution rather than the normal distribution, especially when the population standard deviation is unknown and the sample standard deviation is used as an estimate.

Step-by-Step Construction of the Confidence Interval

Step 1: Identify the Appropriate Distribution

Given the sample size ($N=28$), and the sample standard deviation ($S=16$), the sample is small enough to warrant the use of the t-distribution. The t-distribution accounts for additional uncertainty due to smaller samples.

Degrees of freedom (df):
$df = N - 1 = 28 - 1 = 27$

Step 2: Find the Critical t-Value

The critical t-value (t) corresponds to the desired confidence level and degrees of freedom. For a 90% confidence interval, the significance level (α) is:

$\alpha = 1 - 0.90 = 0.10$

Since the confidence interval is two-tailed, each tail has $\alpha/2 = 0.05$.

Using a t-distribution table or calculator for $df=27$ and $\alpha/2=0.05$:

$t \approx 1.703$

(Note: Exact t-values may vary slightly depending on the source, but 1.703 is typical for $df=27$ and 90% confidence level.)

Step 3: Calculate the Standard Error (SE)

The Standard Error (SE) measures the variability of the sample mean:

$SE = S / \sqrt{N}$

Plugging in the values:

$SE = 16 / \sqrt{28} \approx 16 / 5.2915 \approx 3.023$

Step 4: Calculate the Margin of Error (ME)

The Margin of Error tells us how much the sample mean might vary from the true population mean:

$ME = t \times SE$

$ME \approx 1.703 \times 3.023 \approx 5.147$

Step 5: Construct the Confidence Interval

The confidence interval is calculated as:

Lower Bound: $\bar{X} - ME = 30 - 5.147 \approx 24.853$
Upper Bound: $\bar{X} + ME = 30 + 5.147 \approx 35.147$

Therefore, the 90% confidence interval for the population mean (μ) is approximately (24.85, 35.15).

Interpreting the Confidence Interval

This interval suggests that, based on our sample data, we are 90% confident that the true population mean lies between approximately 24.85 and 35.15. It's important to clarify that this does not mean there is a 90% probability that μ is within this specific interval; rather, if we were to repeat the sampling process numerous times, approximately 90% of the constructed intervals would contain the true population mean.

Additional Considerations

When to Use the t-Distribution

The t-distribution is appropriate when:

- The population standard deviation (σ) is unknown.
- The sample size is small (typically less than 30).
- The sample data are approximately normally distributed or the sampling distribution of the mean is normal.

Verifying Assumptions

Before relying heavily on the confidence interval, consider:

- Normality of Data: For small samples, check whether the data are approximately normal (via plots or tests).
- Random Sampling: Ensure the sample is random to make valid inferences.
- Independence: Data points should be independent of each other.

Adjustments for Larger Samples

If N had been larger (e.g., over 30), the Central Limit Theorem suggests the sampling distribution of the mean would be approximately normal, and you might use the z-distribution instead of the t-distribution. In such cases, the critical z-value for 90% confidence is approximately 1.645.

Summary of the Process

Step	Description	Calculation / Value
1	Determine the appropriate distribution	t-distribution with df=27
2	Find the critical t-value	$t \approx 1.703$
3	Calculate Standard Error (SE)	$16 / \sqrt{28} \approx 3.023$
4	Compute Margin of Error (ME)	$1.703 \times 3.023 \approx 5.147$
5	Construct the interval	$(30 - 5.147, 30 + 5.147) \approx (24.85, 35.15)$

Final Thoughts

Constructing confidence intervals is a vital tool in statistical inference, providing a range within which the true population parameter likely resides. In cases like $N=28$, $\bar{X}=30$, and $S=16$, using the t-distribution ensures that the additional uncertainty due to small sample size is properly accounted for. Remember, the confidence level (here 90%) reflects the reliability of the method over many repetitions, not the probability for a specific interval.

By following these steps and understanding the underlying assumptions, you can confidently construct and interpret confidence intervals in various research contexts, making your analysis both rigorous and meaningful.

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