If The Given Matrix Is Invertible, Find Its Inverse. 5 3 A= 5 5 Find The Inverse. Select The Correct

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Understanding matrix invertibility and how to compute the inverse of a matrix is a fundamental concept in linear algebra. When given a specific matrix, such as

$$
A = \begin{bmatrix} 5 & 3 \\ 5 & 5 \end{bmatrix},
$$

determining whether it is invertible and, if so, finding its inverse, is a key skill. This article provides a comprehensive guide to understanding matrix invertibility, steps to find the inverse of a 2x2 matrix, and applying these principles to the given matrix A. Whether you're a student preparing for exams or a professional solving linear algebra problems, this guide will help clarify the process.

Understanding Matrix Invertibility

Before diving into calculations, it’s essential to understand what it means for a matrix to be invertible and the significance of the inverse.

What Is a Matrix Inverse?

A matrix A has an inverse A^{-1} if and only if:

$$
A \times A^{-1} = A^{-1} \times A = I,
$$

where I is the identity matrix of the same size as A. The inverse matrix essentially "undoes" the effect of A, similar to how reciprocals work for numbers.

Conditions for Invertibility

A square matrix A is invertible (also called non-singular) if:
- Its determinant $\det(A)$ is non-zero.
- It has full rank (for 2x2 matrices, rank 2).

- It does not have linear dependence among its rows or columns.

If $\det(A) = 0$, the matrix is singular, and an inverse does not exist.

Calculating the Inverse of a 2x2 Matrix

Since the given matrix A is 2x2, the process of finding the inverse is straightforward.

Formula for the Inverse of a 2x2 Matrix

Given a matrix:

$$
A = \begin{bmatrix} a & b \\ c & d \end{bmatrix},
$$

its inverse (if it exists) is:

$$
A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix},
$$

where the determinant $\det(A)$ is calculated as:

$$
\det(A) = ad - bc.
$$

Important: The determinant must be non-zero for the inverse to exist.

Step-by-Step Procedure

1. Calculate the determinant $\det(A)$.
2. Verify invertibility: If $\det(A) \neq 0$, proceed.
3. Construct the adjugate matrix: Swap a and d, negate b and c.
4. Multiply the adjugate by $1/\det(A)$.

Applying the Method to the Given Matrix A

Let's now apply these steps to the matrix:

\[
A = \begin{bmatrix} 5 & 3 \\ 5 & 5 \end{bmatrix}.
\]

Step 1: Calculate the determinant

\[
\det(A) = (5)(5) - (3)(5) = 25 - 15 = 10.
\]

Since the determinant is 10 (which is not zero), the matrix is invertible.

Step 2: Verify invertibility

Because \(\det(A) \neq 0\), \(A\) is invertible, and we can proceed to find the inverse.

Step 3: Construct the adjugate matrix

Swap the diagonal entries and negate the off-diagonal entries:

\[
\text{Adjugate of } A = \begin{bmatrix} 5 & -3 \\ -5 & 5 \end{bmatrix}.
\]

Step 4: Multiply by \(1/\det(A)\)

\[
A^{-1} = \frac{1}{10} \times \begin{bmatrix} 5 & -3 \\ -5 & 5 \end{bmatrix} =
\begin{bmatrix} 0.5 & -0.3 \\ -0.5 & 0.5 \end{bmatrix}.
\]

Final answer:

\[
\boxed{
A^{-1} = \begin{bmatrix} 0.5 & -0.3 \\ -0.5 & 0.5 \end{bmatrix}
}
\]

Summary of Key Points

- To determine if a 2x2 matrix is invertible, calculate the determinant.
- If the determinant is non-zero, the inverse exists and can be found using the formula involving the adjugate and determinant.
- For matrix $A = \begin{bmatrix} 5 & 3 \\ 5 & 5 \end{bmatrix}$, the inverse is $\begin{bmatrix} 0.5 & -0.3 \\ -0.5 & 0.5 \end{bmatrix}$.

Additional Tips and Common Mistakes

Tips for Calculating Matrix Inverses

- Always verify the determinant before attempting to find the inverse.
- Double-check arithmetic calculations when computing determinants and inverse matrices.
- Use a calculator or software for complex matrices to avoid calculation errors.

Common Mistakes to Avoid

- Assuming a matrix is invertible without checking the determinant.
- Swapping or negating entries incorrectly when forming the adjugate.
- Forgetting to divide by the determinant, leading to incorrect inverse matrices.

Practical Applications of Matrix Inverses

Understanding how to compute matrix inverses is not just an academic exercise; it has real-world applications, including:

- Solving systems of linear equations.
- Computer graphics transformations.
- Cryptography algorithms.
- Engineering computations involving linear systems.
- Data science tasks such as regression analysis.

Conclusion

In conclusion, determining whether a matrix is invertible hinges on calculating its determinant. For the given matrix:

$$
A = \begin{bmatrix} 5 & 3 \\ 5 & 5 \end{bmatrix},
$$

the determinant is 10, confirming invertibility. Applying the standard formula for 2x2 matrices yields the inverse:

$$
A^{-1} = \begin{bmatrix} 0.5 & -0.3 \\ -0.5 & 0.5 \end{bmatrix}.
$$

Mastering this process enhances your ability to handle linear algebra problems efficiently and accurately.

FAQs

1. **What if the determinant is zero?**
   The matrix is singular and does not have an inverse.

2. **Can I use software to find the inverse?**
   Yes, tools like MATLAB, NumPy (Python), or graphing calculators can compute matrix inverses quickly and accurately.

3. **Is the inverse always a 2x2 matrix?**
   No, the inverse exists for square matrices of any size (if invertible), but the formula differs for larger matrices.

By understanding the principles and steps outlined above, you are well-equipped to find the inverse of any invertible 2x2 matrix confidently.

Frequently Asked Questions

Given the matrix A = [[5, 3], [5, 5]], is A invertible?

Yes, matrix A is invertible because its determinant is non-zero.

How do you find the inverse of a 2x2 matrix A = [[a, b], [c, d]]?

The inverse is given by (1/det(A)) [[d, -b], [-c, a]], where det(A) = ad - bc.

Calculate the determinant of matrix A = [[5, 3], [5, 5]].

det(A) = (5)(5) - (3)(5) = 25 - 15 = 10.

What is the inverse of matrix A = [[5, 3], [5, 5]]?

Since det(A) = 10, the inverse is (1/10) [[5, -3], [-5, 5]] = [[0.5, -0.3], [-0.5, 0.5]].

Why is the determinant important when finding the inverse of a matrix?

The determinant indicates whether a matrix is invertible; if it's zero, the matrix does not have an inverse.

Is the inverse of the matrix [[5, 3], [5, 5]] unique?

Yes, every invertible matrix has a unique inverse.

Can the inverse of matrix A = [[5, 3], [5, 5]] be used to solve linear equations?

Yes, once you find the inverse, you can use it to solve systems of linear equations involving matrix A.

What are the steps to verify the inverse of a matrix?

Multiply the matrix by its supposed inverse; if the result is the identity matrix, the inverse is correct.

If the matrix is not invertible, what does that imply?

It implies that the determinant is zero and the matrix does not have an inverse.

How does the inverse of a 2x2 matrix differ from that of larger matrices?

The inverse of a 2x2 matrix can be calculated with a simple formula, whereas larger matrices require methods like row reduction or adjugate and determinant calculations.

Additional Resources

If The Given Matrix Is Invertible, Find Its Inverse: A Step-by-Step Guide to Solving the Matrix A = [[5, 3], [5, 5]]

In the world of linear algebra, finding the inverse of a matrix is a fundamental skill that allows us to solve systems of equations, analyze transformations, and understand the properties of matrices more deeply. When a matrix is invertible, it means there exists another matrix, called its inverse, which, when multiplied with the original, yields the identity matrix. This property is crucial because it enables us to "undo" transformations represented by matrices and solve equations efficiently.

In this guide, we will walk through the process of finding the inverse of the matrix A = [[5, 3], [5, 5]], assuming it is invertible. We will explore the criteria for invertibility, the step-by-step calculation of the inverse, and how to verify our result. Whether you're a student preparing for exams or a professional working on matrix operations, this comprehensive approach aims to clarify the process and deepen your understanding.

Understanding When a Matrix Is Invertible

Before diving into calculations, it’s essential to determine whether the matrix in question is invertible. For a 2x2 matrix:

$$
A = \begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
$$

the invertibility depends on its determinant:

$$
\det(A) = ad - bc
$$

- If $\det(A) \neq 0$, the matrix is invertible.
- If $\det(A) = 0$, the matrix is not invertible (singular).

Calculating the Determinant of A

Given:

$$
A = \begin{bmatrix}
5 & 3 \\
5 & 5
\end{bmatrix}
$$

Calculate:

$$
\det(A) = (5)(5) - (3)(5) = 25 - 15 = 10
$$

Since $\det(A) = 10 \neq 0$, matrix A is invertible.

Step-by-Step Process to Find the Inverse of a 2x2 Matrix

Once confirmed that the matrix is invertible, the inverse can be computed using a well-known formula for 2x2 matrices:

$$
A^{-1} = \frac{1}{\det(A)} \times \begin{bmatrix}
d & -b \\
-c & a
\end{bmatrix}
$$

Step 1: Identify elements a, b, c, d

From the matrix:

$$
A = \begin{bmatrix}
5 & 3 \\
5 & 5
\end{bmatrix}
$$

we have:

- $a = 5$
- $b = 3$
- $c = 5$
- $d = 5$

Step 2: Compute the determinant

As previously calculated:

$$
\det(A) = 10
$$

Step 3: Swap the elements on the main diagonal and change the signs of the off-diagonal elements

Construct the matrix:

$$
\begin{bmatrix}
d & -b \\
-c & a
\end{bmatrix} = \begin{bmatrix}
5 & -3 \\
-5 & 5
\end{bmatrix}
$$

Step 4: Multiply by the reciprocal of the determinant

$$
A^{-1} = \frac{1}{10} \times \begin{bmatrix}
5 & -3 \\
-5 & 5
\end{bmatrix}
$$

which simplifies to:

$$
A^{-1} = \begin{bmatrix}
\frac{5}{10} & -\frac{3}{10} \\
-\frac{5}{10} & \frac{5}{10}
\end{bmatrix}
$$

or more simply:

$$
A^{-1} = \begin{bmatrix}
0.5 & -0.3 \\
-0.5 & 0.5
\end{bmatrix}
$$

Verifying the Inverse

To ensure our calculation is correct, multiply the original matrix A by the computed inverse A^{-1} and verify that the result is the identity matrix:

$$
A \times A^{-1} = I
$$

Calculating:

\[
\begin{bmatrix}
5 & 3 \\
5 & 5
\end{bmatrix}
\times
\begin{bmatrix}
0.5 & -0.3 \\
-0.5 & 0.5
\end{bmatrix}
\]

First row, first column:

\[
(5)(0.5) + (3)(-0.5) = 2.5 - 1.5 = 1
\]

First row, second column:

\[
(5)(-0.3) + (3)(0.5) = -1.5 + 1.5 = 0
\]

Second row, first column:

\[
(5)(0.5) + (5)(-0.5) = 2.5 - 2.5 = 0
\]

Second row, second column:

\[
(5)(-0.3) + (5)(0.5) = -1.5 + 2.5 = 1
\]

Thus, the product is:

\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

which confirms that our inverse is correct.

Additional Tips and Common Pitfalls

1. Always check the determinant before attempting to find the inverse.

A zero determinant indicates a singular matrix, which doesn't have an inverse.

2. Be mindful of the order of operations.

Matrix multiplication is not commutative; ensure you multiply correctly when verifying.

3. For larger matrices (beyond 2x2), the inverse calculation involves more complex methods such as Gaussian elimination or the adjugate matrix method.

4. Use precise fractional forms when possible to avoid rounding errors during verification.

Summary: Key Takeaways

- Invertibility of a matrix hinges on its determinant being non-zero.
- For a 2x2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the inverse is:

$$
A^{-1} = \frac{1}{ad - bc} \times \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}
$$

- In the case of $A = \begin{bmatrix} 5 & 3 \\ 5 & 5 \end{bmatrix}$, the inverse is:

$$
A^{-1} = \begin{bmatrix} 0.5 & -0.3 \\ -0.5 & 0.5 \end{bmatrix}
$$

- Always verify your inverse by multiplying it with the original matrix to confirm you obtain the identity matrix.

Final Thoughts

Mastering the process of finding the inverse of matrices, especially small 2x2 matrices, is a valuable skill that forms the foundation for more advanced topics in linear algebra. By understanding the determinant's role, applying the standard formula, and verifying your results, you can confidently tackle similar problems. Remember, practice makes perfect—so challenge yourself with different matrices to solidify your understanding and develop intuition for matrix operations.

If The Given Matrix Is Invertible, Find Its Inverse is not just a routine calculation; it's a gateway to understanding how matrices function as transformations and how we can manipulate them to solve real-world problems. Keep practicing, and you'll find that matrix algebra becomes an intuitive and powerful tool in your mathematical toolkit.

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