If Three Times A Number Is At Least Four More Than Eight Times The Number, Which Of The Following Is

If Three Times A Number Is At Least Four More Than Eight Times The Number, Which Of The Following Is a common type of algebraic problem that challenges students to translate words into mathematical expressions and inequalities. Understanding how to approach such problems is essential for mastering algebra and developing problem-solving skills applicable in various real-world contexts. This article will explore the structure of these problems, step-by-step methods to solve them, and tips to improve your algebraic reasoning.

Understanding the Problem Statement

Before diving into solving the problem, it’s important to carefully analyze and interpret the statement. The phrase:

> "If three times a number is at least four more than eight times the number..."

contains key information that can be expressed mathematically.

Breaking Down the Statement

- "Three times a number" indicates 3 times some unknown number, which we can denote as x.
- "At least" signifies a greater than or equal to relationship.
- "Four more than eight times the number" means 8 x plus 4.

Putting this together, the sentence translates into an inequality:

$3x \geq 8x + 4$

This inequality is the core mathematical expression derived from the problem statement.

Formulating the Mathematical Inequality

Once the key components are identified, the next step is to formalize the inequality:

Step 1: Assign a Variable

Let x be the unknown number.

Step 2: Translate the Words into an Expression

- "Three times a number" → 3x
- "At least" → ≥ (greater than or equal to)
- "Four more than eight times the number" → 8x + 4

Step 3: Write the Inequality

Putting these together:

$3x \geq 8x + 4$

This is the primary inequality to solve.

Solving the Inequality

The goal is to find all values of x that satisfy the inequality.

Step 1: Bring all terms to one side

Subtract 8x from both sides:

$3x - 8x \geq 4$

which simplifies to:

$-5x \geq 4$

Step 2: Isolate x

Divide both sides by -5. Remember, dividing by a negative number reverses the inequality sign:

$x \leq -4 / 5$

Thus, the solution set is all real numbers less than or equal to -0.8.

Interpreting the Solution

The inequality $x \leq -0.8$ indicates that any number less than or equal to -0.8 satisfies the original statement: "Three times a number is at least four more than eight times the number."

Visualizing the Solution

A simple number line can illustrate the solution:

- Mark -0.8 as a point.

- Shade all numbers to the left of -0.8, including -0.8 itself.

Any value of x within this region makes the inequality true.

Checking the Solution with Examples

To ensure the solution is correct, test values:

- For x = -1:

3(-1) = -3

8(-1) + 4 = -8 + 4 = -4

Is -3 ≥ -4? Yes, so x = -1 satisfies the inequality.

- For x = 0:

3(0) = 0

8(0) + 4 = 4

Is 0 ≥ 4? No, so x = 0 does not satisfy the inequality.

- For x = -0.8:

3(-0.8) = -2.4

8(-0.8) + 4 = -6.4 + 4 = -2.4

Is -2.4 ≥ -2.4? Yes, equality holds.

This confirms the boundary point is included in the solution.

Key Concepts in Solving Inequalities

Understanding inequalities involves several fundamental concepts:

1. Variable Isolation

Just like equations, inequalities require isolating the variable to find its range of values.

2. Reversing Inequality Signs

When multiplying or dividing both sides by a negative number, always reverse the inequality sign.

3. Solution Set Representation

Solutions can be expressed in various forms:

- Interval notation: (-∞, -0.8]
- Graphical representation: Number line shading to indicate all solutions.

Additional Example Problems

Practicing similar problems can deepen understanding.

Example 1:

If five times a number is less than or equal to ten more than twice the number, what is the inequality?

Solution:

- "Five times a number" → 5x
- "Less than or equal to" → ≤
- "Ten more than twice the number" → 2x + 10

Inequality:

$5x \leq 2x + 10$

Subtract 2x from both sides:

$3x \leq 10$

Divide both sides by 3:

$x \leq 10/3 \approx 3.33$

Solutions are all real numbers less than or equal to approximately 3.33.

Example 2:

If seven times a number is at least fifteen less than three times the number, find the inequality.

Solution:

- "Seven times a number" → 7x
- "At least" → ≥
- "Fifteen less than three times the number" → 3x - 15

Inequality:

$7x \geq 3x - 15$

Subtract 3x from both sides:

$4x \geq -15$

Divide both sides by 4:

$x \geq -15/4 = -3.75$

Solutions include all numbers greater than or equal to -3.75.

Real-World Applications of Inequalities

Inequalities are not just academic exercises; they are vital in real-world scenarios:

- Budgeting: Ensuring expenses stay below a certain limit.
- Engineering: Maintaining safety margins.
- Statistics: Establishing confidence intervals.
- Economics: Analyzing profit or cost constraints.

For example, suppose a manufacturer wants to keep production costs below a certain threshold; inequalities help model these constraints.

Tips for Solving Inequalities Effectively

- Always perform inverse operations carefully.
- Remember to reverse the inequality when multiplying or dividing by a negative.
- Check your solutions by substituting back into the original inequality.
- Use graphing tools or number lines to visualize solutions.
- Practice with a variety of problems to build confidence and understanding.

Conclusion

Transforming word problems into algebraic inequalities is a fundamental skill that enables problem-solving across mathematics and real-life situations. In the specific case of "If three times a number is at least four more than eight times the number," the key steps involve translating the statement into an inequality, solving for the variable, and interpreting the solution. Mastering these steps enhances your algebraic reasoning and prepares you for more complex mathematical challenges. Keep practicing different problem types, and over time, you'll develop a strong intuition for translating words into equations and inequalities.

Frequently Asked Questions

What does the inequality 'Three times a number is at least four more than eight times the number' translate to mathematically?

It translates to $3x \geq 8x + 4$.

How do you set up an inequality based on the statement 'Three times a number is at least four more than eight times the number'?

You set it as $3x \geq 8x + 4$.

What is the first step to solve the inequality $3x \geq 8x + 4$?

Subtract 8x from both sides to get $-5x \geq 4$.

What is the solution to the inequality $-5x \geq 4$?

Divide both sides by -5, remembering to flip the inequality sign: $x \leq -4/5$.

What range of values for x satisfies the inequality 'Three times a number is at least four more than eight times the number'?

Any x less than or equal to -4/5.

Which of the following options correctly represents the

solution set for the problem: 'x ≤ -4/5', 'x ≥ -4/5', 'x < -4/5', or 'x > -4/5'?

x ≤ -4/5.

If you are asked to find 'which of the following' options for the inequality, what should you compare?

Compare each option to the solution set x ≤ -4/5 to see which is correct.

Why is it important to flip the inequality sign when dividing both sides of an inequality by a negative number?

Because dividing by a negative reverses the inequality direction, changing ≥ to ≤ or vice versa.

Additional Resources

Understanding the Mathematical Problem: "If Three Times A Number Is At Least Four More Than Eight Times The Number, Which Of The Following Is"

When approaching algebraic inequalities, it’s important to carefully translate words into mathematical expressions, analyze the relationships, and determine the solution set. The problem presented here is a classic example of an inequality involving multiple terms with a variable, which requires precise interpretation and methodical solving.

Introduction to the Problem

The problem statement reads:
“If three times a number is at least four more than eight times the number, which of the following is…”

While the question appears incomplete, the core of this type of problem generally involves interpreting a comparative relationship between two expressions involving an unknown number. The key phrase here is “at least”, which in mathematical terms indicates a “greater than or equal to” relationship.

The typical structure can be summarized as:

> Three times a number ≥ Four more than eight times the number.

This naturally translates into an inequality and forms the basis for solving the problem.

Step-by-Step Breakdown of the Problem

1. Define the Variable

- Let the unknown number be represented by a variable, commonly x.
- The problem involves three times this number: $3x$.
- It also involves eight times this number: $8x$.

2. Identify the Relationship

- The phrase “at least” implies a “greater than or equal to” relationship:

$$
3x \geq \text{(something)}
$$

- The phrase “Four more than eight times the number” translates to:

$$
8x + 4
$$

- Combining these, the inequality becomes:

$$
3x \geq 8x + 4
$$

This is the core inequality derived from the problem statement.

Translating Words into Mathematical Inequality

Key Components

- "Three times a number": $3x$
- "Four more than eight times the number": $8x + 4$
- "Is at least": $\geq$

Constructed Inequality

$$
3x \geq 8x + 4
$$

This inequality now serves as the foundation for solving the problem.

Solving the Inequality

Step 1: Bring all terms to one side

$$
3x - 8x \geq 4
$$

Simplify:

$$
-5x \geq 4
$$

Step 2: Isolate x

Dividing both sides by -5 to solve for x:

$$
x \leq \frac{4}{-5}
$$

Recall that dividing by a negative number reverses the inequality sign:

$$
x \leq -\frac{4}{5}
$$

3. Final Solution Set

The solution to the inequality is:

$$
x \leq -\frac{4}{5}
$$

This means any number less than or equal to -0.8 satisfies the original statement.

Interpreting the Solution

What does $x \leq -\frac{4}{5}$ signify?

- The values of x that satisfy the inequality are all numbers less than or equal to -0.8.
- This set includes numbers like -1, -1.5, -2, and so on, extending infinitely in the negative direction, as well as -0.8 itself.

Graphical Representation

- On a number line, represent the solution with a solid dot at -0.8, indicating inclusion.
- Shade the region to the left of -0.8, encompassing all smaller numbers.

Implications in Real-World Contexts

- While the problem is abstract, such inequalities often model scenarios where a certain quantity (represented by x) must be less than or equal to a particular threshold.
- For example, if x represented a measurement (say, temperature, speed, or dosage), the solution indicates the acceptable range.

Additional Considerations and Common Pitfalls

1. Sign Reversal When Dividing by Negative

- It’s crucial to remember that dividing or multiplying both sides of an inequality by a negative number reverses the inequality sign.
- Failing to do so leads to incorrect solutions.

2. Checking the Solution

- Pick a value satisfying the solution set, for example, -1, and verify:

$$
3(-1) = -3
$$
$$
8(-1) + 4 = -8 + 4 = -4
$$

Check if:

$$
-3 \geq -4
$$

which is true.
Hence, the solution set is valid.

- Conversely, pick 0, which does not satisfy $x \leq -0.8$:

$$
3(0) = 0
$$
$$
8(0) + 4 = 4
$$

Check:

$$
0 \geq 4
$$

which is false.
This confirms the correctness of the solution.

Exploring Related Variations and Extensions

1. What if the inequality was “less than”?

- Change $\geq$ to $<$, leading to:

$$
3x < 8x + 4
$$

- Solving:

$$
3x - 8x < 4
$$
$$
-5x < 4
$$

- Divide by -5, reversing the inequality:

$$
x > -\frac{4}{5}
$$

- Solution set: all x greater than -0.8.

2. Handling equality cases

- When the inequality is “at least,” the solution includes the boundary point $x = -\frac{4}{5}$.

3. Real-world applications

- Such inequalities are common in budgeting, physics, and engineering, where upper or lower bounds are specified.
- For example, if a process requires a parameter x to be less than or equal to a certain threshold to prevent failure, the solution set guides operational limits.

Understanding the Broader Context of Inequalities

1. Importance in Mathematics

- Inequalities are foundational in mathematical analysis, optimization, and problem-solving.
- They help describe constraints and feasible solutions in real-world scenarios.

2. Types of Inequalities

- Linear inequalities (like the one discussed)
- Quadratic inequalities
- Polynomial inequalities
- Rational inequalities
- Absolute value inequalities

3. Techniques for Solving Inequalities

- Simplification of expressions
- Isolating the variable
- Reversing inequalities when multiplying/dividing by negatives
- Graphical interpretation
- Testing points within the solution region

Conclusion

In sum, the problem “If three times a number is at least four more than eight times the number, which of the following is...” revolves around translating a verbal statement into an algebraic inequality, solving it carefully, and interpreting the solution. The key steps involve identifying the correct expressions, correctly manipulating the inequality with attention to the sign changes, and understanding the solution set both numerically and graphically.

The solution $x \leq -\frac{4}{5}$ indicates that any number less than or equal to -0.8 satisfies the given condition. This type of problem exemplifies the importance of precise interpretation of language, rigorous algebraic manipulation, and thoughtful analysis. Mastery of such inequalities is fundamental not only in academic settings but also in numerous practical applications where constraints and thresholds are common.

Additional Tips for Solving Similar Problems:

- Always clearly define your variables.
- Carefully translate words into algebraic expressions.
- Pay attention to keywords like “at least,” “more than,” “less than,” “not less than,” etc., as they determine the inequality sign.
- When solving, remember the special rule about dividing or multiplying by negative numbers.
- Verify your solutions by substitution or graphical reasoning.
- Practice with a variety of problems to develop intuition and speed.

By understanding these principles and working through the problem systematically, you develop confidence in handling algebraic inequalities and interpreting their solutions in both abstract and real-world contexts.

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