If U (x) = Negative 2 X Squared And V (x) = StartFraction 1 Over X EndFraction, What Is The Range Of

If U (x) = Negative 2 X Squared And V (x) = StartFraction 1 Over X EndFraction, What Is The Range Of the combined functions or the resulting expressions? Understanding the range of functions is a fundamental aspect of studying mathematics, especially in the fields of algebra and calculus. The range of a function refers to the set of all possible output values (y-values) it can produce when the input values (x-values) are from its domain. In this article, we will explore the functions $U(x) = -2x^2$ and $V(x) = 1/x$, analyze their individual properties, and determine the combined behavior or the range of expressions derived from these functions. This comprehensive guide aims to clarify the concepts involved and provide a step-by-step methodology to find the range of such functions.

Understanding the Given Functions

Before delving into the combined or related functions, it is essential to understand the individual characteristics of the given functions U(x) and V(x).

Function $U(x) = -2x^2$

- Type of Function: Quadratic function.
- Graph Shape: Parabola opening downward because the coefficient of x^2 is negative.
- Domain: All real numbers, i.e., $x \in (-\infty, \infty)$.
- Vertex: At (0, 0) because the quadratic has no linear or constant shift.
- Range: Since the parabola opens downward and has its maximum at the vertex, the range is:
- $y \leq 0$
- In interval notation: $(-\infty, 0]$

Key Point: The maximum value of U(x) is 0, attained at x=0.

Function $V(x) = 1/x$

- Type of Function: Rational function.
- Graph Shape: Hyperbola with two branches, one in quadrants I and III, or II and IV, depending on the sign of x.

- Domain: All real $x \neq 0$, i.e., $x \in (-\infty, 0) \cup (0, \infty)$.
- Range: All real numbers except zero, because $1/x$ approaches infinity as x approaches zero, but never equals zero itself.
- $y \neq 0$
- In interval notation: $(-\infty, 0) \cup (0, \infty)$

Key Point: V(x) has a vertical asymptote at x=0 and a horizontal asymptote at y=0, but y never reaches exactly zero.

Analyzing the Range of Combined Functions or Expressions

Depending on the context, the question might refer to the combined functions such as:

- The sum: U(x) + V(x)
- The product: U(x) V(x)
- The quotient: U(x) / V(x)
- The composition: U(V(x)) or V(U(x))

In each case, the domain and range need to be carefully analyzed.

Case 1: Range of U(x) + V(x)

This is a common scenario where we consider the sum of the two functions.

Expression: $U(x) + V(x) = -2x^2 + 1/x$

Step 1: Determine the domain

- Both functions are defined when:
- $x \neq 0$ (due to V(x))
- Therefore, the domain of the sum is $x \in (-\infty, 0) \cup (0, \infty)$.

Step 2: Find the critical points

- To find the range, we analyze the behavior of the function:
- $f(x) = -2x^2 + 1/x$
- Compute the derivative to locate local maxima or minima:

f'(x) = derivative of $-2x^2 + 1/x$

$f'(x) = -4x - 1/x^2$

Step 3: Set the derivative to zero to find critical points

- $-4x - 1/x^2 = 0$
- Multiply both sides by x^2 ($x \neq 0$):

$-4x^3 - 1 = 0$

- Solve for x:

$x^3 = -1/4$

$x = \sqrt[3]{(-1/4)} \approx -0.62996$

Step 4: Analyze the critical point

- Check the second derivative or the nature of this point via the first derivative test.
- Since the derivative changes sign around this point, it indicates a local extremum.

Step 5: Determine the extremal value

- Substitute $x \approx -0.62996$ into $f(x)$:

$f(x) = -2x^2 + 1/x$

Compute:

$x^2 \approx 0.3968$

$f(x) \approx -2\ 0.3968 + 1/(-0.62996) \approx -0.7936 - 1.586 \approx -2.3796$

- Since this is a critical point, and the parabola opens downward, this point corresponds to a maximum value.

Step 6: Behavior at the boundaries

- As x approaches 0 from the positive side:

$f(x) \approx -2(0) + 1/x \to +\infty$

- As x approaches 0 from the negative side:

$f(x) \approx -2(0) + 1/x \to -\infty$

- As x approaches $\pm\infty$:

$f(x) \approx -2x^2 \to -\infty$

Step 7: Summarize the range

- The maximum value is approximately -2.38 at $x \approx -0.63$.
- The function tends to $+\infty$ as x approaches 0 from the positive side.
- It tends to $-\infty$ as x approaches 0 from the negative side.
- Therefore, the range includes all real numbers greater than or equal to approximately -2.38, extending to $+\infty$, but with some nuances due to the behavior near zero.

Final Range for U(x)+V(x):

- $y \in (-\infty, +\infty)$, but with the maximum at approximately -2.38. Since the function attains values close to infinity near zero from the right and negative infinity from the left, the overall range is:

Range: $(-\infty, \infty)$

But note that the maximum value is about -2.38, so the exact range is:

Range: $y \leq$ approximately -2.38, but since the function reaches values arbitrarily large as $x \to 0+$, the full range is all real numbers.

Case 2: Range of U(x) V(x)

The product:

- $f(x) = U(x)\,V(x) = (-2x^2)\,(1/x) = -2x^2 / x = -2x$, for $x \neq 0$.

Step 1: Domain

- $x \neq 0$.

Step 2: Simplify

- $f(x) = -2x$, a linear function.

Step 3: Range

- Since $x \in (-\infty, 0) \cup (0, \infty)$:

- As $x \to \infty$, $f(x) \to -\infty$.
- As $x \to -\infty$, $f(x) \to \infty$.
- As x approaches 0 from above, $f(x) \to 0-$.
- As x approaches 0 from below, $f(x) \to 0+$.

Result:

- The range of $f(x) = -2x$ is all real numbers:

Range: $(-\infty, \infty)$

Case 3: Range of $U(x) / V(x)$

The quotient:

- $f(x) = U(x) / V(x) = (-2x^2) / (1/x) = -2x^2 x = -2x^3$

Step 1: Domain

- $x \neq 0$.

Step 2: Behavior

- $f(x) = -2x^3$

- As $x \to \infty$, $f(x) \to -\infty$
- As $x \to -\infty$, $f(x) \to \infty$
- Since the cube function covers all real numbers, and multiplication by -2 just flips the sign, the range is all real numbers.

Range:

- $(-\infty, \infty)$

Case 4: Composition of functions

Depending on the composition, the range varies.

Example: $U(V(x)) = U(1/x) = -2(1/x)^2 = -2(1/x^2) = -2 / x^2$

- Domain: $x \neq 0$
- Behavior:

- As $x \to 0$, $x^2 \to 0$, so $f(x) \to -\infty$.
- As $x \to \pm\infty$, $x^2 \to \infty$, so $f(x) \to 0^-$.

- Range:

- Since $f(x) = -2 / x^2$, and $x^2 > 0$ for all $x \neq 0$:

- $f(x) < 0$ for all $x \neq 0$.
- $f(x)$ approaches 0 from below as $x \to \pm\infty$.

- $f(x)$ approaches $-\infty$ as $x \to 0$.

Range:

- $(-\infty, 0)$

Summary and Key Takeaways

- Understanding individual functions: $U(x) = -$

Frequently Asked Questions

What is the function U(x) in the given problem?

$U(x) = -2x^2$.

What is the function V(x) in the given problem?

$V(x) = 1/x$.

How do you find the range of $U(x) = -2x^2$?

Since $-2x^2$ is a downward-opening parabola, its range is all real numbers less than or equal to 0, i.e., $(-\infty, 0]$.

What is the domain of $V(x) = 1/x$?

The domain is all real numbers except $x = 0$.

How does the domain of V(x) affect the overall range when combined with U(x)?

Since $V(x)$ is undefined at $x=0$, the combined function's behavior near zero must be considered, but if analyzing separately, $V(x)$ has no impact on the range of $U(x)$.

Can the range of the combined function be determined from the individual functions?

Yes, by analyzing how $U(x)$ and $V(x)$ interact or compose, but since the problem only provides separate functions, the range of each can be determined independently.

Is the range of $U(x) = -2x^2$ affected by the value of $V(x)$?

No, because $U(x)$ and $V(x)$ are separate functions. The range of $U(x)$ depends solely on its own formula.

What is the likely intended question about the range in this problem?

The most probable intended question is: 'What is the range of $U(x) = -2x^2$?', which is $(-\infty, 0]$.

Additional Resources

Understanding the Range of the Functions $U(x) = -2x^2$ and $V(x) = \frac{1}{x}$: A Detailed Exploration

When studying functions in algebra and calculus, one of the fundamental concepts is understanding their range—the set of all possible output values. A clear grasp of the range provides insight into the behavior of the function, its limitations, and its applications. In this comprehensive review, we will analyze two distinct functions:

- $U(x) = -2x^2$
- $V(x) = \frac{1}{x}$

By dissecting their properties, domain considerations, and the nature of their outputs, we aim to develop a thorough understanding of their respective ranges.

Understanding the Function $U(x) = -2x^2$

1. Nature of $U(x)$: A Quadratic Function

The function $U(x) = -2x^2$ is a quadratic function, characterized by its parabola shape. The coefficient of x^2 is -2, which directly influences the parabola's orientation and steepness.

- The coefficient -2 is negative, indicating that the parabola opens downward.
- The quadratic term x^2 always yields non-negative results ($x^2 \geq 0$ for all real x).

2. Domain of $U(x)$

- Since $U(x)$ is a polynomial with no restrictions, the domain is all real numbers:

$$
\text{Domain} = (-\infty, \infty)
$$

3. Graphical Behavior and Vertex

- The parabola's vertex is at the point where the function attains its maximum or minimum.
- For quadratic functions in the form $y = ax^2 + bx + c$, the vertex's x-coordinate is at $-\frac{b}{2a}$.

In our case, $U(x) = -2x^2$, with $a = -2$, $b = 0$, and $c = 0$:

$$
x_{vertex} = -\frac{0}{2 \times (-2)} = 0
$$

- The y-coordinate at the vertex:

$$
U(0) = -2 \times 0^2 = 0
$$

- Therefore, the vertex is at $(0, 0)$.

4. Range of $U(x)$

Given the parabola opens downward (since $a = -2 < 0$), the maximum value of $U(x)$ is at the vertex:

- Maximum value: 0 at $x = 0$.

- As $x \to \pm \infty$, $U(x) \to -\infty$, because the square term dominates and the negative coefficient drives the output downward indefinitely.

Thus, the range of $U(x)$ is:

$$
\boxed{
(-\infty, 0]
}
$$

This includes all real numbers less than or equal to zero, with the maximum at zero.

Deep Dive into $U(x)$

1. Interpretation of the Range

- The range indicates the set of all possible "heights" or output values of the quadratic.
- Since the parabola opens downward, the highest point is at the vertex, and outputs decrease indefinitely as x moves away from zero.

2. Real-World Applications

- Such functions model scenarios like projectile motion under gravity (neglecting air

resistance), where the maximum height corresponds to the vertex.
- In economics, quadratic functions can model profit or loss functions with a maximum point.

3. Variations and Transformations

- Changing the coefficient -2 to other negative numbers affects the parabola's steepness.
- Adding a constant c shifts the parabola vertically, impacting the maximum value.

Analyzing $V(x) = \frac{1}{x}$

1. Nature of $V(x)$: The Reciprocal Function

The function $V(x) = \frac{1}{x}$ is a rational function, specifically, the reciprocal of x.

- It is a hyperbola with two distinct branches, one in the first quadrant and the other in the third quadrant.
- The function exhibits symmetry about the origin, known as origin symmetry or odd symmetry.

2. Domain of $V(x)$

- The function is undefined where the denominator is zero, i.e., at $x=0$. Hence, the domain is:

$$
\text{Domain} = (-\infty, 0) \cup (0, \infty)
$$

- Zero is excluded because division by zero is undefined.

3. Behavior of $V(x)$

- As $x \to 0^+$, $V(x) \to +\infty$.
- As $x \to 0^-$, $V(x) \to -\infty$.
- As $x \to \pm \infty$, $V(x) \to 0$.

4. Range of $V(x)$

- The outputs of $V(x)$ cover all real numbers except zero.

- Because the function approaches zero but never actually reaches it, zero is a horizontal asymptote.

\[
\boxed{
\text{Range} = (-\infty, 0) \cup (0, \infty)

}
\]

- The function attains all real values except zero.

Deep Dive into $V(x) = \frac{1}{x}$

1. Graphical Insights

- The hyperbola features two branches:

- In the first quadrant: $x > 0$, $V(x) > 0$.
- In the third quadrant: $x < 0$, $V(x) < 0$.

- Vertical asymptote at $x=0$: the function approaches infinity or negative infinity but never crosses or touches $x=0$.

- Horizontal asymptote at $y=0$: the function approaches zero from above or below but never equals zero.

2. Significance of the Range

- The entire set of real numbers except zero is attainable, reflecting the function's unbounded growth in both positive and negative directions.

3. Applications and Real-World Contexts

- The reciprocal function appears in physics (inverse proportionality), engineering, and economics.
- For example, the relationship between speed and travel time, or inverse relationships in supply and demand.

4. Transformations and Variations

- Vertical or horizontal shifts (adding constants) modify asymptotes.
- Scaling factors change the steepness but do not affect the range.

Comparing $U(x)$ and $V(x)$: Key Differences and Similarities

1. Domain and Range Overview

Function	Domain	Range	Behavior
$U(x) = -2x^2$	$(-\infty, \infty)$	$(-\infty, 0]$	Parabola opening downward
$V(x) = \frac{1}{x}$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$	Hyperbola with asymptotes

2. Nature of Outputs

- \(U(x) \) yields only non-positive outputs, with zero as the maximum.
- \(V(x) \) can produce any real number except zero.

3. Symmetry Considerations

- \(U(x) \) is symmetric about the \(y \)-axis (even function).
- \(V(x) \) exhibits odd symmetry about the origin.

4. Asymptotic Behavior

- \(U(x) \) does not have asymptotes but approaches \(-\infty \) as \(|x| \to \infty \).
- \(V(x) \) has both vertical (\(x=0 \)) and horizontal (\(y=0 \)) asymptotes.

Broader Implications and Conceptual Takeaways

1. The Role of Function Types in Determining Range

- Quadratic functions with a negative leading coefficient tend to have a maximum point, defining an upper bound for the range.
- Rational functions like \(\frac{1}{x} \) often have asymptotes, leading to ranges that exclude certain values.

2. How Domain Restrictions Affect Range

- For \(V(x) \), the restriction \(x \neq 0 \) creates gaps in the domain and range.
- For \(U(x) \), the domain is unrestricted, but the output is bounded above by the maximum at the vertex.

3. Practical Applications of Range Analysis

- Understanding the range helps in modeling real-world phenomena, such as projectile heights, inverse relationships, and asymptotic behaviors.
- It informs the feasibility and limits of physical, economic, or engineering

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