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Introduction to Uniform Distribution and Transformation of Random Variables

Understanding the behavior of transformed random variables is fundamental in probability theory and statistics. When a random variable undergoes a function transformation, such as taking the absolute value, its probability distribution (specifically, the probability density function or PDF) changes accordingly. This article explores this concept by examining a uniformly distributed random variable X over the interval $[1, 1]$ and analyzing the distribution of $Y = |X|$.

Clarifying the Uniform Distribution over the Interval [1, 1]

Understanding the Distribution's Nature

The statement "If X is uniformly distributed between 1 and 1" appears to describe a degenerate case. Typically, a uniform distribution over an interval $[a, b]$, where $a < b$, is characterized by a constant probability density over that entire interval. However, when $a = b$, the distribution collapses to a single point, effectively becoming a degenerate distribution.

Key points:

- Degenerate Distribution: When the interval collapses to a single point, say c, the distribution assigns probability 1 to $X = c$ and 0 elsewhere.
- Implication: The probability density function (PDF) becomes a Dirac delta function at c.

Therefore:

- If the interval is $[1, 1]$, then X is degenerate at the point 1.
- The PDF of X is:

$$
f_X(x) = \delta(x - 1)
$$

where $\delta(\cdot)$ is the Dirac delta function.

Assuming the Intended Distribution: Uniform Between 1 and 2

Given the degenerate case is trivial and not insightful, it's common in problems like this to consider the uniform distribution over a non-zero interval, such as $[1, 2]$.

Hence, we assume:

$$
X \sim \text{Uniform}(1, 2)
$$

with the PDF:

$$
f_X(x) =
\begin{cases}
\frac{1}{2 - 1} = 1, & x \in [1, 2] \\
0, & \text{otherwise}
\end{cases}
$$

Note: If the original problem indeed meant $[1, 1]$, then $Y = |X|$ is deterministic, and the distribution of Y is degenerate at 1. But for richer analysis, the above assumption is more instructive.

Finding the Distribution of $Y = |X|$ When $X \sim \text{Uniform}(1, 2)$

Step 1: Understanding $Y = |X|$

- Since X takes values in $[1, 2]$, which are positive, the absolute value operation does not alter the values.

- Therefore:

$$
Y = |X| = X
$$

- This implies that:

$$
Y \sim \text{Uniform}(1, 2)
$$

- The PDF of Y is identical to that of X:

$$
f_Y(y) =
\begin{cases}
1, & y \in [1, 2] \\
0, & \text{otherwise}
\end{cases}
$$

In this case, the distribution of Y is straightforward.

Step 2: Considering the Case of $X \sim \text{Uniform}(-a, a)$

To explore more general scenarios, let's suppose:

$$
X \sim \text{Uniform}(-a, a)
$$

where $a > 0$.

- The PDF:

$$
f_X(x) =
\begin{cases}
\frac{1}{2a}, & x \in [-a, a] \\
0, & \text{otherwise}
\end{cases}
$$

- The transformation:

$$
Y = |X|
$$

- Since X is symmetric about zero, Y takes values in $[0, a]$.

Step 3: Deriving the PDF of Y

- For $y \in [0, a]$, the possible X values satisfying $|X| = y$ are:

$$
X = y \quad \text{and} \quad X = -y
$$

- The probability mass associated with each is:

$$
f_X(y) = \frac{1}{2a} \quad \text{for} \quad y \in [0, a]
$$

- The distribution of Y can be obtained via the change of variables:

$$
f_Y(y) = f_X(y) + f_X(-y) = \frac{1}{2a} + \frac{1}{2a} = \frac{2}{2a} = \frac{1}{a}
$$

for $y \in (0, a)$, noting that at $y=0$, the probability density is derived from the limit.

- At $y=0$, the PDF is:

$$
f_Y(0) = \frac{1}{a}
$$

but since the point has zero measure in continuous distributions, the density at zero is the same as elsewhere in the interval.

Therefore, the PDF of $Y = |X|$ when $X \sim \text{Uniform}(-a, a)$ is:

$$
f_Y(y) =
\begin{cases}
\frac{1}{a}, & y \in [0, a] \\
0, & \text{otherwise}
\end{cases}
$$

This is a uniform distribution over $[0, a]$.

Summary of Key Results

Degenerate Distribution at a Single Point

- When $X \sim \text{Uniform}(1, 1)$, it is degenerate at 1.
- $Y = |X|$ is deterministic: $Y = 1$.
- The PDF of Y is a Dirac delta at 1:

$$
f_Y(y) = \delta(y - 1)
$$

Uniform Distribution Over $[a, b]$ with $a \geq 1$

- If $X \sim \text{Uniform}(a, b)$, with $1 \leq a < b$,
- Then $Y = |X|$ has PDF:

$$
f_Y(y) =
\begin{cases}
\frac{1}{b - a} & y \in [a, b] \quad \text{(if } a \geq 0) \\
\text{more complex if } a < 0
\end{cases}
$$

- For symmetric distributions over $[-a, a]$, the PDF of Y is uniform over $[0, a]$:

$$
f_Y(y) = \frac{1}{a}, \quad y \in [0, a]
$$

Conclusion and Final Remarks

The distribution of the transformed variable $Y = |X|$ heavily depends on the initial distribution of X. In the special case where X is degenerate at a point (such as 1), the distribution of Y is trivial—a point mass at that value. When X is uniformly distributed over an interval $[a, b]$ with $a \geq 1$, the distribution of Y reflects the range of X and can be directly derived.

In practical applications, understanding how transformations like the absolute value affect distributions is essential for modeling and inference. Whether dealing with symmetric distributions or degenerate cases, the principles outlined here provide a systematic approach to deriving the PDFs of transformed variables.

Frequently Asked Questions

What is the probability density function (PDF) of a uniform random variable X between 1 and 1?

Since X is uniformly distributed between 1 and 1, it is a degenerate distribution at the point 1, meaning P(X=1)=1 and the PDF is a delta function at x=1.

How do you define the new variable Y = |X| when X is uniformly distributed between 1 and 1?

Since X takes the value 1 with probability 1, Y=|X| is also 1 with probability 1, making its distribution degenerate at 1.

What is the PDF of Y = |X| given that X is uniformly distributed between 1 and 1?

The PDF of Y is a degenerate distribution at y=1, expressed as f_Y(y) = δ(y - 1), where δ is the Dirac delta function.

Are there any other values Y can take, given X is between 1 and 1?

No, since X is always 1, Y=|X| is always 1, so Y can only take the value 1.

Can the PDF of Y be expressed as a standard distribution?

No, because Y is a degenerate random variable at 1, its PDF is a delta function, not a standard distribution like uniform or normal.

What implications does the degenerate distribution have for statistical analysis?

It means there is no variability in Y; all probability mass is concentrated at 1, so standard statistical measures like variance are zero.

If the distribution of X was between a and b (not 1 and 1), how would you find the PDF of Y=|X|?

You would analyze the distribution of X over [a, b], then transform it to find the distribution of Y=|X|

by considering the absolute value mapping, accounting for whether the interval includes negative values.

Does the absolute value transformation change the distribution if X is always positive?

No, if X is always positive, then Y=|X| is the same as X, so the distribution of Y is identical to that of X.

What is the key takeaway about the distribution of Y when X is degenerate at a point?

Y will also be degenerate at the same point, resulting in a delta function PDF concentrated at that value.

Additional Resources

Uniform Distribution and Transformation: Deriving the PDF of Y = |X| for a Uniform Random Variable

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Introduction

In probability theory and statistical analysis, understanding how transformations of random variables affect their distributions is fundamental. One common transformation involves taking the absolute value of a random variable, which often appears in fields ranging from signal processing to econometrics. When the original variable exhibits a uniform distribution, the resulting distribution after such a transformation exhibits unique characteristics that merit detailed exploration.

This article delves into the case where the random variable \(X\) follows a uniform distribution on the interval \([1, 1]\). Although this particular interval suggests a degenerate case (since the interval's endpoints are the same), it appears to be a typographical error or a conceptual placeholder. For the purposes of this discussion, we will interpret the problem as considering \(X\) uniformly distributed over \([-1, 1]\), a standard and well-studied interval in probability theory. We will then derive the probability density function (PDF) of the transformed variable \(Y = |X|\).

Understanding the Uniform Distribution on \([-1, 1]\)

Definition and Properties

The uniform distribution on the interval \([-1, 1]\) is a continuous probability distribution where every number within the interval is equally likely. Its defining characteristics include:

- PDF: \(f_X(x) = \frac{1}{b - a} = \frac{1}{2}\), for \(x \in [-1, 1]\)
- Support: \([-1, 1]\)
- Cumulative Distribution Function (CDF): \(F_X(x) = \frac{x + 1}{2}\), for \(x \in [-1, 1]\)

This distribution is symmetric about zero, which simplifies many calculations involving transformations like the absolute value.

Significance of the Support

Choosing $[-1, 1]$ is typical in many analytical contexts, especially when considering symmetry. The uniform distribution over this interval provides an ideal setting for examining the effect of the absolute value transformation because the symmetry simplifies the derivation of the new distribution.

Transformation of Random Variables: The Case of $Y = |X|$

Why Transformations Matter

Transformations of random variables are central in probability and statistics because they allow modeling of complex phenomena and the derivation of new distributions from known ones. In this case, the transformation $Y = |X|$ effectively maps the original domain $[-1, 1]$ into $[0, 1]$, focusing on the magnitude of X.

The Absolute Value Transformation

The absolute value function is a non-linear transformation that reflects negative values onto positive territory. When applied to a symmetric distribution like the uniform distribution on $[-1, 1]$, it produces a distribution concentrated on $[0, 1]$ with a mass that accounts for both positive and negative parts of the original distribution.

Deriving the PDF of $Y = |X|$

Step 1: Identify the Range of Y

Since X ranges from -1 to 1, the absolute value $Y = |X|$ will range from 0 to 1. Therefore:

$$
Y \in [0, 1]
$$

Step 2: Express the Distribution of Y

The key to deriving $f_Y(y)$ is understanding the relationship between the events:

$$
Y \leq y \quad \text{and} \quad Y = |X|
$$

Because the absolute value operation is symmetric about zero, for each $y \in [0, 1]$:

- The pre-image under the transformation is $\{-y, y\}$

- The probability that $Y \leq y$ involves integrating over the segments $[-y, y]$

Step 3: Use the PDF of X

Given that X is uniformly distributed on $[-1, 1]$ with:

$$
f_X(x) = \frac{1}{2}, \quad x \in [-1, 1]
$$

The probability that $|X| \leq y$ is:

$$
P(Y \leq y) = P(|X| \leq y) = P(-y \leq X \leq y)
$$

Calculating this probability:

$$
P(|X| \leq y) = \int_{-y}^{y} f_X(x) \, dx = \int_{-y}^{y} \frac{1}{2} \, dx = \frac{1}{2} \times (2y) = y
$$

This indicates that the cumulative distribution function (CDF) of Y is:

$$
F_Y(y) = P(Y \leq y) = y, \quad y \in [0,1]
$$

Step 4: Derive the PDF of Y

Since the CDF $F_Y(y) = y$ is differentiable on $[0, 1]$, the PDF is:

$$
f_Y(y) = \frac{d}{dy} F_Y(y) = 1, \quad y \in [0,1]
$$

This is a uniform distribution on $[0, 1]$.

Summary of Results

The transformation of a uniformly distributed variable $X \sim U(-1, 1)$ under the absolute value operation yields a new variable $Y = |X|$ with a uniform distribution over $[0, 1]$. Its PDF is succinctly expressed as:

$$
\boxed{
f_Y(y) = \begin{cases}
1, & 0 \leq y \leq 1 \\

0, & \text{otherwise}
\end{cases}
}
\]

This result is both elegant and intuitive: since \(X\) is equally likely at any point in \([-1, 1]\), the absolute value, which folds negative values onto positive ones, results in a distribution with equal density over \([0, 1]\).

Broader Implications and Applications

Symmetry and Distribution Transformation

The derivation underscores the importance of symmetry in distributions. When the original distribution is symmetric about zero, transformations like the absolute value often produce simpler, well-understood distributions.

Practical Applications

- Signal Processing: Absolute values of signals often represent magnitudes or energies.
- Econometrics: Modeling absolute deviations or errors often involves similar transformations.
- Machine Learning: Feature transformations that involve absolute values can influence model behavior and interpretability.

Limitations and Considerations

While the derivation assumes a uniform distribution over \([-1, 1]\), different underlying distributions or intervals will modify the resulting distribution of the transformed variable. For example, if \(X\) were normally distributed, the distribution of \(|X|\) would follow a folded normal distribution, which is more complex.

Conclusion

In conclusion, the transformation of a uniform random variable over \([-1, 1]\) under the absolute value results in a uniform distribution over \([0, 1]\). The derivation hinges on symmetry and the properties of the uniform distribution, illustrating a straightforward yet powerful example of how transformations influence probability distributions.

Understanding these transformations is crucial for statisticians and data scientists, providing insights into how data manipulations impact underlying probability models. This knowledge not only facilitates better modeling but also enhances interpretation across diverse fields such as engineering, finance, and artificial intelligence.

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