# II) A 55-kg Woman And A 72-kg Man Stand 10.0 m Apart On Frictionless Ice. (a) How Far From The Woman

## Introduction to Conservation of Momentum and Center of Mass

### Understanding the Scenario

The problem involves a 55-kg woman and a 72-kg man standing on frictionless ice, initially 10.0 meters apart. Since the ice is frictionless, there are no external horizontal forces acting on the system, and the principle of conservation of momentum applies. When they push off against each other, they will recoil in opposite directions, but the center of mass of the system will remain stationary. The key question is: How far from the woman does the man end up after they push off?

This scenario exemplifies fundamental physics concepts such as conservation of momentum, center of mass, and relative motion. Understanding these principles allows us to analyze how the two individuals move relative to each other and determine their final positions on the ice.

## Fundamental Principles Underlying the Problem

### Conservation of Momentum

In the absence of external forces, the total momentum of the system remains constant. Initially, both are stationary, so the total initial momentum is zero. When they push apart, their momenta are equal in magnitude and opposite in direction, ensuring total momentum remains zero.

### Center of Mass Concept

The center of mass (CM) of a system is the weighted average position of all parts, considering their masses. Since no external forces act horizontally, the CM remains stationary. This provides a powerful tool for analyzing their final positions after the push.

## Step-by-Step Analysis of the Problem

### Initial Conditions

- Mass of woman, $m_w = 55\,kg$
- Mass of man, $m_m = 72\,kg$
- Initial separation, $d_{initial} = 10.0\,m$
- Initial velocities, $v_{w,i} = v_{m,i} = 0\,m/s$

### Final Conditions

- Let $x_w$ be the distance the woman moves from her original position.
- Let $x_m$ be the distance the man moves from his original position.
- Since they push off, they will move in opposite directions along the line connecting them.

### Applying Conservation of Momentum

- The total momentum before push: 0
- After push, total momentum must still be 0:

$$
m_w v_w + m_m v_m = 0
$$

$$
\Rightarrow m_w v_w = - m_m v_m
$$

- The negative sign indicates motion in opposite directions.

### Relating Velocities and Displacements

- Using the relation:

$$
v_w = - \frac{m_m}{m_w} v_m
$$

- Since the velocities are related, and assuming the push occurs over a very short time, the displacements $x_w$ and $x_m$ are proportional to their velocities:

$$
x_w = v_w t
$$

$$
x_m = v_m t
$$

- Eliminating $v_w$ and $v_m$:

$$
x_w = - \frac{m_m}{m_w} x_m
$$

Note: The negative sign indicates that if the woman moves in one direction, the man moves in the opposite.

## Calculating the Final Positions

### Using the Center of Mass Frame

Since the initial CM is stationary, it must remain so:

$$
m_w x_{w,final} + m_m x_{m,final} = 0
$$

- Let's choose the initial position of the woman as zero:

$$
x_{w,initial} = 0
$$

$$
x_{m,initial} = 10\,m
$$

- After movement:

$$
x_{w} = x_{w,initial} + x_w
$$

$$
x_{m} = x_{m,initial} + x_m
$$

Applying the CM condition:

$$
m_w x_w + m_m (x_{m,initial} + x_m) = 0
$$

But because their initial positions are fixed and the system is internal, the relative positions are more straightforwardly analyzed by considering their displacements from initial positions.

Alternatively, focusing on their relative displacement:

$$
\text{Initial separation} = 10\,m
$$

$$
\text{Final separation} = (x_{m,initial} + x_m) - (x_{w,initial} + x_w)
$$

Since the initial positions are fixed:

$$
\text{Final separation} = (10 + x_m) - x_w
$$

Using the relation between $x_w$ and $x_m$:

$$
x_w = - \frac{m_m}{m_w} x_m
$$

Plugging into the final separation:

$$
\text{Final separation} = 10 + x_m - \left( - \frac{m_m}{m_w} x_m \right) = 10 + x_m + \frac{m_m}{m_w} x_m

\]

$$
= 10 + x_m \left( 1 + \frac{m_m}{m_w} \right)
$$

$$
= 10 + x_m \left( \frac{m_w + m_m}{m_w} \right)
$$

This indicates that the final separation depends on $x_m$, which is related to the velocities after push.

## Using Momentum Conservation to Find $x_m$ and $x_w$

From earlier, the ratio of displacements is:

$$
x_w = - \frac{m_m}{m_w} x_m
$$

Because the total momentum in the system is zero:

$$
m_w v_w = - m_m v_m
$$

and assuming the velocities are related to displacements over the same short time interval $t$:

$$
v_w = \frac{x_w}{t}
$$

$$
v_m = \frac{x_m}{t}
$$

Rearranged:

$$
m_w \frac{x_w}{t} = -m_m \frac{x_m}{t}
$$

$$
m_w x_w = - m_m x_m
$$

which matches the displacement relation above.

Now, the key is to find the magnitude of the displacements. To do this, we need to consider the impulse or the relative velocities resulting from the push.

However, since the problem asks "How far from the woman?" and assuming the push is impulsive and the individuals recoil with velocities determined by their masses, the analysis simplifies to the following:

- The center of mass remains stationary.
- The displacements of each individual are inversely proportional to their masses because their momenta are equal and opposite.

Thus:

$$
x_w = \frac{m_m}{m_w + m_m} \times D_{initial}
$$
$$
x_m = \frac{m_w}{m_w + m_m} \times D_{initial}
$$

This relation is derived from the principle that after pushing off, the individuals have moved such that the CM remains at the same point, and the displacements are proportional to their masses.

Final Calculation

$$
x_w = \frac{m_m}{m_w + m_m} \times D_{initial} = \frac{72}{55 + 72} \times 10\,m
$$

$$
x_w = \frac{72}{127} \times 10\,m \approx 0.5677 \times 10\,m \approx 5.677\,m
$$

Similarly,

$$
x_m = \frac{m_w}{m_w + m_m} \times D_{initial} = \frac{55}{127} \times 10\,m \approx 0.4331 \times 10\,m \approx 4.331\,m
$$

Interpretation:

- The woman moves approximately 5.68 meters away from her original position.
- The man moves approximately 4.33 meters away from his original position.

Since the initial separation was 10 meters, the final separation between them is:

$$
\text{Final separation} = 10\,m - (x_w + x_m) \approx 10\,m - (5.68\,m + 4.33\,m) = 10\,m - 10.01\,m \approx -0.01\,m
$$

This negligible negative value indicates they essentially end up very close to each other, consistent with conservation principles and the initial separation.

How Far From the Woman Is the Man After Push Off?

The question asks: How far from the woman is the man?

- Since the woman moves roughly 5.68 meters in one direction,
- and the man moves roughly 4.33 meters in the opposite direction,

The distance between them after the push is approximately:

$$
\boxed{
x_{final} = x_w + x_m \approx 5.68\,m + 4.33\,m \approx 10.01\,m
}
$$

which aligns

# Frequently Asked Questions

### What is the main physical principle involved when a woman and a man stand on frictionless ice and move apart?

The main principle is conservation of momentum, which states that the total momentum of an isolated system remains constant when no external forces act on it.

### How do you calculate the distance the woman moves when the man moves on frictionless ice?

You set the initial total momentum to zero and use the relation $m_1 x_1 = m_2 x_2$, where $m_1$ and $m_2$ are the masses and $x_1$ and $x_2$ are the displacements. Since the total separation is 10.0 m, you can find the woman's distance from her starting point by solving for $x_1$.

### What is the initial setup for solving the problem of the woman and man on ice?

Initially, both are stationary, so total momentum is zero. When they move apart, their momenta are equal in magnitude and opposite in direction, maintaining the total momentum at zero.

### How do the masses of the woman and man influence their respective movements on ice?

The heavier person (the man) will move less than the lighter person (the woman) because their displacements are inversely proportional to their masses, ensuring momentum conservation.

### What is the approximate distance the woman moves from her original position if they are 10 m apart and the woman weighs 55 kg while the man weighs 72 kg?

Using conservation of momentum, the woman moves approximately 3.9 meters from her initial position while the man moves about 5.9 meters, maintaining the total separation of 10 meters.

# Additional Resources

Ice Dynamics and Center of Mass: An Expert Exploration of a Woman and Man on Frictionless Ice

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# Introduction: The Fascinating World of Frictionless Motion

Imagine a scenario where two individuals, a woman weighing 55 kg and a man weighing 72 kg, stand on a perfectly frictionless ice surface, separated by a distance of 10 meters. This seemingly simple setup embodies fundamental principles of physics—specifically Newtonian mechanics, conservation of momentum, and the center of mass concept. Such problems are not just academic exercises; they serve as excellent models to understand real-world phenomena, from the motion of spacecraft in space to the dynamics of ice skaters and astronauts.

In this article, we will delve into the detailed physics governing this scenario, focusing on calculating how far the woman moves when they begin to push off each other. We will not only provide a step-by-step derivation of the solution but also explore the underlying principles, assumptions, and implications of such a system. Whether you're a physics student, educator, or curious reader, this comprehensive review aims to clarify the intricacies involved in this fascinating problem.

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# Understanding the Physical Principles

Before diving into calculations, it's essential to grasp the fundamental concepts that underpin the problem:

## Conservation of Momentum

- Definition: The total momentum of an isolated system remains constant if no external forces act upon it.
- Relevance: Since the ice is frictionless and, assuming no external pushes, the total momentum before and after the push remains zero.

## Center of Mass (CM)

- Definition: The point representing the average position of mass in a system.
- Key Point: For an isolated system with no external forces, the center of mass remains stationary.

## Internal Forces and External Forces

- Internal Forces: The push between the woman and the man.
- External Forces: Since the ice is frictionless, external horizontal forces are negligible; gravity acts vertically and does not influence horizontal motion.

These principles form the backbone of our analysis: the internal push causes the two individuals to move in opposite directions, conserving the total momentum and keeping the center of mass stationary.

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# Setting Up the Problem: Known Data and Assumptions

| Parameter | Value | Notes |
|-----------|--------|--------|
| Mass of woman | $m_w = 55\, \text{kg}$ | Known |
| Mass of man | $m_m = 72\, \text{kg}$ | Known |
| Initial separation | $d = 10.0\, \text{m}$ | Known |
| Friction | Zero (frictionless ice) | Assumption |
| Initial velocities | $v_{w,i} = v_{m,i} = 0$ | Rest initially |
| Final velocities | $v_{w,f}$, $v_{m,f}$ | To be calculated |
| Distance moved | $x_w$ (woman), $x_m$ (man) | To be determined |

Assumptions:
- Both individuals push off simultaneously with equal and opposite internal forces.
- The push is impulsive; their initial and final velocities are the primary focus.
- No external horizontal forces act during the push.
- The ice surface ensures no energy loss through friction.

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## Analyzing the Motion: Conservation of Momentum

Since the system is isolated horizontally, the total momentum before and after the push is conserved:

$$
\text{Initial momentum} = 0
$$
$$
\text{Final momentum} = m_w v_{w,f} + m_m v_{m,f} = 0
$$

From this, it follows:

$$
m_w v_{w,f} = - m_m v_{m,f}
$$

or

$$
v_{w,f} = - \frac{m_m}{m_w} v_{m,f}
$$

The negative sign indicates that the woman and the man move in opposite directions.

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## Determining the Final Velocities and Displacements

Key insight: The internal push results in the individuals moving in opposite directions, but the center of mass remains stationary.

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## Center of Mass Condition

The position of the center of mass (CM) at any time is:

$$
X_{CM} = \frac{m_w x_w + m_m x_m}{m_w + m_m}
$$

Since initially both are stationary, and the entire system is isolated, the CM remains at its initial position:

$$
X_{CM, initial} = \frac{m_w \times 0 + m_m \times 10\, \text{m}}{m_w + m_m} = \frac{72 \times 10}{55 + 72}
$$

Calculating:

$$
X_{CM, initial} = \frac{720}{127} \approx 5.67\, \text{m}
$$

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## Displacements and the Stationary Center of Mass

Let's define:

- $x_w$: the displacement of the woman from her initial position.
- $x_m$: the displacement of the man from his initial position.

Since the initial positions are:

- Woman: $x_{w,i} = 0$
- Man: $x_{m,i} = 10\, \text{m}$

And their final positions are:

- Woman: $x_{w,f} = x_w$
- Man: $x_{m,f} = 10 + x_m$

The center of mass condition implies:

$$
X_{CM} = \frac{m_w x_w + m_m (10 + x_m)}{m_w + m_m} = 5.67\, \text{m}
$$

Substituting known values:

$$
\frac{55 x_w + 72 (10 + x_m)}{127} = 5.67
$$

Multiplying both sides by 127:

$$
55 x_w + 720 + 72 x_m = 127 \times 5.67
$$

Calculating the right side:

$$
127 \times 5.67 \approx 127 \times 5.67 = 720.09
$$

Now, the equation becomes:

$$
55 x_w + 72 x_m = 720.09 - 720 = 0.09
$$

This simplifies to:

$$
55 x_w + 72 x_m = 0.09

---

## Relating Displacements via Velocities

From conservation of momentum, the velocities are related:

$$
v_{w,f} = - \frac{m_m}{m_w} v_{m,f}
$$

Assuming the push occurs over a short time interval $\Delta t$:

$$
x_w = v_{w,f} \Delta t
$$

$$
x_m = v_{m,f} \Delta t
$$

Thus:

$$
x_w = - \frac{m_m}{m_w} x_m
$$

which leads to:

$$
x_w = - \frac{72}{55} x_m \approx -1.309 x_m
$$

The negative sign indicates the woman moves in the opposite direction to the man.

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## Calculating the Displacements

Substitute this relation into the CM equation:

$$
55 x_w + 72 x_m = 0.09
$$

Replacing $x_w$:

$$
55 \times (-1.309 x_m) + 72 x_m = 0.09
$$

$$
-72.0 x_m + 72 x_m = 0.09
$$

Simplify:

$$
( -72.0 + 72 ) x_m = 0.09
$$

$$
0 \times x_m = 0.09
$$

This suggests an inconsistency, highlighting that our initial assumption of displacements proportional to velocities over the same time interval is problematic because the push is impulsive and the actual displacements depend on the impulse applied, not just velocity.

Alternative approach: Since the push is impulsive, and no external forces act, the key is that the center of mass remains fixed. The individual displacements are inversely proportional to their masses, and the total shift of the two bodies must compensate to keep the CM stationary.

Given the initial CM position and the fact that the two individuals move in opposite directions, the displacement of the woman can be derived directly from the conservation of momentum and the initial positions.

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## Final Calculation: How Far the Woman Moves

Given the symmetry and the conservation principles, the displacement of the woman, $x_w$, can be directly calculated using the relation:

$$
m_w x_w + m_m x_m = 0
$$

since the total shift of the system's CM is zero, and the initial CM is at 5.67 m.

Recall:

$$
x_m = - \frac{m_w}{m_m} x_w
$$

The total movement of the woman relative to her initial position is $x_w$, and her final position relative to the initial is:

$$
x_{w,\text{final}} = x_w
$$

Similarly, the man moves:

$$
x_{m,\text{final}} = 10 +
$$

# Ii A 55 Kg Woman And A 72 Kg Man Stand 10 0 M Apart On Frictionless Ice A How Far From The Woman

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