Image Transcription TextQuestion 3(a) Calculate The Angles That Vector H = 3ax + 5ay - 8az Makeswith

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Understanding Vector Angles and Their Significance

When working with vectors in physics and engineering, one fundamental task is to determine the angles that a given vector makes with the coordinate axes or with other vectors. Such angles provide critical insights into the vector's orientation in space, help analyze forces, velocities, or other quantities modeled as vectors, and are essential in solving many real-world problems.

In this article, we focus on the specific problem of calculating the angles that the vector H = 3ax + 5ay - 8az makes with the coordinate axes. This task involves understanding vector components, dot products, and vector magnitudes, which are foundational concepts in vector calculus and physics.

Breaking Down the Problem

The problem statement is:
Calculate the angles that vector H = 3ax + 5ay - 8az makes with the coordinate axes.

This entails determining three separate angles:

- The angle between H and the x-axis (denoted as θx)
- The angle between H and the y-axis (denoted as θy)
- The angle between H and the z-axis (denoted as θz)

Understanding how to find these angles involves the following key concepts:

- Vector components
- Dot product of vectors
- Magnitude (or length) of a vector
- The formula relating the dot product and the angle between vectors

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Fundamental Concepts for Calculating Vector Angles

Vector Components

A vector in three-dimensional space can be expressed in component form as:

$$
\vec{H} = H_x \mathbf{a}_x + H_y \mathbf{a}_y + H_z \mathbf{a}_z
$$

In this problem:

$$
\vec{H} = 3\mathbf{a}_x + 5\mathbf{a}_y - 8\mathbf{a}_z
$$

where:

- $H_x = 3$,
- $H_y = 5$,
- $H_z = -8$.

Magnitude of a Vector

The magnitude (or length) of vector H is calculated as:

$$
|\vec{H}| = \sqrt{H_x^2 + H_y^2 + H_z^2}
$$

For our vector:

$$
|\vec{H}| = \sqrt{3^2 + 5^2 + (-8)^2} = \sqrt{9 + 25 + 64} = \sqrt{98}
$$

which simplifies to:

$$
|\vec{H}| = \sqrt{98} \approx 9.899
$$

Dot Product and Angle Between Vectors

The angle θ between two vectors $\vec{A}$ and $\vec{B}$ is given by the formula:

$$
\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}
$$

where:

- $\vec{A} \cdot \vec{B}$ is the dot product
- $|\vec{A}|$ and $|\vec{B}|$ are the magnitudes of the vectors

When calculating the angle that H makes with the coordinate axes, the other vector is the unit vector along each axis:

- $\mathbf{a}_x = (1, 0, 0)$
- $\mathbf{a}_y = (0, 1, 0)$
- $\mathbf{a}_z = (0, 0, 1)$

Calculating the Angles with Coordinate Axes

Angle with the x-axis (θ_x)

The unit vector along the x-axis is:

$$
\mathbf{a}_x = (1, 0, 0)
$$

The dot product $\vec{H} \cdot \mathbf{a}_x$ is:

$$
(3)(1) + (5)(0) + (-8)(0) = 3
$$

Using the dot product formula:

$$
\cos \theta_x = \frac{\vec{H} \cdot \mathbf{a}_x}{|\vec{H}| \times |\mathbf{a}_x|} = \frac{3}{9.899 \times 1} \approx \frac{3}{9.899} \approx 0.303
$$

Therefore:

$$
\theta_x = \arccos(0.303) \approx 72.3^\circ
$$

Angle with the y-axis (θ_y)

The unit vector along the y-axis:

$$
\mathbf{a}_y = (0, 1, 0)
$$

Dot product:

$$
(3)(0) + (5)(1) + (-8)(0) = 5
$$

Calculate:

$$
\cos \theta_y = \frac{5}{9.899 \times 1} \approx 0.505
$$

Thus:

\[

$$
\theta_y = \arccos(0.505) \approx 59.8^\circ
$$

Angle with the z-axis (θ_z)

The unit vector along the z-axis:

$$
\mathbf{a}_z = (0, 0, 1)
$$

Dot product:

$$
(3)(0) + (5)(0) + (-8)(1) = -8
$$

Calculate:

$$
\cos \theta_z = \frac{-8}{9.899 \times 1} \approx -0.808
$$

Therefore:

$$
\theta_z = \arccos(-0.808) \approx 143.1^\circ
$$

Note that the angle with the negative z-axis is greater than 90°, indicating that H points downward relative to the z-axis.

Summary of Results

Axis	Vector Component	Calculated Cosine	Angle ($^{\circ}$)
x-axis	$H_x = 3$	0.303	approximately 72.3°
y-axis	$H_y = 5$	0.505	approximately 59.8°
z-axis	$H_z = -8$	-0.808	approximately 143.1°

Practical Applications of Calculating Vector Angles

Understanding the angles a vector makes with the axes is vital across various fields:

- Physics: To analyze forces, velocities, and accelerations in three-dimensional space.
- Engineering: For designing components where directional properties matter.

- Computer Graphics: To determine object orientations and lighting directions.
- Navigation: To compute directions and inclinations in 3D space.

Calculating these angles allows for a better understanding of the vector's spatial orientation and behavior within a system.

Additional Considerations and Tips

- Always verify the units of your angles — typically, the inverse cosine function returns angles in degrees or radians depending on the calculator or software used.
- Be cautious with negative components; they indicate directionality (e.g., downward in the case of the z-component).
- Ensure the vector and axis vectors are in the same coordinate system and units for accurate calculations.
- For vectors with components close to zero, small numerical errors can affect the calculated angles; consider using high-precision calculations when necessary.

Conclusion

Calculating the angles that a vector makes with the coordinate axes is a fundamental task in vector analysis, providing critical insights into the vector's spatial orientation. For the specific vector H = 3ax + 5ay - 8az, the angles with the x, y, and z axes are approximately 72.3°, 59.8°, and 143.1°, respectively. These calculations rely on understanding vector components, magnitudes, and the dot product formula, which together form the backbone of vector mathematics in physics and engineering.

By mastering these techniques, students and professionals can better analyze and interpret vector quantities in diverse applications, from mechanical systems to computer graphics.

Keywords: vector analysis, angle calculation, vector components, dot product, vector magnitude, coordinate axes, physics, engineering, spatial orientation

Frequently Asked Questions

How do you calculate the angle between vector H = 3ax + 5ay - 8az and the x-axis?

Use the dot product formula: cosθ = (H · x⍰) / (|H| |x⍰ |). Since x⍰ = 1,0,0, the dot product is 3. Calculate the magnitude of H: |H| = ⍰ (3² + 5² + (-8)²) = ⍰ (9 + 25 + 64) = ⍰ 98. Then, cosθ = 3 / ⍰ 98, and θ = arccos(3 / ⍰ 98).

What is the process to find the angle between vector H and the y-axis?

Calculate the dot product of H with the y-unit vector (0,1,0): H · y⍰ = 5. Find |H| as before: ⍰ 98. Then, cosθ

= 5 / ⍰ 98, and θ = arccos(5 / ⍰ 98).

How can I determine the angle between vector H and the z-axis?

Calculate the dot product with the z-unit vector (0,0,1): H · z⍰ = -8. Use |H| = ⍰ 98. Then, cosθ = -8 / ⍰ 98, and θ = arccos(-8 / ⍰ 98).

What is the significance of the angles between vector H and the coordinate axes?

These angles help understand the orientation of vector H in 3D space, indicating how much the vector aligns or deviates from each principal axis.

How do I interpret the angles calculated between vector H and the axes?

Angles close to 0° suggest the vector is aligned with that axis, while angles near 90° indicate it is perpendicular. Negative cosine values imply the vector points in the opposite direction along that axis.

Additional Resources

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Understanding the Problem: Calculating Angles Between a Vector and Coordinate Axes

When working with vectors in three-dimensional space, a fundamental task is to determine the angles that a given vector makes with the coordinate axes. In this case, we are asked to find the angles that the vector H = 3ax + 5ay - 8az makes with the x, y, and z axes. This is a common problem in vector analysis, physics, and engineering, as understanding the orientation of a vector relative to axes provides insights into its directional properties and how it interacts within a coordinate system.

Why is calculating these angles important?
- It helps in visualizing the vector's orientation in space.
- It aids in resolving vectors into components aligned with axes.
- It's essential in applications like force analysis, navigation, and computer graphics.

Step-by-Step Guide to Calculating the Angles

1. Recall the Vector Components and the Dot Product Formula

The vector H is given as:

$$H = 3a_x + 5a_y - 8a_z$$

which can be represented as:

$$\mathbf{H} = (3, 5, -8)$$

The coordinate axes are represented by unit vectors:

- $\mathbf{i} = (1, 0, 0)$ (x-axis)
- $\mathbf{j} = (0, 1, 0)$ (y-axis)
- $\mathbf{k} = (0, 0, 1)$ (z-axis)

The key to finding the angles is the dot product between the vector and the axes.

The dot product between two vectors $\mathbf{A}$ and $\mathbf{B}$ is:

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos \theta$$

where:

- $|\mathbf{A}|$ and $|\mathbf{B}|$ are the magnitudes of the vectors.
- θ is the angle between the vectors.

Since each axis vector is a unit vector, the dot product simplifies to:

$$\mathbf{H} \cdot \text{axis} = |\mathbf{H}| \times 1 \times \cos \theta$$

which simplifies to:

$$\cos \theta = \frac{\mathbf{H} \cdot \text{axis}}{|\mathbf{H}|}$$

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2. Calculate the Magnitude of Vector H

The magnitude (length) of H is:

$$|\mathbf{H}| = \sqrt{(3)^2 + (5)^2 + (-8)^2}$$

Calculating step-by-step:

- $3^2 = 9$
- $5^2 = 25$
- $(-8)^2 = 64$

Sum these:

$$9 + 25 + 64 = 98$$

Thus,

$$|\mathbf{H}| = \sqrt{98} = \sqrt{49 \times 2} = 7 \sqrt{2} \approx 9.899$$

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3. Compute Dot Products with Each Axis

Now, find the dot product of H with each axis:

- With x-axis ($\mathbf{i}$):

$$\mathbf{H} \cdot \mathbf{i} = (3)(1) + (5)(0) + (-8)(0) = 3$$

- With y-axis ($\mathbf{j}$):

$$\mathbf{H} \cdot \mathbf{j} = (3)(0) + (5)(1) + (-8)(0) = 5$$

- With z-axis ($\mathbf{k}$):

$$\mathbf{H} \cdot \mathbf{k} = (3)(0) + (5)(0) + (-8)(1) = -8$$

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4. Calculate the Angles Using the Cosine Formula

Using:

$$\cos \theta = \frac{\mathbf{H} \cdot \text{axis}}{|\mathbf{H}|}$$

we get:

- Angle with x-axis (θ_x):

$$\cos \theta_x = \frac{3}{7 \sqrt{2}}$$

- Angle with y-axis (θ_y):

$$\cos \theta_y = \frac{5}{7 \sqrt{2}}$$

- Angle with z-axis (θ_z):

$$\cos \theta_z = \frac{-8}{7 \sqrt{2}}$$

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5. Find the Actual Angles

Calculate each angle by taking the inverse cosine (arccos):

- $\theta_x = \arccos \left(\frac{3}{7 \sqrt{2}} \right)$
- $\theta_y = \arccos \left(\frac{5}{7 \sqrt{2}} \right)$
- $\theta_z = \arccos \left(\frac{-8}{7 \sqrt{2}} \right)$

Let's compute approximate values:

First, evaluate the denominators:

$$7 \sqrt{2} \approx 7 \times 1.4142 \approx 9.899$$

Angles in degrees:

- θ_x:

$$\cos \theta_x = \frac{3}{9.899} \approx 0.303$$

$$\theta_x \approx \arccos(0.303) \approx 72.3^\circ$$

- θ_y:

$$\cos \theta_y = \frac{5}{9.899} \approx 0.505$$

$$\theta_y \approx \arccos(0.505) \approx 59.5^\circ$$

- θ_z:

$$\cos \theta_z = \frac{-8}{9.899} \approx -0.808$$

$$\theta_z \approx \arccos(-0.808) \approx 143.1^\circ$$

Final Results: The Angles That Vector H Makes with the Axes

Axis	Calculated Cosine	Approximate Angle (degrees)
x-axis	0.303	72.3°
y-axis	0.505	59.5°
z-axis	-0.808	143.1°

Note: The negative cosine value with the z-axis indicates that the vector makes an obtuse angle (>90°), which aligns with the negative component in the z-direction.

Visual Interpretation and Applications

Understanding these angles allows us to visualize the orientation of H in three-dimensional space:

- The vector is inclined at approximately 72.3° to the x-axis.
- It makes an angle of about 59.5° with the y-axis.
- It is oriented at roughly 143.1° with the z-axis, indicating it points somewhat away from the positive z-direction due to its negative component.

This kind of analysis is vital in many fields:

- Physics: For force resolution and understanding directional effects.
- Engineering: In structural analysis and component design.
- Computer Graphics: To compute lighting, shading, and object orientation.
- Navigation: To determine directions relative to a fixed coordinate system.

Summary

Calculating the angles that a vector makes with coordinate axes involves:

1. Expressing the vector in component form.
2. Computing its magnitude.
3. Calculating dot products with each axis.
4. Using the dot product formula to find the cosine of the angles.
5. Applying inverse cosine to find the angles in degrees.

By following these steps, you can analyze any vector's orientation relative to the axes, which is a foundational skill in vector calculus and many applied sciences.

Final Thoughts

Mastering the process of calculating angles between vectors and axes enhances spatial reasoning and analytical skills. Practice with various vectors and coordinate systems will deepen your understanding and prepare you for more advanced topics like vector projections, cross products, and three-dimensional transformations.

If you're working on vector problems regularly, consider developing a quick reference sheet with the formulas, steps, and example calculations to streamline your process and improve accuracy.

Remember: Visualize the problem, break it down into manageable steps, and verify your results to develop confidence in your vector analysis skills!

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