In Problems 1-22 Solve The Given Differential Equation By Separation Of Variables. 1. Dy/dx = Sin 5x

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Introduction to Differential Equations and Separation of Variables

Differential equations are fundamental in mathematics and applied sciences, describing how quantities change with respect to each other. They are equations involving derivatives, which represent rates of change. Among various methods to solve differential equations, separation of variables is a powerful technique, especially suitable for equations where variables can be algebraically separated on different sides of the equation.

In this article, we will focus on solving the differential equation:

$$\frac{dy}{dx} = \sin 5x$$

using the separation of variables method. This particular equation is a straightforward example that illustrates the core principles of the method and demonstrates how to approach similar problems efficiently.

Understanding the Differential Equation: $\frac{dy}{dx} = \sin 5x$

Before diving into the solution process, let's analyze the structure of the differential equation:

- The derivative $\frac{dy}{dx}$ is expressed explicitly in terms of the independent variable x.
- The right-hand side, $\sin 5x$, is a function solely of x.

Since the equation is separable, we can think of it as:

$$dy = \sin 5x \, dx$$

which indicates that the change in y depends only on x, and the equation can be integrated directly once separated.

Step-by-Step Solution Using Separation of Variables

Step 1: Rewrite the Differential Equation

Begin by expressing the differential equation in a form that isolates dy and dx:

$$ \frac{dy}{dx} = \sin 5x $$

which can be rewritten as:

$$ dy = \sin 5x \, dx $$

This form indicates that the differential change in y is directly proportional to $\sin 5x$.

Step 2: Integrate Both Sides

To find the general solution, integrate both sides:

$$ \int dy = \int \sin 5x \, dx $$

The left side integrates straightforwardly:

$$ y = \int \sin 5x \, dx + C $$

where C is the constant of integration.

Step 3: Compute the Integral $\int \sin 5x \, dx$

The integral of $\sin 5x$ with respect to x can be computed using substitution:

- Let $u = 5x$, then $du = 5 dx \Rightarrow dx = \frac{du}{5}$.

Rewriting the integral:

$$ \int \sin 5x \, dx = \int \sin u \cdot \frac{du}{5} = \frac{1}{5} \int \sin u \, du $$

The integral:

$$ \int \sin u \, du = -\cos u + D $$

Putting it all together:

$$ \int \sin 5x \, dx = -\frac{1}{5} \cos u + D $$

Substituting back $u = 5x$:

$$ \int \sin 5x \, dx = -\frac{1}{5} \cos 5x + D $$

Since D is an arbitrary constant, it can be absorbed into the overall constant C.

Step 4: Write the General Solution

Combining the results:

$$ y = -\frac{1}{5} \cos 5x + C $$

This is the general solution to the differential equation, representing a family of functions parameterized by the constant C.

Interpreting the Solution

The solution:

$$ y(x) = -\frac{1}{5} \cos 5x + C $$

has several notable features:

- It involves a cosine function scaled by $\frac{1}{5}$, indicating the amplitude of oscillation.
- The variable C shifts the entire graph vertically, representing initial conditions or specific solutions.
- The period of the cosine function is:

$$ T = \frac{2\pi}{5} $$

which is derived from the coefficient of x inside the cosine, indicating how frequently the oscillations occur.

Applications of the Solution

Understanding and solving such differential equations has wide applications in fields like physics, engineering, biology, and economics.

Some practical scenarios include:

- Oscillatory systems: Modeling systems with periodic behavior, such as pendulums or electrical circuits.
- Population dynamics: When growth or decline depends on periodic factors.
- Signal processing: Analyzing waveforms and oscillations.

The explicit solution allows for precise analysis of these systems, especially when initial conditions are known.

Additional Examples and Practice Problems

To solidify understanding, here are similar problems that employ separation of variables:

1. Solve $\frac{dy}{dx} = \cos 3x$
2. Solve $\frac{dy}{dx} = e^{2x}$
3. Solve $\frac{dy}{dx} = y \sin x$
4. Solve $\frac{dy}{dx} = \frac{1}{x}$

Each problem involves integrating the right-hand side after separation, emphasizing the versatility of the method.

Summary and Key Takeaways

- Separation of variables is a straightforward method for solving differential equations where variables can be separated on different sides.
- The key steps include rewriting the differential equation, integrating both sides, and applying initial conditions if available.
- The solution to $\frac{dy}{dx} = \sin 5x$ is $y = -\frac{1}{5} \cos 5x + C$.
- Recognizing the structure of the differential equation simplifies the solution process.

- These solutions are crucial in modeling periodic phenomena and systems exhibiting oscillatory behavior.

Conclusion

Mastering the separation of variables technique enables students and professionals to solve a broad class of differential equations efficiently. The problem \(\frac{dy}{dx} = \sin 5x\) exemplifies how direct integration transforms a differential equation into a functional relationship, providing insights into the behavior of the modeled system. Practice with similar problems will deepen understanding and enhance problem-solving skills in differential equations.

For further learning, consider exploring differential equations with more complex functions, initial conditions, and applications in various scientific fields.

Frequently Asked Questions

What is the differential equation given in Problem 1?

The differential equation is Dy/dx = Sin 5x.

How do you approach solving Dy/dx = Sin 5x using separation of variables?

Since the equation is already separable, we can write it as dy = Sin 5x dx and then integrate both sides.

What is the integral of Sin 5x with respect to x?

The integral of Sin 5x dx is -1/5 Cos 5x + C.

What is the general solution to the differential equation Dy/dx = Sin 5x?

The general solution is y = -1/5 Cos 5x + C, where C is the constant of integration.

Why is separation of variables applicable to this differential equation?

Because the derivative Dy/dx can be written as a product of functions of y and x separately, allowing us to integrate both sides independently.

Are there any initial conditions provided for this problem?

No, the problem does not specify initial conditions, so the solution includes an arbitrary constant C.

What are common mistakes to avoid when solving this differential equation?

Common mistakes include forgetting to integrate both sides correctly, neglecting the constant of integration, or attempting to separate variables when the equation isn't separable.

Can this method be applied to non-separable differential equations?

No, separation of variables is only applicable to equations that can be written in the form dy/dx = g(x)h(y), where variables can be separated on opposite sides of the equation.

Additional Resources

Comprehensive Review of Solving Differential Equations by Separation of Variables: Case Study of Dy/dx = Sin 5x

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Introduction to Differential Equations and Separation of Variables

Differential equations form a fundamental part of mathematical modeling across numerous scientific disciplines, including physics, engineering, biology, and economics. They describe how a quantity changes with respect to another, often time or space, and thus encapsulate dynamics within systems.

Separation of Variables is one of the most straightforward and widely used methods for solving certain classes of differential equations. It involves algebraically manipulating the equation to isolate variables on opposite sides, enabling the integration of both sides separately. This technique is especially effective for first-order ordinary differential equations (ODEs) where variables can be separated naturally.

Understanding the Problem: Dy/dx = Sin 5x

The differential equation under consideration is:

$$
\frac{dy}{dx} = \sin 5x
$$

This is a first-order ordinary differential equation, linear in form, with the derivative of y expressed explicitly as a function of x.

Key features of this equation:

- The right side depends only on x, not on y, simplifying the process.
- The equation is separable because it can be written as:

$$
dy = \sin 5x \, dx
$$

which clearly separates the differential of y and the differential of x.

Step-by-Step Solution Using Separation of Variables

Step 1: Rewrite the Differential Equation

Express the differential equation to isolate dy and dx:

$$
dy = \sin 5x \, dx
$$

This step is crucial because it aligns with the principle of separation of variables—variables are now on different sides.

Step 2: Integrate Both Sides

Integrate both sides with respect to their variables:

$$
\int dy = \int \sin 5x \, dx
$$

This yields:

$$
y = \int \sin 5x \, dx + C
$$

where C is the constant of integration.

Step 3: Compute the Integral $\int \sin 5x \, dx$

Calculating the integral involves recognizing it as a straightforward substitution problem:

- Let $u = 5x$, so $du = 5 dx$ or $dx = \frac{du}{5}$.

- Substituting into the integral:

$$
\int \sin 5x \, dx = \int \sin u \cdot \frac{du}{5} = \frac{1}{5} \int \sin u \, du
$$

- The integral $\int \sin u \, du = -\cos u + D$, but since we're integrating indefinite integrals, constants are absorbed into C.

- Therefore:

$$
\int \sin 5x \, dx = -\frac{1}{5} \cos u + C = -\frac{1}{5} \cos 5x + C
$$

Step 4: Write the General Solution

Putting it all together:

$$
y = -\frac{1}{5} \cos 5x + C
$$

This is the general solution to the differential equation.

Analyzing the Solution and Its Features

1. Nature of the Solution:

- The solution involves a cosine function scaled by $-\frac{1}{5}$, indicating that the solution y oscillates with respect to x.
- The constant C represents the family of solutions, reflecting initial conditions or boundary values.

2. Graphical Interpretation:

- The graph of y versus x is a shifted cosine wave with amplitude $\frac{1}{5}$.
- The phase shift is determined by the initial conditions, which can specify where on the wave the solution begins.

3. Physical and Applied Contexts:

- Such solutions often model oscillatory phenomena, such as pendulum swings, alternating currents, or wave functions.
- The linearity and simplicity of the integral make it accessible for various applications, especially where sinusoidal inputs are involved.

General Principles Derived from the Example

1. Recognizing Separable Equations:

- Equations where the derivative can be written as the product of a function of x and a function of y or solely in terms of x are prime candidates.
- In this case, the right side depends only on x, simplifying the process.

2. Methodology:

- Always aim to express the differential as $dy = \text{(function of } x) \, dx$.
- Integrate both sides independently, applying substitution if necessary.
- Include the constant of integration to represent the general solution.

3. Integration Techniques:

- Use substitution when the integrand involves composite functions.
- Recall standard integrals, such as $\int \sin u \, du = -\cos u + D$.
- Be attentive to the chain rule when integrating functions like $\sin 5x$.

Applications and Extensions

1. Handling More Complex Equations:

- The method demonstrated scales to more complex equations where variables can be separated, such as:

$$
\frac{dy}{dx} = g(x) \cdot h(y)
$$

- In such cases, the integral becomes:

$$
\int \frac{1}{h(y)} \, dy = \int g(x) \, dx
$$

2. Initial Value Problems:

- When initial conditions $y(x_0) = y_0$ are specified, the constant C can be determined explicitly.
- This allows for particular solutions tailored to specific scenarios.

3. Limitations of Separation of Variables:

- Not all differential equations are separable; some require alternative methods such as integrating factors, substitution, or numerical approaches.
- Equations involving multiple y and x terms intertwined in complex ways may not be directly separable.

Deeper Insights into the Mathematical Process

1. The Importance of Recognizing Separable Equations:

- The skill of identifying whether an equation is separable is foundational in differential equations.
- Recognizing the form $dy/dx = f(x)$ makes the process straightforward.

2. The Role of Substitution:

- When integrating functions like $\sin 5x$, substitution simplifies the process.
- Substitution reduces complicated integral forms to standard integrals.

3. Fundamental Theorem of Calculus:

- The solution hinges on the fundamental theorem, linking differentiation and integration.
- Integrating the right side to find the antiderivative directly yields the solution.

4. Constants of Integration:

- The constant C encapsulates the entire family of solutions, reflecting the generality of indefinite integrals.
- Physical interpretations often relate C to initial conditions.

Practice and Application Exercises

1. Varying the Function:

- Solve $dy/dx = \cos 3x$ using separation of variables.
- Solution: $y = \frac{1}{3} \sin 3x + C$.

2. Initial Condition Application:

- Find the particular solution to $dy/dx = \sin 5x$ with $y(0) = 2$.
- Solution: $y = -\frac{1}{5} \cos 5x + 2$.

3. Real-World Modeling:

- Model a physical system where the rate of change of y is proportional to $\sin 2x$. Write the general solution.

Summary and Final Thoughts

The differential equation $dy/dx = \sin 5x$ exemplifies the power of the separation of variables method. By recognizing the structure of the equation, isolating variables, and performing straightforward integrations, we obtain an elegant solution that captures the oscillatory nature of the problem.

Key takeaways include:

- The importance of algebraic manipulation to facilitate integration.
- The utility of substitution techniques in handling composite functions.
- The insight that solutions to such equations often describe oscillatory phenomena, with applications across sciences.

Mastering this method provides a foundation for tackling more complex differential equations, understanding their behaviors, and applying them effectively in modeling real-world systems.

In conclusion, the separation of variables technique, as demonstrated through solving $dy/dx = \sin 5x$, is an essential tool in the mathematician's toolkit. Its simplicity, elegance, and broad applicability make it a cornerstone method in the study of differential equations, opening pathways to deeper understanding and more advanced problem-solving strategies.

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