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In This Question, You Are Asked To Find The Arc-length Of The Curve Y = (4x+2)^3/2 From X = 1 To X = The concept of arc length is fundamental in calculus, serving as a measure of the distance along a curve between two points. Calculating the arc length of a given function involves integrating the infinitesimal distances along the curve, which requires a good understanding of derivatives and integrals. In this article, we will explore the detailed process of finding the arc length of the curve $y = (4x + 2)^{3/2}$ from $x = 1$ to a specified upper limit, demonstrating the steps involved and providing insights into the underlying mathematical principles.

Understanding the Concept of Arc Length

What Is Arc Length?

Arc length refers to the measurement of a curve's length between two points. Unlike straight lines, curves are not straightforward to measure directly, so calculus offers a systematic approach to determine their lengths using integral calculus.

Mathematical Formula for Arc Length

For a function $y = f(x)$ that is continuous and differentiable on the interval $[a, b]$, the arc length L from $x = a$ to $x = b$ is given by:

$$
L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \, dx
$$

This formula stems from the Pythagorean theorem, considering the infinitesimal changes in x and y along the curve.

Step-by-Step Solution to Find the Arc Length of $y = (4x + 2)^{3/2}$ from $x=1$ to $x= \text{upper limit}$

In this section, we will methodically compute the arc length for the given function. Since the question appears incomplete regarding the upper limit, we'll assume it to be a variable $x = c$, where $c > 1$. The process, however, remains the same regardless of the specific upper limit.

Step 1: Find the Derivative $\frac{dy}{dx}$

Given the function:

$$
y = (4x + 2)^{3/2}
$$

Apply the chain rule to differentiate:

$$
\frac{dy}{dx} = \frac{3}{2} (4x + 2)^{1/2} \times 4 = 6 (4x + 2)^{1/2}
$$

Explanation:

- The outer function is $u^{3/2}$, with $u = 4x + 2$.
- Derivative of $u^{3/2}$ with respect to u is $\frac{3}{2} u^{1/2}$.
- Multiply by the derivative of $u = 4x + 2$, which is 4.

Result:

$$
\frac{dy}{dx} = 6 \sqrt{4x + 2}
$$

Step 2: Set Up the Arc Length Integral

Using the formula:

$$
L = \int_{x=1}^{x=c} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \, dx
$$

Substitute $\frac{dy}{dx}$:

$$
L = \int_1^c \sqrt{1 + [6 \sqrt{4x + 2}]^2} \, dx
$$

Simplify inside the square root:

$$
L = \int_1^c \sqrt{1 + 36 (4x + 2)} \, dx
$$

Distribute:

$$
L = \int_1^c \sqrt{1 + 36 \times (4x + 2)} \, dx
$$

$$
L = \int_1^c \sqrt{1 + 36 \times 4x + 36 \times 2} \, dx
$$

$$
L = \int_1^c \sqrt{1 + 144x + 72} \, dx
$$

Combine constant terms:

$$
L = \int_1^c \sqrt{144x + 73} \, dx
$$

Step 3: Simplify and Integrate

The integral reduces to:

$$
L = \int_1^c \sqrt{144x + 73} \, dx
$$

To evaluate this integral, apply substitution:

Let:

$$
u = 144x + 73
$$

Then:

$$
du = 144 \, dx \quad \Rightarrow \quad dx = \frac{du}{144}
$$

Change of limits:

- When $x=1$:

$$
u = 144(1) + 73 = 144 + 73 = 217
$$

- When $x=c$:

$$
u = 144c + 73
$$

The integral becomes:

$$
L = \int_{u=217}^{u=144c + 73} \sqrt{u} \times \frac{1}{144} \, du
$$

Simplify:

$$
L = \frac{1}{144} \int_{217}^{144c + 73} u^{1/2} \, du
$$

Now, integrate:

$$
\int u^{1/2} \, du = \frac{2}{3} u^{3/2}
$$

Therefore:

$$
L = \frac{1}{144} \times \frac{2}{3} \left[u^{3/2} \right]_{217}^{144c + 73}
$$

Simplify constants:

$$
L = \frac{2}{432} \left[(144c + 73)^{3/2} - 217^{3/2} \right]
$$

which reduces to:

$$
L = \frac{1}{216} \left[(144c + 73)^{3/2} - 217^{3/2} \right]
$$

Interpreting the Result and Completing the Calculation

The arc length L from $x=1$ to $x=c$ for the given function is:

$$
L = \frac{1}{216} \left[(144c + 73)^{3/2} - 217^{3/2} \right]
$$

To find the specific length for a particular upper limit, substitute the value of c. For example, if the question specifies $x=c=2$, then:

$$
L = \frac{1}{216} \left[(144 \times 2 + 73)^{3/2} - 217^{3/2} \right]
$$

Calculating:

$$
144 \times 2 = 288
$$

$$
288 + 73 = 361
$$

$$
L = \frac{1}{216} \left[361^{3/2} - 217^{3/2} \right]
$$

Since:

$$
361^{3/2} = (361)^{1} \times \sqrt{361} = 361 \times 19 = 6859
$$

and

$$
217^{3/2} = 217 \times \sqrt{217}
$$

Calculate $\sqrt{217} \approx 14.730$:

$$
217 \times 14.730 \approx 3198.31
$$

Thus,

$$
L \approx \frac{1}{216} (6859 - 3198.31) = \frac{1}{216} \times 3660.69 \approx 16.94
$$

This provides an approximate arc length between $x=1$ and $x=2$.

Summary of the Process to Find the Arc Length

- Identify the function $y = (4x + 2)^{3/2}$.
- Calculate its derivative $\frac{dy}{dx}$ using the chain rule.
- Set up the arc length integral using the formula $L = \int \sqrt{1 + (\frac{dy}{dx})^2} \, dx$.
- Simplify the integrand and perform substitution to evaluate the integral.
- Express the result in terms of the upper limit c, or evaluate for specific bounds.

Additional Tips for Solving Arc Length Problems

1. **Always verify the differentiability:** Ensure the function is continuous and differentiable over the interval.
2. **Use substitution wisely:** Recognize forms that fit standard integrals, like u^{n}, to simplify calculations.
3. **Check the domain:** Confirm the limits of integration are appropriate for the problem context.
4. **Approximate**

Frequently Asked Questions

How do I set up the integral to find the arc length of the curve y = (4x + 2)^{3/2} from x = 1 to x = a?

To find the arc length, use the formula L = ⍰ from x=1 to x=a of ⍰ (1 + [dy/dx]^2) dx. First, compute dy/dx, then substitute into the integrand and evaluate the integral.

What is the derivative dy/dx of y = (4x + 2)^{3/2}?

Using the chain rule, dy/dx = (3/2) (4x + 2)^{1/2} 4 = 6 (4x + 2)^{1/2}.

How do I simplify the expression for the arc length integral with dy/dx = 6 (4x + 2)^{1/2}?

The integrand becomes ⍰ (1 + [dy/dx]^2) = ⍰ (1 + 36(4x + 2)). Simplify inside the square root to get ⍰ (1 + 36(4x + 2)).

What substitution can be used to evaluate the integral for the arc length?

Let u = 4x + 2. Then, du/dx = 4, so dx = du/4. Rewrite the integral in terms of u to simplify.

What are the limits of integration when substituting u = 4x + 2 from x=1 to x=a?

When x=1, u=4(1)+2=6; when x=a, u=4a+2. So, the limits become u=6 to u=4a+2.

How do I evaluate the arc length integral after substitution for the curve y = (4x + 2)^{3/2} from x=1 to x=a?

The integral becomes L = ⍰ from u=6 to u=4a+2 of (1/4) ⍰ (1 + 36u) du. Integrate this expression to find the arc length.

Additional Resources

Arc-Length Calculation of the Curve \(y = (4x+2)^{3/2} \): An In-Depth Exploration

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Introduction

In the realm of calculus, one of the fundamental and fascinating topics is the determination of the arc-length of a curve. Whether you're a student delving into the intricacies of calculus or a seasoned mathematician exploring the depths of geometric analysis, understanding how to compute the length of a given curve is both an essential skill and a gateway to more advanced concepts.

Today, we explore a specific problem: finding the arc-length of the curve $y = (4x+2)^{3/2}$ from $x = 1$ to a given upper limit $x = a$. This problem not only exemplifies the application of integral calculus but also underscores the importance of precise substitution and differentiation techniques.

Setting the Stage: Understanding the Curve

Before jumping into the calculations, let's understand the nature of the curve:

- Equation: $y = (4x+2)^{3/2}$
- Domain: The problem specifies x from 1 to some upper bound a. For the sake of completeness, we'll consider a as a variable, but later, you can substitute any specific value.

This curve is a composition of linear and power functions, which makes it a perfect candidate for substitution techniques to simplify the derivative and the integral involved in arc-length calculations.

The Concept of Arc-Length in Calculus

The arc-length L of a curve $y = f(x)$ from $x = x_1$ to $x = x_2$ is given by the integral:

$$
L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx
$$

This integral accounts for the infinitesimal segments along the curve, summing their lengths to find the total arc-length.

To solve this for our specific function, we need:

1. The derivative $\frac{dy}{dx}$.
2. The expression $1 + \left(\frac{dy}{dx}\right)^2$.
3. The integral of its square root over the given interval.

Step 1: Differentiating $y = (4x+2)^{3/2}$

The differentiation process involves the chain rule. Let's compute $\frac{dy}{dx}$:

$$
\frac{dy}{dx} = \frac{d}{dx} \left((4x+2)^{3/2} \right)
$$

Applying the chain rule:

\[

$$
\frac{dy}{dx} = \frac{3}{2} (4x+2)^{1/2} \times \frac{d}{dx}(4x+2)
$$

Since $\frac{d}{dx}(4x+2) = 4$, the derivative simplifies to:

$$
\boxed{
\frac{dy}{dx} = 6 (4x+2)^{1/2}
}
$$

This derivative is crucial for the next step, where we evaluate the integrand of the arc-length formula.

Step 2: Computing $1 + \left(\frac{dy}{dx}\right)^2$

Now, square the derivative:

$$
\left(\frac{dy}{dx}\right)^2 = [6 (4x+2)^{1/2}]^2 = 36 (4x+2)
$$

Add 1 to it:

$$
1 + \left(\frac{dy}{dx}\right)^2 = 1 + 36 (4x+2)
$$

Distribute:

$$
= 1 + 36 \times 4x + 36 \times 2 = 1 + 144x + 72
$$

Combine like terms:

$$
\boxed{
1 + \left(\frac{dy}{dx}\right)^2 = 144x + 73
}
$$

This expression simplifies the integrand of the arc-length integral significantly.

Step 3: Setting up the Arc-Length Integral

The arc-length L from $x = 1$ to $x = a$ is:

$$
L = \int_{1}^{a} \sqrt{144x + 73} \, dx
$$

This integral appears manageable with substitution, as it involves a square root of a linear function.

Step 4: Substitution to Simplify the Integral

Let's perform an appropriate substitution to evaluate this integral:

- Substitution: $u = 144x + 73$

Then, the differential:

$$
du = 144 \, dx \Rightarrow dx = \frac{du}{144}
$$

Change of limits:

- When $x = 1$:

$$
u_1 = 144 \times 1 + 73 = 144 + 73 = 217
$$

- When $x = a$:

$$
u_2 = 144a + 73
$$

Rewriting the integral:

$$
L = \int_{u=217}^{u=144a + 73} \sqrt{u} \times \frac{1}{144} \, du
$$

which simplifies to:

$$
L = \frac{1}{144} \int_{217}^{144a + 73} u^{1/2} \, du
$$

Step 5: Evaluating the Integral

The integral of $u^{1/2}$ is:

$$
\int u^{1/2} \, du = \frac{2}{3} u^{3/2}
$$

Applying the limits:

$$
L = \frac{1}{144} \times \frac{2}{3} \left[(144a + 73)^{3/2} - 217^{3/2} \right]
$$

Simplify the constants:

$$
L = \frac{2}{3 \times 144} \left[(144a + 73)^{3/2} - 217^{3/2} \right]
$$

$$
L = \frac{1}{216} \left[(144a + 73)^{3/2} - 217^{3/2} \right]
$$

Final Expression for Arc-Length

$$
\boxed{
\text{Arc-length } L = \frac{1}{216} \left[(144a + 73)^{3/2} - 217^{3/2} \right]
}
$$

This formula allows you to compute the arc-length of the curve $y = (4x+2)^{3/2}$ from $x = 1$ to any arbitrary $x = a$.

Practical Example: Computing the Arc-Length from $x=1$ to $x=2$

Suppose you want the arc-length from $x=1$ to $x=2$. Plug in $a = 2$:

- Calculate $u = 144 \times 2 + 73 = 288 + 73 = 361$.

Now, evaluate:

$$
L = \frac{1}{216} \left[361^{3/2} - 217^{3/2} \right]
$$

Recall:

$$
u^{3/2} = u \times \sqrt{u}
$$

- $\sqrt{361} = 19$, so:

$$
361^{3/2} = 361 \times 19 = 361 \times 19
$$

Calculate:

$$
361 \times 19 = (360 + 1) \times 19 = 360 \times 19 + 1 \times 19 = 6840 + 19 = 6859
$$

Similarly, for 217:

- $\sqrt{217} \approx 14.73$, but since 217 isn't a perfect square, we'll leave it as is or approximate:

$$
217^{3/2} = 217 \times \sqrt{217} \approx 217 \times 14.73 \approx 3197.4
$$

Finally, compute L:

$$
L \approx \frac{1}{216} (6859 - 3197.4) = \frac{1}{216} \times 3661.6 \approx 16.94
$$

Thus, the arc-length from $x=1$ to $x=2$ approximately equals 16.94 units.

Additional Insights and Applications

Why is this calculation significant?

- It demonstrates how substitution simplifies complex integrals involving square roots of linear functions.
- The process exemplifies the importance of chain rule differentiation in setting up integral formulas.
- The method can be adapted for different functions with similar structures, making it a versatile tool in calculus.

Potential applications include:

- Designing curves in engineering with precise length requirements.
- Analyzing the length of biological structures modeled mathematically.
- Calculating the length of paths or roads in geographic information systems.

Conclusion

Calculating the arc-length of a curve like $y = (4x+2)^{3/2}$ from $x=1$

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