Kindly Solve Legibly. (step-by-step)If S (x) = 6x^5-5x^4 + 3x^3 7x^2 + 9x 14 Then Find F^(n) (x) For

Kindly Solve Legibly. (step-by-step) If S(x) = 6x^5 - 5x^4 + 3x^3 + 7x^2 + 9x + 14 Then Find F^(n)(x) For

Introduction

Understanding how to differentiate functions repeatedly is a fundamental skill in calculus. In this guide, we will carefully analyze the given function S(x) = 6x^5 - 5x^4 + 3x^3 + 7x^2 + 9x + 14 and determine the nth derivative, denoted as F^(n)(x). We will approach this step-by-step, ensuring clarity and precision to help you grasp the process thoroughly.

Understanding the Problem

Before diving into derivatives, it’s crucial to understand what is being asked:
- Function Provided: S(x) = 6x^5 - 5x^4 + 3x^3 + 7x^2 + 9x + 14
- Goal: Find the nth derivative of S(x), i.e., F^(n)(x)
- Key Point: The nth derivative is the derivative taken n times successively.

Knowing how derivatives behave, especially polynomial derivatives, helps us predict the pattern for higher-order derivatives.

Step 1: Find the First Few Derivatives

Calculating the first, second, and third derivatives will reveal the pattern.

1. First derivative, F'(x)

Using the power rule (d/dx of x^k = kx^{k-1}):

- d/dx of 6x^5 = 6 5x^{4} = 30x^{4}
- d/dx of -5x^4 = -5 4x^{3} = -20x^{3}
- d/dx of 3x^3 = 3 3x^{2} = 9x^{2}
- d/dx of 7x^2 = 7 2x^{1} = 14x
- d/dx of 9x = 9
- d/dx of 14 (constant) = 0

Therefore:

$F'(x) = 30x^{4} - 20x^{3} + 9x^{2} + 14x + 9$

2. Second derivative, $F''(x)$

Differentiate $F'(x)$:

- d/dx of $30x^{4} = 30 \ 4x^{3} = 120x^{3}$
- d/dx of $-20x^{3} = -20 \ 3x^{2} = -60x^{2}$
- d/dx of $9x^{2} = 9 \ 2x = 18x$
- d/dx of $14x = 14$
- d/dx of $9 = 0$

Thus:

$F''(x) = 120x^{3} - 60x^{2} + 18x + 14$

3. Third derivative, $F'''(x)$

Differentiate $F''(x)$:

- d/dx of $120x^{3} = 120 \ 3x^{2} = 360x^{2}$
- d/dx of $-60x^{2} = -60 \ 2x = -120x$
- d/dx of $18x = 18$
- d/dx of $14 = 0$

Result:

$F'''(x) = 360x^{2} - 120x + 18$

Step 2: Recognize the Pattern in Derivatives

Observe the derivatives calculated so far:

Derivative	Expression	Degree of Polynomial
F(x)	6x^5 - 5x^4 + 3x^3 + 7x^2 + 9x + 14	5
F'(x)	30x^4 - 20x^3 + 9x^2 + 14x + 9	4
F''(x)	120x^3 - 60x^2 + 18x + 14	3
F'''(x)	360x^2 - 120x + 18	2

From the above, the degree of the polynomial decreases by 1 with each derivative, which is typical for polynomial functions. The coefficients multiply by the current degree at each differentiation.

Pattern in coefficients:

- The leading coefficient: 6 (degree 5) → derivative coefficient: 6 5 = 30
- Next: 30 (degree 4) → derivative coefficient: 30 4 = 120

- Next: 120 (degree 3) → derivative coefficient: 120 3 = 360

Similarly, for other terms, the pattern continues with multiplication by the current degree.

Key observations:

- The constant term (14) becomes zero after the first derivative.
- The derivatives of polynomial terms are zero once the degree reduces to zero or below.

Step 3: Derivatives of Each Term and General Pattern

Breaking down the derivatives term-by-term:

1. 6x^5:
- 1st derivative: 6 5x^4 = 30x^4
- 2nd: 30 4x^3 = 120x^3
- 3rd: 120 3x^2 = 360x^2
- 4th: 360 2x = 720x
- 5th: 720 1 = 720
- 6th: 720 0 = 0

2. -5x^4:
- 1st: -5 4x^3 = -20x^3
- 2nd: -20 3x^2 = -60x^2
- 3rd: -60 2x = -120x
- 4th: -120 1 = -120
- 5th: 0

3. 3x^3:
- 1st: 3 3x^2 = 9x^2
- 2nd: 9 2x = 18x
- 3rd: 18 1 = 18
- 4th: 0

4. 7x^2:
- 1st: 7 2x = 14x
- 2nd: 14 1 = 14
- 3rd: 0

5. 9x:
- 1st: 9
- 2nd: 0

6. 14 (constant):
- 1st: 0

This breakdown shows that:

- The nth derivative of x^k is zero if $n > k$.
- The nth derivative of a polynomial term x^k is given by:

$$
\frac{k!}{(k - n)!} x^{k - n}
$$

when $n \leq k$; otherwise, it is zero.

Step 4: Deriving the General Formula for the nth Derivative

Given the pattern, the nth derivative of the entire polynomial S(x) is the sum of the nth derivatives of each term:

$$
F^{(n)}(x) = \sum_{i} \text{coeff}_i \times \frac{k_i!}{(k_i - n)!} x^{k_i - n}
$$

where:
- coeff_i is the coefficient of the ith term in S(x),
- k_i is the degree of that term.

Applying this to each term in S(x):

Term	Coefficient	Degree k	nth derivative (if $n \leq k$)
$6x^5$	6	5	$6 \times \frac{5!}{(5 - n)!} x^{5 - n}$
$-5x^4$	-5	4	$-5 \times \frac{4!}{(4 - n)!} x^{4 - n}$
$3x^3$	3	3	$3 \times \frac{3!}{(3 - n)!} x^{3 - n}$
$7x^2$	7	2	$7 \times \frac{2!}{(2 - n)!} x^{2 - n}$
9x	9	1	$9 \times \frac{1!}{(1 - n)!} x^{1 - n}$
14	14	0	derivative is zero for $n \geq 1$

Note: For terms where $n > k$, the

Frequently Asked Questions

How do you find the nth derivative of the function $S(x) = 6x^5 - 5x^4 + 3x^3 - 7x^2 + 9x - 14$ step-by-step?

To find the nth derivative, differentiate S(x) repeatedly, reducing the power of each term by 1 each time. For example:

1. First derivative: $S'(x) = d/dx [6x^5 - 5x^4 + 3x^3 - 7x^2 + 9x - 14] = 30x^4 - 20x^3 + 9x^2 - 14x + 9$.
2. Second derivative: $S''(x) = d/dx [30x^4 - 20x^3 + 9x^2 - 14x + 9] = 120x^3 - 60x^2 + 18x - 14$.
3. Continue differentiating until the nth derivative is obtained, noting that each differentiation reduces the power by 1 and multiplies by the original power.
4. For the nth derivative, use the general formula for derivatives of power functions: if $f(x) = ax^k$, then $f^{(n)}(x) = a\ k\ (k-1)\ \dots\ (k-n+1)\ x^{k-n}$ for $k \geq n$, else zero.

What is the general formula for the nth derivative of a polynomial term like 6x^5 in the given function?

The nth derivative of $6x^5$ is given by:

$$f^{(n)}(x) = 6\ 5\ 4\ \dots\ (5 - n + 1)\ x^{5 - n}$$

for $n \leq 5$. Specifically:
- For n=1: $6\ 5\ x^{4} = 30x^4$
- For n=2: $6\ 5\ 4\ x^{3} = 120x^3$
- For n=3: $6\ 5\ 4\ 3\ x^{2} = 360x^2$
- For n=4: $6\ 5\ 4\ 3\ 2\ x^{1} = 720x$
- For n=5: $6\ 5\ 4\ 3\ 2\ 1 = 720$
- For $n > 5$: derivative is zero.

How do derivatives of polynomial terms with decreasing powers contribute to calculating F^(n)(x) of the given function?

Each polynomial term's nth derivative is computed based on the factorial pattern of its original power. Terms with powers less than n will have zero derivatives beyond that point. To find $F^{(n)}(x)$, differentiate each term n times, applying the power rule iteratively. For example, for $3x^3$, the third derivative is $3\ 3\ 2\ x^{0} = 18$, and the fourth derivative would be zero. Summing all these derivatives gives the nth derivative of the entire function.

What is the final expression for F^{(n)}(x) of the function S(x) = 6x^5 - 5x^4 + 3x^3 - 7x^2 + 9x - 14?

The nth derivative $F^{(n)}(x)$ is obtained by differentiating each term n times:

$F^{(n)}(x) =$
- For $6x^5$: $6\ 5\ 4\ \dots\ (5 - n + 1)\ x^{5 - n}$ if $n \leq 5$, else 0.
- For $-5x^4$: $-5\ 4\ 3\ \dots\ (4 - n + 1)\ x^{4 - n}$ if $n \leq 4$, else 0.

- For 3x^3: 3 3 2 x^{3 - n} if n ≤ 3, else 0.
- For -7x^2: -7 2 x^{2 - n} if n ≤ 2, else 0.
- For 9x: 9 1 x^{1 - n} if n ≤ 1, else 0.
- For constant -14: derivative is zero for n ≥ 1.

Putting it together:

F^{(n)}(x) = [720 x^{5 - n}] - [480 x^{4 - n}] + [18 x^{3 - n}] - [14 x^{2 - n}] + [9 x^{1 - n}] ,
with the understanding that any term with a negative exponent is zero, so the derivatives are zero when n exceeds the original degree of the term.

Additional Resources

Kindly Solve Legibly. (step-by-step) If S(x) = 6x^5 - 5x^4 + 3x^3 + 7x^2 + 9x + 14, then find $F^{(n)}(x)$ for ...

Introduction: The Significance of Derivatives in Mathematics

In the realm of calculus, derivatives serve as the foundational tool for understanding how functions behave—how they change and evolve at any given point. Whether analyzing the velocity of a moving object, the rate of change in economics, or the slope of a curve, derivatives enable mathematicians and scientists to quantify change precisely. The task at hand involves a polynomial function and its derivatives, a classic exercise that exemplifies the power of differentiation rules and the systematic approach necessary to unravel the behavior of complex functions.

The problem statement centers on a polynomial function S(x):

$$
S(x) = 6x^5 - 5x^4 + 3x^3 + 7x^2 + 9x + 14
$$

The goal is to find the n-th derivative of a related function $F(x)$, implied to be $S(x)$ itself, or perhaps a function derived from it, depending on the context. Since the problem directly provides the polynomial, the key is to understand how to differentiate polynomials repeatedly and identify patterns, especially for higher-order derivatives.

This article aims to systematically explore the differentiation process, apply it to the given polynomial, and derive a general formula or pattern for the n-th derivative, offering clarity and insight into the mechanics of differentiation.

Understanding the Function and Its Derivatives

The Polynomial Function S(x)

The given function:

$$
S(x) = 6x^5 - 5x^4 + 3x^3 + 7x^2 + 9x + 14
$$

is a degree 5 polynomial, which implies that its derivatives will reduce in degree with each differentiation until reaching a constant or zero.

The process of differentiation involves applying the power rule:

$$
\frac{d}{dx} [ax^n] = a \cdot n \cdot x^{n-1}
$$

Applying this rule to each term of $S(x)$, we can find the first derivative $S'(x)$, then the second $S''(x)$, and so on, until the derivatives become zero.

Step-by-Step Differentiation of the Polynomial

1. First Derivative $S'(x)$

Applying the power rule to each term:

- $6x^5 \Rightarrow 6 \times 5 \times x^{4} = 30x^{4}$
- $-5x^4 \Rightarrow -5 \times 4 \times x^{3} = -20x^{3}$
- $3x^3 \Rightarrow 3 \times 3 \times x^{2} = 9x^{2}$
- $7x^2 \Rightarrow 7 \times 2 \times x^{1} = 14x$
- $9x \Rightarrow 9 \times 1 \times x^{0} = 9$
- $14 \Rightarrow$ derivative of a constant is zero.

Thus,

\[

S'(x) = 30x^{4} - 20x^{3} + 9x^{2} + 14x + 9
\]

---

2. Second Derivative \(S''(x) \)

Differentiating \(S'(x) \):

- \(30x^{4} \Rightarrow 30 \times 4 \times x^{3} = 120x^{3} \)
- \(-20x^{3} \Rightarrow -20 \times 3 \times x^{2} = -60x^{2} \)
- \(9x^{2} \Rightarrow 9 \times 2 \times x^{1} = 18x \)
- \(14x \Rightarrow 14 \times 1 \times x^{0} = 14 \)
- \(9 \Rightarrow \) derivative of constant zero.

Result:

\[
S''(x) = 120x^{3} - 60x^{2} + 18x + 14
\]

---

3. Third Derivative \(S'''(x) \)

Differentiate \(S''(x) \):

- \(120x^{3} \Rightarrow 120 \times 3 \times x^{2} = 360x^{2} \)
- \(-60x^{2} \Rightarrow -60 \times 2 \times x^{1} = -120x \)
- \(18x \Rightarrow 18 \times 1 \times x^{0} = 18 \)
- \(14 \Rightarrow 0 \)

Result:

\[
S'''(x) = 360x^{2} - 120x + 18
\]

---

4. Fourth Derivative \(S^{(4)}(x) \)

Differentiate \(S'''(x) \):

- \(360x^{2} \Rightarrow 360 \times 2 \times x^{1} = 720x \)

- $-120x \Rightarrow -120 \times 1 \times x^{0} = -120$
- $18 \Rightarrow 0$

Result:

$$
S^{(4)}(x) = 720x - 120
$$

---

5. Fifth Derivative $S^{(5)}(x)$

Differentiate $S^{(4)}(x)$:

- $720x \Rightarrow 720 \times 1 \times x^{0} = 720$
- $-120 \Rightarrow 0$

Result:

$$
S^{(5)}(x) = 720
$$

---

6. Sixth Derivative $S^{(6)}(x)$

Differentiate $S^{(5)}(x)$:

- $720 \Rightarrow 0$

Result:

$$
S^{(6)}(x) = 0
$$

Beyond this point, all higher derivatives are zero.

Pattern Recognition and General Formula for the

n-th Derivative

Having computed derivatives step-by-step, the key observation is that the derivatives of a polynomial reduce in degree by one with each differentiation, and the coefficients follow factorial-like patterns involving the original coefficients and powers.

Key observations:

- For a polynomial $a x^{m}$, the n-th derivative is zero if $n > m$.
- The n-th derivative of $a x^{m}$ (for $n \leq m$) is:

$$
a \times m \times (m-1) \times (m-2) \times \dots \times (m - n + 1) \times x^{m - n}
$$

which can be written as:

$$
a \times \frac{m!}{(m - n)!} \times x^{m - n}
$$

- If $n > m$, the derivative is zero.

Applying this to each term of $S(x)$:

$$
S^{(n)}(x) = \sum_{i} c_i \times \frac{m_i!}{(m_i - n)!} \times x^{m_i - n}
$$

where c_i and m_i are the coefficients and exponents of the original polynomial.

Explicit Formula for $F^{(n)}(x)$ (or $S^{(n)}(x)$)

Given the polynomial:

$$
S(x) = 6x^{5} - 5x^{4} + 3x^{3} + 7x^{2} + 9x + 14
$$

The n-th derivative, $S^{(n)}(x)$, is:

```
\[
S^{(n)}(x) = \left\{
\begin{aligned}
& 0, & \text{if } n > 5 \\
& 6 \times \frac{5!}{(5 - n)!} x^{5 - n} - 5 \times \frac{4!}{(4 - n)!} x^{4 - n} + 3 \times \frac{3!}{(3 - n)!} x^{3 - n} + 7 \times \frac{2!}{(2 - n)!} x^{2 - n} + 9 \times \frac{1!}{(1 - n)!} x^{1 - n} + 14 \times \frac{0!}{(0 - n)!} x^{0 - n}, & \text{if } n \leq 5
\end{aligned}
\right.
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Kindly Solve Legibly Step By Step If S X 6x 5 5x 4 3x 3 7x 2 9x 14 Then Find F N X For

Kindly Solve Legibly Step By Step If S X 6x 5 5x 4 3x 3 7x 2 9x 14 Then Find F N X For

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