Let A=[22 18].[24 20]Find Two Different Diagonal Matrices D And The Corresponding Matrix S Such That

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In linear algebra, the process of diagonalizing a matrix is fundamental for simplifying many matrix operations, including powers, exponentials, and solving systems of equations. Given a matrix \(A\), identifying its diagonal form and the corresponding similarity transformation is a crucial step. This article explores how to find two different diagonal matrices \(D\) and their associated matrices \(S\) such that they satisfy the relation \(A = S D S^{-1}\). Specifically, we analyze the matrix \(A = \begin{bmatrix} 22 & 18 \\ 24 & 20 \end{bmatrix}\) and demonstrate the process of diagonalization, including the computation of eigenvalues, eigenvectors, and the resulting similarity transformations.

Understanding the Matrix \(A\)

Before diving into the diagonalization process, let's understand the structure and properties of the matrix:

\[
A = \begin{bmatrix} 22 & 18 \\ 24 & 20 \end{bmatrix}
\]

This matrix is a 2x2 real matrix with elements that suggest potential eigenvalues and eigenvectors that can be computed systematically.

Key Properties of \(A\)

- It is a real matrix.
- It is not symmetric, so eigenvalues may be complex, but in this case, they are likely real because the matrix is close to a multiple of the identity matrix.
- The trace and determinant provide useful information for eigenvalues calculations.

Calculating Eigenvalues of \(A\)

Eigenvalues are solutions to the characteristic equation:

\[
\det(A - \lambda I) = 0
\]

where \(I\) is the identity matrix, and \(\lambda\) is an eigenvalue.

Characteristic Equation

Set up the equation:

\[
\det \begin{bmatrix} 22 - \lambda & 18 \\ 24 & 20 - \lambda \end{bmatrix} = 0
\]

Calculate the determinant:

\[
(22 - \lambda)(20 - \lambda) - (18)(24) = 0
\]

Expanding:

\[
(22 \times 20) - 22\lambda - 20\lambda + \lambda^2 - 432 = 0
\]

Simplify:

\[
440 - 42 \lambda + \lambda^2 - 432 = 0
\]

\[
\lambda^2 - 42 \lambda + 8 = 0
\]

The quadratic equation:

\[
\lambda^2 - 42 \lambda + 8 = 0
\]

Eigenvalues Calculation

Using the quadratic formula:

\[

$$
\lambda = \frac{42 \pm \sqrt{(-42)^2 - 4 \times 1 \times 8}}{2}
$$

Compute discriminant:

$$
(-42)^2 - 4 \times 8 = 1764 - 32 = 1732
$$

Calculate square root:

$$
\sqrt{1732} \approx 41.6
$$

Thus, eigenvalues are:

$$
\lambda_1 = \frac{42 + 41.6}{2} \approx \frac{83.6}{2} = 41.8
$$
$$
\lambda_2 = \frac{42 - 41.6}{2} \approx \frac{0.4}{2} = 0.2
$$

Approximate eigenvalues:

$$
\boxed{
\lambda_1 \approx 41.8,\quad \lambda_2 \approx 0.2
}
$$

Finding Eigenvectors Corresponding to Eigenvalues

For each eigenvalue, find a non-zero vector v such that:

$$
A v = \lambda v
$$

Eigenvector for $\lambda_1 \approx 41.8$

Substitute λ_1 into $A - \lambda I$:

$$
A - 41.8 I = \begin{bmatrix} 22 - 41.8 & 18 \\ 24 & 20 - 41.8 \end{bmatrix} = \begin{bmatrix} -19.8 & 18 \\ 24 & -21.8 \end{bmatrix}
$$

Set up the homogeneous system:

$$
\begin{cases}
-19.8 x + 18 y = 0 \\
24 x - 21.8 y = 0
\end{cases}
$$

From the first equation:

$$
-19.8 x + 18 y = 0 \Rightarrow y = \frac{19.8}{18} x \approx 1.1 x
$$

Choose $x = 1$, then:

$$
y \approx 1.1
$$

Thus, an eigenvector:

$$
v_1 \approx \begin{bmatrix} 1 \\ 1.1 \end{bmatrix}
$$

Eigenvector for $\lambda_2 \approx 0.2$

Similarly, compute:

$$
A - 0.2 I = \begin{bmatrix} 22 - 0.2 & 18 \\ 24 & 20 - 0.2 \end{bmatrix} = \begin{bmatrix} 21.8 & 18 \\ 24 & 19.8 \end{bmatrix}
$$

Set up the homogeneous system:

$$
\begin{cases}
21.8 x + 18 y = 0 \\
24 x + 19.8 y = 0
\end{cases}
$$

From the first:

$$
18 y = -21.8 x \Rightarrow y = -\frac{21.8}{18} x \approx -1.21 x
$$

Choose $x = 1$, then:

$$
y \approx -1.21
$$

Eigenvector:

$$
v_2 \approx \begin{bmatrix} 1 \\ -1.21 \end{bmatrix}
$$

Normalizing Eigenvectors

For convenience, normalize the eigenvectors:

$$
v_1^{(norm)} = \frac{1}{\sqrt{1^2 + 1.1^2}} \begin{bmatrix} 1 \\ 1.1 \end{bmatrix} \approx \frac{1}{\sqrt{1 + 1.21}} \begin{bmatrix} 1 \\ 1.1 \end{bmatrix} \approx \frac{1}{\sqrt{2.21}} \begin{bmatrix} 1 \\ 1.1 \end{bmatrix}
$$

Similarly for v_2:

$$
v_2^{(norm)} \approx \frac{1}{\sqrt{1^2 + 1.21^2}} \begin{bmatrix} 1 \\ -1.21 \end{bmatrix} \approx \frac{1}{\sqrt{1 + 1.4641}} \begin{bmatrix} 1 \\ -1.21 \end{bmatrix} \approx \frac{1}{\sqrt{2.4641}} \begin{bmatrix} 1 \\ -1.21 \end{bmatrix}
$$

Constructing the Diagonal Matrices D and the Matrices S

Having found approximate eigenvalues and eigenvectors, we can now construct the diagonal matrices D and the invertible matrices S.

First Diagonal Matrix D_1

Choose D_1 to have the eigenvalues on its diagonal:

$$
D_1 = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \approx \begin{bmatrix} 41.8 & 0 \\ 0 & 0.2 \end{bmatrix}
$$

Corresponding Matrix S_1

Construct matrix S_1 with the normalized eigenvectors as columns:

$$
S_1 \approx \begin{bmatrix}
\frac{1}{\sqrt{2.21}} & \frac{1}{\sqrt{2.4641}} \\
\frac{1.1}{\sqrt{2.21}} & -\frac{1.21}{\sqrt{2.4641}}
\end{bmatrix}
$$

Calculating the approximate numerical values:

$$
S_1 \approx \begin{bmatrix}
0.673 & 0.638 \\
0.741 & -0.778
\end{bmatrix}
$$

Verify that:

$$
A \approx S_1 D_1 S_1^{-1}
$$

by computing $S_1 D_1 S_1^{-1}$ and comparing with original A.

Finding an Alternative Diagonal Matrix D_2

To find a second diagonal matrix D_2, we can choose different eigenvalues or different orderings of the same eigenvalues.

Suppose we swap the order of eigenvalues

Frequently Asked Questions

Given A =
[22 18]
[24 20], how can we find two different diagonal matrices D1 and D2 such that A = S D1 S^{-1} and A = S D2 S^{-1} for some invertible matrix S?

To find such D and S, we perform the eigen-decomposition of A. First, compute the eigenvalues of A, which will form the diagonals of D. Then, find the eigenvectors to construct S. Since different sets of eigenvectors can lead to different D matrices, choosing different bases for eigenvectors can produce different diagonal matrices D1 and D2 satisfying A = S D S^{-1}.

What are the eigenvalues of the matrix A =
[22 18]
[24 20], and how do they influence the diagonal matrices D1 and D2?

The eigenvalues of A are found by solving det(A - λI) = 0. For A, the characteristic polynomial is λ^2 - 42λ + 0 = 0, giving eigenvalues λ = 0 and λ = 42. These eigenvalues form the diagonals of the diagonal matrices D1 and D2, which are similar to A.

How can different choices of eigenvectors lead to different diagonal matrices D and different matrices S for the same matrix A?

Since eigenvectors are determined up to a scalar multiple, selecting different eigenvectors (or different bases for eigenspaces) results in different invertible matrices S. Consequently, the diagonal matrices D obtained via similarity transformations are different, reflecting different bases but representing the same linear operator.

Is it possible for two different diagonal matrices D1 and D2 to satisfy A = S D1 S^{-1} and A = T D2 T^{-1} for different invertible matrices S and T?

Yes. For the same matrix A, different eigenvector bases lead to different similarity transformations, producing distinct diagonal matrices D1 and D2 with the same eigenvalues but different eigenvectors, and corresponding matrices S and T.

What role does the invertible matrix S play in the similarity transformation A = S D S^{-1}?

Matrix S contains the eigenvectors of A arranged as its columns. It transforms the diagonal matrix D (containing eigenvalues) into A via similarity. Different choices of S (based on different eigenvector bases) yield different D matrices, illustrating the non-uniqueness of eigenvector bases.

How do you verify that a given diagonal matrix D and matrix S satisfy A = S D S^{-1} for the matrix A provided?

To verify, compute S D S^{-1} and check if it equals A. If the product matches A, then D and S satisfy the similarity relation. This confirms that D contains eigenvalues and S contains the corresponding eigenvectors (or basis vectors) for A.

Additional Resources

Let A = [22 18; 24 20] Find Two Different Diagonal Matrices D and the Corresponding Matrix S Such That — this problem is a classic example of exploring matrix diagonalization, a fundamental concept in linear algebra that has wide-ranging applications in engineering, computer science, and applied mathematics. Understanding how to find diagonal matrices that are similar to a given matrix and the corresponding transformation matrices is essential for simplifying complex matrix operations, analyzing eigenvalues and eigenvectors, and solving systems of differential equations, among other tasks.

In this comprehensive guide, we will walk through step-by-step how to find two different diagonal matrices D and the matrices S such that $A = S D S^{-1}$. We will explore the underlying theories, the calculation process, and practical insights to deepen your understanding of diagonalization.

Understanding the Problem

Given the matrix:

$$
A = \begin{bmatrix}
22 & 18 \\
24 & 20
\end{bmatrix}
$$

we are asked to find two different diagonal matrices D and their corresponding matrices S such that:

$$
A = S D S^{-1}
$$

This essentially involves diagonalizing A, which requires:

- Finding the eigenvalues of A,
- Finding the eigenvectors corresponding to those eigenvalues,
- Constructing D with the eigenvalues on its diagonal,
- Constructing S with the eigenvectors as its columns,
- Ensuring S is invertible.

Why Diagonalize?

Diagonalization simplifies many matrix operations. For example, computing A^n becomes straightforward once A is diagonalized:

$$
A^n = S D^n S^{-1}
$$

where D^n is simply the diagonal matrix with each eigenvalue raised to the power n.

Step 1: Find the Eigenvalues of A

The eigenvalues λ satisfy:

$$
\det(A - \lambda I) = 0
$$

where I is the identity matrix.

So, compute:

$$
A - \lambda I = \begin{bmatrix}
22 - \lambda & 18 \\
24 & 20 - \lambda
\end{bmatrix}
$$

The characteristic polynomial:

$$
\det(A - \lambda I) = (22 - \lambda)(20 - \lambda) - (18)(24)
$$

Calculate:

$$
(22 - \lambda)(20 - \lambda) = (22 \times 20) - 22 \lambda - 20 \lambda + \lambda^2 = 440 - 42 \lambda + \lambda^2
$$

And:

$$
18 \times 24 = 432
$$

Putting it all together:

$$
\lambda^2 - 42 \lambda + 440 - 432 = 0
$$

Simplify:

\[

\lambda^2 - 42 \lambda + 8 = 0
\]

Now, solve the quadratic:

\[
\lambda^2 - 42 \lambda + 8 = 0
\]

Using the quadratic formula:

\[
\lambda = \frac{42 \pm \sqrt{(42)^2 - 4 \times 1 \times 8}}{2}
\]

Calculate discriminant:

\[
(42)^2 - 4 \times 8 = 1764 - 32 = 1732
\]

Since:

\[
\sqrt{1732} \approx 41.62
\]

Therefore:

\[
\lambda_{1,2} = \frac{42 \pm 41.62}{2}
\]

Compute:

- For the plus:

\[
\lambda_1 = \frac{42 + 41.62}{2} \approx \frac{83.62}{2} \approx 41.81
\]

- For the minus:

\[
\lambda_2 = \frac{42 - 41.62}{2} \approx \frac{0.38}{2} \approx 0.19
\]

Eigenvalues:

\[
\boxed{

\lambda_1 \approx 41.81, \quad \lambda_2 \approx 0.19
}
\]

Step 2: Find Eigenvectors

Next, for each eigenvalue, find the corresponding eigenvector \(\mathbf{v} \) satisfying:

\[
(A - \lambda I) \mathbf{v} = \mathbf{0}
\]

Eigenvector for \(\lambda_1 \approx 41.81 \):

Compute:

\[
A - 41.81 I \approx \begin{bmatrix}
22 - 41.81 & 18 \\
24 & 20 - 41.81
\end{bmatrix} = \begin{bmatrix}
-19.81 & 18 \\
24 & -21.81
\end{bmatrix}
\]

Set up the system:

\[
-19.81 x + 18 y = 0
\]
\[
24 x - 21.81 y = 0
\]

From the first equation:

\[
-19.81 x + 18 y = 0 \Rightarrow y = \frac{19.81}{18} x \approx 1.10 x
\]

Choose \(x = 1 \):

\[
\Rightarrow y \approx 1.10
\]

Eigenvector:

$$
\mathbf{v}_1 \approx \begin{bmatrix}
1 \\
1.10
\end{bmatrix}
$$

Eigenvector for $\lambda_2 \approx 0.19$:

Compute:

$$
A - 0.19 I \approx \begin{bmatrix}
22 - 0.19 & 18 \\
24 & 20 - 0.19
\end{bmatrix} = \begin{bmatrix}
21.81 & 18 \\
24 & 19.81
\end{bmatrix}
$$

Set up the system:

$$
21.81 x + 18 y = 0
$$

$$
24 x + 19.81 y = 0
$$

From the first:

$$
y = -\frac{21.81}{18} x \approx -1.21 x
$$

Choose $x = 1$:

$$
y \approx -1.21
$$

Eigenvector:

$$
\mathbf{v}_2 \approx \begin{bmatrix}
1 \\
-1.21
\end{bmatrix}
$$

Step 3: Construct D and S

Matrix D:

The diagonal matrix with eigenvalues:

$$
D = \begin{bmatrix}
\lambda_1 & 0 \\
0 & \lambda_2
\end{bmatrix}
$$

Since the eigenvalues are approximate, for better accuracy, we can choose exact forms or normalized eigenvectors.

Two different D matrices:

1. First $D^{(1)}$:

$$
D^{(1)} = \begin{bmatrix}
41.81 & 0 \\
0 & 0.19
\end{bmatrix}
$$

2. Second $D^{(2)}$:

Suppose we swap the order of eigenvalues:

$$
D^{(2)} = \begin{bmatrix}
0.19 & 0 \\
0 & 41.81
\end{bmatrix}
$$

Matrix S:

Construct S with eigenvectors as columns:

$$
S^{(1)} = \begin{bmatrix}
1 & 1 \\
1.10 & -1.21
\end{bmatrix}

\]

For better clarity, normalize eigenvectors:

- Eigenvector for \(\lambda_1 \):

\[
\mathbf{v}_1 = \begin{bmatrix}
1 \\
1.10
\end{bmatrix}
\]

Normalize:

\[
\|\mathbf{v}_1\| = \sqrt{1^2 + 1.10^2} \approx \sqrt{1 + 1.21} = \sqrt{2.21} \approx 1.49
\]

Normalized:

\[
\mathbf{v}_1^{(norm)} \approx \frac{1}{1.49} \begin{bmatrix} 1 \\ 1.10 \end{bmatrix} \approx \begin{bmatrix} 0.67 \\ 0.74 \end{bmatrix}
\]

- Eigenvector for \(\lambda_2 \):

\[
\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1.21 \end{bmatrix}
\]

Normalize:

\[
\|\mathbf{v}_2\| = \sqrt{1^2 + (-1.21)^2} \approx \sqrt{1 + 1.46} = \sqrt{2.46} \approx 1.57
\]

Normalized:

\[
\mathbf{v}_2^{(norm)} \approx \frac{1}{1.57} \begin{bmatrix} 1 \\ -1.21 \end{bmatrix} \approx \begin{bmatrix}

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