Let A Be A 7 X 6 Matrix Such That Ax = 0 Has Only The Trivial Solution. Find The Rank And Nullity Of

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Understanding the properties of matrices is fundamental in linear algebra, especially when analyzing solutions to systems of linear equations. In this article, we will delve into the specific case where matrix A is a 7 x 6 matrix, and the homogeneous system Ax = 0 has only the trivial solution. This scenario offers valuable insights into the rank, nullity, and the underlying structure of the matrix.

Introduction to Matrix Dimensions and Linear Systems

Before exploring the specific problem, it's essential to understand some foundational concepts:

Matrix Dimensions

- A matrix A with dimensions m x n means it has m rows and n columns.
- For our case, A is a 7 x 6 matrix: 7 rows and 6 columns.

Homogeneous System Ax = 0

- This system involves finding all vectors x such that when multiplied by A, results in the zero vector.
- The trivial solution is x = 0, meaning all components of x are zero.
- Non-trivial solutions exist if the nullity (dimension of the null space) is greater than zero.

Implications of Ax = 0 Having Only the Trivial Solution

When the homogeneous system Ax = 0 has only the trivial solution, it indicates a particular property of matrix A:

Injectivity of the Transformation

- The linear transformation represented by A is injective (one-to-one).
- No non-zero vector x maps to the zero vector under this transformation.

Full Column Rank

- The matrix A has full column rank, which equals the number of columns n.
- Since A is 7 x 6, the maximum possible rank is 6.

Matrix Rank and Nullity Relationship

- The Rank-Nullity Theorem states:

rank(A) + nullity(A) = n

WHERE:
- RANK(A) IS THE DIMENSION OF THE COLUMN SPACE OF A.
- NULLITY(A) IS THE DIMENSION OF THE NULL SPACE (SOLUTION SPACE OF Ax=0).

Given that the system has only the trivial solution, the nullity must be zero.

Determining the Rank of Matrix A

Based on the above insights, let's analyze the rank:

Maximum Possible Rank for A

- Since A has 6 columns, the maximum rank is 6.
- For the homogeneous system to have only the trivial solution, the rank must be equal to the number of columns.

Why Must the Rank Be 6?

- A lower rank (less than 6) would imply a non-zero nullity, leading to non-trivial solutions.
- Therefore, to ensure Ax=0 only has the trivial solution, rank(A) = 6.

Conclusion on Rank

- Rank(A) = 6

Calculating the Nullity of Matrix A

With the rank determined, we can find the nullity:

Applying the Rank-Nullity Theorem

- Recall: rank(A) + nullity(A) = n
- Plugging in the known values:

6 + nullity(A) = 6

- Solving for nullity(A):

nullity(A) = 6 - 6 = 0

Interpretation of Nullity

- Nullity is zero, meaning the null space contains only the zero vector.
- The system Ax=0 has only the trivial solution, confirming the initial condition.

Summary of Findings

- Rank of A: 6
- Nullity of A: 0

Significance of These Results

Understanding the rank and nullity provides insights into the structure of the matrix and the solution space:

Implications for the System of Equations

- Since rank(A) = 6, the system is consistent and has a unique solution for every right-hand side if it were non-homogeneous.
- The null space's triviality indicates no free variables; the system is fully determined within the bounds of the columns.

Impact on the Linear Transformation

- The transformation represented by A is injective from R^6 to R^7.
- The image (column space) of A is a 6-dimensional subspace of R^7.

Conclusion and Final Remarks

In linear algebra, the relationship between the dimensions of the matrix, its rank, and nullity is crucial for understanding the solutions to systems of equations. For a 7 x 6 matrix A such that Ax = 0 has only the trivial solution, the matrix must have full column rank, which is 6. Consequently, the nullity is zero, indicating no non-trivial solutions exist.

This case exemplifies a fundamental principle: when the homogeneous system associated with a matrix has only the trivial solution, the matrix is of full column rank, and the nullity is zero. Such matrices are injective transformations and play a vital role in various applications, including data science, engineering, and computational mathematics.

By grasping these concepts, students and practitioners can better analyze linear systems, determine invertibility, and understand the structure of solutions in higher-dimensional spaces.

Frequently Asked Questions

Given a 7 x 6 matrix A where Ax = 0 has only the trivial solution, what is the rank of A?

The rank of A is 6 because the nullity is zero, and by the Rank-Nullity Theorem, rank + nullity = number of columns (6).

For a 7 x 6 matrix A with only the trivial solution to Ax = 0, what is the nullity of A?

The nullity of A is 0 because Ax = 0 has only the trivial solution, implying the null space contains only the zero vector.

What does the condition 'Ax = 0 has only the trivial solution' imply about the matrix A?

It implies that A has full column rank, meaning all columns are linearly independent, and the nullity is zero.

Can a 7 x 6 matrix have full column rank? Why or why not?

Yes, a 7 x 6 matrix can have full column rank (which is 6), because the number of columns is less than or equal to the number of rows, allowing independent columns.

Using the Rank-Nullity Theorem, how do you find the rank and nullity for the given matrix?

Since the nullity is 0 (only trivial solution), the rank is equal to the number of columns, which is 6.

What is the significance of the matrix having only the trivial solution for Ax=0 in terms of invertibility?

Since the matrix is not square, it isn't invertible, but having only the trivial solution indicates the columns are linearly independent and the matrix has full column rank.

Is the matrix A necessarily of full row rank? Why or why not?

No, because A has more rows (7) than columns (6), and full row rank would require rank 6; since the rank is 6, it is of full column rank but not necessarily full row rank.

What is the maximum possible rank of matrix A under the given conditions?

The maximum possible rank of A is 6, since there are 6 columns, and the columns are linearly independent.

If the nullity of A is zero, what does that tell us about the solutions to Ax = b for an arbitrary vector b?

It indicates that the homogeneous system has only the trivial solution, but for the nonhomogeneous system, solutions exist only if b is in the column space of A; the system may be consistent or inconsistent depending on b.

Summarize: For a 7 x 6 matrix A where Ax=0 has only the trivial solution, what are the rank and nullity?

The rank of A is 6, and the nullity is 0.

Additional Resources

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Introduction

In the realm of linear algebra, matrices serve as fundamental tools to understand systems of linear equations, transformations, and vector spaces. Among the critical properties of matrices are their rank and nullity, which provide essential insights into the solution spaces of associated equations. When examining a matrix A, especially one characterized by particular properties such as the nature of its solutions, it becomes crucial to determine these attributes accurately.

This article delves into a specific problem involving a 7×6 matrix A, where the system $Ax = 0$ admits only the trivial solution. The goal is to analyze what this condition signifies about the matrix's rank and nullity. Through detailed explanations, mathematical principles, and logical reasoning, we will explore how the properties of such a matrix lead us to precise conclusions regarding its rank and nullity.

Understanding the Basic Concepts

Before we analyze the problem, it is essential to clarify the core concepts involved:

1. The Matrix Dimensions

- Size of the Matrix A:
The matrix A is a 7×6 matrix, which means it has 7 rows and 6 columns. This indicates a linear transformation from a 6-dimensional vector space $\mathbb{R}^6$ to a 7-dimensional space $\mathbb{R}^7$.

2. The Homogeneous System $Ax = 0$

- Trivial and Non-trivial Solutions:
The solutions to the homogeneous system $Ax = 0$ are vectors $x \in \mathbb{R}^6$ such that when multiplied by A, they produce the zero vector.

- Trivial Solution:
The solution $x = 0$ (the zero vector) is always a solution to homogeneous systems. If this is the only solution, the system is said to have a trivial solution only.

3. Matrix Rank and Nullity

- Rank of a Matrix ($\operatorname{rank}(A)$):
The maximum number of linearly independent columns (or rows) in A. It indicates the dimension of the column space (or row space).
- The rank can range from 0 up to the smaller of the number of rows or columns, i.e., from 0 to 6 for this matrix.

- Nullity of a Matrix ($\operatorname{nullity}(A)$):
The dimension of the null space (solution space of $Ax=0$). It measures the number of free variables in the system.

- The Rank-Nullity Theorem:
For an $m \times n$ matrix A,
$$
\operatorname{rank}(A) + \operatorname{nullity}(A) = n
$$
where n is the number of columns.

Analyzing the Given Conditions

The Core Condition: $Ax = 0$ Has Only the Trivial Solution

This condition is a pivotal starting point. It implies that:

- The null space of A contains only the zero vector.
- The nullity of A is zero:

$$
\operatorname{nullity}(A) = 0
$$

Implications of Nullity = 0

Given the Nullity-Nullity Theorem:

$$
\operatorname{rank}(A) + \operatorname{nullity}(A) = n
$$

where $n = 6$ (number of columns). Since $\operatorname{nullity}(A) = 0$:

$$
\operatorname{rank}(A) + 0 = 6
$$

$$
\Rightarrow \operatorname{rank}(A) = 6
$$

This directly indicates that:

- The matrix A has full column rank.

Is the Full Column Rank Possible?

In the context of the matrix dimensions:

- A 7×6 matrix can have at most rank 6 because the rank cannot exceed the smaller of the number of rows or columns.
- Achieving rank 6 means that the 6 columns are linearly independent.

Detailed Explanation of the Results

1. Rank of A:
As established, the rank of A must be 6. This is the maximum possible rank given the number of columns, and it aligns with the condition that the null space contains only the zero vector.

2. Nullity of A:
Since the nullity is determined by the difference between the number of columns and the rank:

$$
\operatorname{nullity}(A) = 6 - \operatorname{rank}(A) = 6 - 6 = 0
$$

This confirms that the null space is trivial, containing only the zero vector.

3. Interpretation in Linear Algebra Context:

- The matrix A is injective when viewed as a linear transformation from $\mathbb{R}^6$ to $\mathbb{R}^7$.
- The fact that $Ax=0$ has only the trivial solution indicates that A has full column rank, and the

columns are linearly independent.

4. Implications for the System and Transformation:

- The linear transformation represented by A is one-to-one (injective) on $\mathbb{R}^6$.
- There are no non-trivial solutions to $Ax=0$, which means the image of A is a 6-dimensional subspace of $\mathbb{R}^7$, embedded within the larger space.

Broader Context and Significance

The Rank-Nullity Theorem in Practice

The problem exemplifies the practical application of the rank-nullity theorem:

- Given: Nullity is zero.
- Therefore: Rank must be equal to the number of columns (6).

This provides a straightforward way to deduce properties of a matrix based on solution space characteristics.

Significance in Applications

Matrices with full column rank are vital in numerous applications:

- Solving Systems of Equations:
When A has full column rank, the system $Ax=b$ has a unique solution for every b in the column space, provided the system is consistent.

- Linear Independence:
The columns form a basis for their span, which is crucial in vector space decompositions, least squares problems, and data analysis.

- Invertibility and Pseudoinverses:
Although A isn't square and thus not invertible in the traditional sense, its full column rank allows for the construction of the Moore-Penrose pseudoinverse, facilitating solutions to least squares problems.

Concluding Summary

To synthesize the insights:

Property	Value	Explanation
Matrix Dimensions	7×6	7 rows, 6 columns
Nullity of A	0	Because $Ax=0$ has only the trivial solution
Rank of A	6	Maximum possible for a 7×6 matrix, matching the number of columns

This analysis confirms that the matrix A is of full column rank, and its nullity is zero. These properties are fundamental in guaranteeing the uniqueness of solutions to linear systems and understanding the structure of the linear transformation represented by A.

Final Remarks

Understanding the interplay between the rank and nullity of matrices is central to linear algebra. In problems where the homogeneous system has only the trivial solution, the matrix's rank necessarily equals the number of columns. Such matrices are instrumental in various fields, including engineering, computer science, and applied mathematics, where ensuring unique solutions and invertibility (or pseudo-invertibility) is crucial.

This problem succinctly demonstrates how solution properties directly inform us about the matrix's structural attributes, reinforcing the elegance and power of linear algebra's foundational theorems.

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