# Let F Be The Function Given By F(x)=5e^(3x^3). For What Positive Value Of A Is The Slope Of The Line

**Let F Be The Function Given By F(x)=5e^(3x^3). For What Positive Value Of A Is The Slope Of The Line**

Understanding the behavior of a function, particularly its slope at various points, is fundamental in calculus. When given a specific function like F(x)=5e^(3x^3), one common question is determining at which point the slope of the tangent line to the graph is equal to a certain value, or in particular, for what positive value of A the slope of the tangent line is equal to A. This involves finding the derivative of the function, analyzing its behavior, and solving for the specific value of A that corresponds to the slope at some point x.

In this comprehensive guide, we will explore the process step-by-step, including how to compute the derivative, interpret the results, and find the positive value of A for which the slope of the tangent line matches A. We will also include insights into related concepts such as the importance of derivatives, the application of the chain rule, and the significance of positive slopes in the context of the function.

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## Understanding the Function F(x)=5e^(3x^3)

Before diving into derivatives and slopes, it’s essential to comprehend the structure of the given function:

- Function form: F(x) = 5e^(3x^3)
- Components:
- The constant multiplier 5
- The exponential function e^(3x^3)
- Behavior:
- The exponential function is known for its rapid growth or decay depending on its exponent.
- The exponent 3x^3 indicates a cubic term multiplied by 3, which influences the curvature and growth rate.

This function is smooth and differentiable everywhere because exponential functions are infinitely differentiable, and polynomial compositions are as well. The key to understanding the slope of the tangent line at any point x is to find the derivative, which gives the instantaneous rate of change of the function.

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# Calculating the Derivative of F(x)

The main step in analyzing the slope of the tangent line is to compute the derivative of F(x). Applying the chain rule carefully is crucial here.

## Applying the Chain Rule

Recall that the derivative of $e^{(u)}$ with respect to x is $e^{(u)}$ multiplied by the derivative of u with respect to x:

$$ \frac{d}{dx} e^{u} = e^{u} \cdot \frac{du}{dx} $$

In our case, $u = 3x^3$, which is a composite function, requiring another application of the chain rule.

Step-by-step differentiation:

1. Identify the inner function $u(x) = 3x^3$
2. Compute du/dx:

$$ \frac{du}{dx} = 9x^2 $$

3. Apply the chain rule:

$$ F'(x) = 5 \cdot e^{u} \cdot \frac{du}{dx} $$

4. Substitute u back:

$$ F'(x) = 5 \cdot e^{3x^3} \cdot 9x^2 $$

5. Simplify:

$$ F'(x) = 45x^2 e^{3x^3} $$

This derivative tells us how the function F(x) changes at any point x, i.e., the slope of the tangent line to the graph at x.

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# Analyzing the Derivative and the Slope

The derivative:

$$F'(x) = 45x^2 e^{3x^3}$$

has several key properties:

- Non-negativity: Since $x^2 \geq 0$ and $e^{3x^3} > 0$ for all real x, $F'(x) \geq 0$. This means F(x) is non-decreasing everywhere.
- Zero slope points: $F'(x) = 0$ when $x = 0$, indicating a potential minimum or flat point.
- Positive slope: For $x \neq 0$, the derivative is positive, indicating that F(x) is increasing on both sides of zero.

Given the nature of the derivative, the slope of the tangent line at any point x is always non-negative, with the minimum at $x=0$ where the slope is zero.

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# Finding the Positive Value of A Corresponding to the Slope

The core question is: For what positive value of A is the slope of the line equal to A? This involves solving the equation:

$$F'(x) = A$$

for a given positive A.

Step 1: Set the derivative equal to A

$$45x^2 e^{3x^3} = A$$

Since $A > 0$, we seek solutions x that satisfy this equation.

Step 2: Express x in terms of A

Rearranging the equation:

$$45 x^2 e^{3x^3} = A$$

This is a transcendental equation, combining polynomial and exponential functions, which generally does not have an explicit algebraic solution. However, we can analyze the behavior and find the positive value of A for which the equation has solutions.

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# Determining the Range of Possible Slopes A

Given the function:

$$ G(x) = 45x^2 e^{3x^3} $$

we note that:

- When $x = 0$:

$$ G(0) = 45 \cdot 0^2 \cdot e^{0} = 0 $$

- When $x \to \infty$:

$$ G(x) \rightarrow \infty $$

because $e^{3x^3}$ grows faster than any polynomial as x approaches infinity.

- When $x \to -\infty$:

$$ G(x) \rightarrow 0 $$

since $x^2$ is positive but $e^{3x^3}$ approaches 0 due to the exponential decay for large negative x.

Key observations:

- The function $G(x)$ reaches a minimum of 0 at $x=0$.
- $G(x)$ is symmetric around $x=0$ in terms of magnitude but not in behavior because the exponential term dominates.

Step 3: Find the maximum or minimum of $G(x)$

To determine the range of A values, analyze the critical points of $G(x)$, i.e., where $G'(x) = 0$.

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# Calculating the Derivative of G(x)

$$ G(x) = 45x^2 e^{3x^3} $$

Differentiate using the product rule:

$$ G'(x) = 45 \left( 2x e^{3x^3} + x^2 \cdot e^{3x^3} \cdot 9x^2 \right) $$

Breaking it down:

- Derivative of $x^2$:

$$ 2x $$

- Derivative of $e^{3x^3}$:

$$ e^{3x^3} \cdot 9x^2 $$

Putting it all together:

$$ G'(x) = 45 \left( 2x e^{3x^3} + x^2 \cdot e^{3x^3} \cdot 9x^2 \right) $$

Factor common terms:

$$ G'(x) = 45 e^{3x^3} \left( 2x + 9x^4 \right) $$

Set $G'(x) = 0$ to find critical points:

$$ 45 e^{3x^3} (2x + 9x^4) = 0 $$

Since $e^{3x^3} \neq 0$, the critical points are determined by:

$$ 2x + 9x^4 = 0 $$

Factor out x:

$$ x (2 + 9x^3) = 0 $$

Solutions:

- $x = 0$
- $2 + 9x^3 = 0 \rightarrow x^3 = -2/9 \rightarrow x = -\sqrt[3]{2/9}$

Thus, critical points are at:

- $x = 0$
- $x = -\sqrt[3]{2/9}$

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## Evaluating G(x) at Critical Points

- At $x=0$:

$$ G(0) = 0 $$

- At $x = -\sqrt[3]{2/9}$:

Let’s denote:

$$ x_c = -\left(\frac{2}{9}\right)^{1/3} $$

Calculate $G(x_c)$:

$$ G(x_c) = 45 x_c^2 e^{3 x_c^3} $$

But since $x_c^3 = -2/9$:

$$ e^{3 x_c^3} = e^{3 \times (-2/9)} = e^{-2/3} $$

And:

$$ x_c^2 = \left( \left(\frac{2}{9}\right)^{1/3} \right)^2 = \left(\frac{2}{9}\right)^{2/3} $$

Thus:

$$ G(x_c) = 45 \times \left(\frac{2}{9}\right)^{2/3} \times e^{-2/3} $$

This value is positive, indicating a local maximum of $G(x)$.

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## Range of the Slope Function G(x)

From the critical points analysis:

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## Frequently Asked Questions

### What is the derivative of the function $F(x) = 5e^{3x^3}$ with respect to x?

The derivative is $F'(x) = 45x^2 e^{3x^3}$.

### How do you find the slope of the tangent line to the curve at a point x?

The slope is given by the derivative $F'(x)$, so you evaluate $F'(x)$ at the point of interest.

### For which positive value of A is the slope of the

## tangent line to F at x = A equal to a specific value?

You set $F'(A)$ equal to that value and solve for A to find the positive value where the slope matches.

## If the slope of the tangent line at x = A is to be equal to 10, what is the value of A?

Set $45A^2 e^{3A^3} = 10$ and solve for A numerically or analytically if possible.

## Is the function F(x) increasing or decreasing for positive x?

Since $F'(x) = 45x^2 e^{3x^3}$ is positive for all $x > 0$, F is increasing on $(0, \infty)$.

## What techniques can be used to solve for A in the equation $45A^2 e^{3A^3} = k$, where k is a positive constant?

Methods include numerical methods like Newton-Raphson or iterative approximation, as closed-form solutions may not exist.

## How does the exponential term $e^{3A^3}$ affect the slope as A increases?

The exponential term grows rapidly as A increases, causing the slope to increase exponentially with A.

## Can the value of A be zero when setting the slope equal to a positive number?

No, because at $A=0$, the slope $F'(0) = 0$, which is not positive. For positive slopes, A must be positive.

## What is the significance of choosing a positive A in this context?

Choosing a positive A ensures the point lies in the domain where the exponential function behaves regularly and the slope is positive.

### How can you verify the value of A once you find it satisfies the slope condition?

Substitute A back into F'(A) and check if it equals the desired slope value to verify correctness.

## Additional Resources

**Let F be the function given by F(x) = 5e^(3x^3). For what positive value of A is the slope of the line?**

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Introduction

Mathematics often challenges us to explore the intricate relationships between functions and their derivatives, revealing the dynamic behaviors underlying seemingly simple formulas. The question, "For what positive value of A is the slope of the line?" prompts a deep dive into calculus, specifically the concepts of differentiation and the behavior of exponential functions. The function in focus, $F(x) = 5e^{3x^3}$, is a fascinating blend of exponential growth modulated by a cubic polynomial in the exponent, making its derivative complex yet insightful. This article aims to dissect this function meticulously, analyze its derivatives, and ultimately determine the positive value of A at which the slope of a particular line—presumably tangent or secant—attains a specified characteristic.

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Understanding the Function $F(x) = 5e^{3x^3}$

The Components of the Function

At first glance, $F(x) = 5e^{3x^3}$ combines two fundamental elements:

1. Constant Multiplier (5): This scalar influences the overall scale of the function but doesn't affect its rate of change directly.
2. Exponential Component $e^{3x^3}$: The core of the function’s growth, exponential functions exhibit rapid increase or decrease, depending on their exponents.

The exponent $3x^3$ indicates that the growth rate is highly sensitive to changes in $x$, especially as $x$ becomes larger or smaller. Since the exponent involves a cubic term, the function exhibits different behaviors in positive and negative domains, with significant implications for its derivative and the slope of tangent lines.

Graphical Intuition

Graphically, $F(x)$ is an exponentially increasing function for positive $x$, with the rate of increase accelerating as $x$ grows. For negative $x$, because $x^3$ becomes negative, the exponential term approaches zero, making $F(x)$ tend toward zero from above. Understanding the shape of this function is essential for interpreting the behavior of its derivatives and the slopes of lines tangent to the curve at various points.

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## Differentiating $F(x)$: Finding the Slope Expression

### Applying the Chain Rule

To analyze the slope of a line related to $F(x)$, we need its derivative $F'(x)$. Differentiation of exponential functions with composite exponents typically involves the chain rule.

Given:

$$F(x) = 5e^{3x^3}$$

The derivative:

$$F'(x) = 5 \cdot \frac{d}{dx} \left( e^{3x^3} \right)$$

Applying the chain rule:

$$F'(x) = 5 \cdot e^{3x^3} \cdot \frac{d}{dx} (3x^3)$$

Calculating:

$$\frac{d}{dx} (3x^3) = 9x^2$$

Thus:

$$F'(x) = 5 \cdot e^{3x^3} \cdot 9x^2 = 45x^2 e^{3x^3}$$

### Interpretation of the Derivative

The derivative $F'(x) = 45x^2 e^{3x^3}$ encapsulates the instantaneous rate of change of the function at any point $x$. Several key observations emerge:

- Non-negativity: Since $45x^2 \geq 0$ for all real $x$, and $e^{3x^3} > 0$, the derivative is always non-negative.
- Zero at $x=0$: When $x=0$, $F'(0) = 0$, indicating a horizontal tangent line at the origin.
- Growth Behavior: For $x \neq 0$, the derivative is positive, indicating that $F(x)$ is increasing in those regions.

This derivative's form suggests that the slope of the tangent line to $F(x)$

$F'(x)$ varies with $x$, reaching a minimum (zero) at $x=0$ and increasing elsewhere.

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## The Quest for the Positive Value of A

### Contextualizing the Question

The core question asks: "For what positive value of A is the slope of the line?" Although the phrasing appears incomplete or ambiguous, it commonly implies a scenario where:

- The slope of the line (possibly tangent or secant) equals a certain value A.
- The task is to find the positive value of A for which the line has a specific property, such as being tangent to the curve, or perhaps where the slope equals A at some point $x$.

Given the context, the most plausible interpretation is:

"At what positive value of $A$ does the tangent line to $F(x)$ have a slope of $A$?"

This interpretation aligns with typical calculus problems involving derivatives and slopes of tangent lines.

### Setting the Derivative Equal to A

If we assume the goal is to find $A$ such that there exists a point $x$ where the slope of the tangent line equals $A$, then:

$$F'(x) = A$$

Substituting:

$$45x^2 e^{3x^3} = A$$

Since $A$ is positive, and the right-hand side involves exponential and quadratic terms, our task reduces to understanding how $A$ varies with $x$.

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## Analyzing the Relationship Between $A$ and $x$

### Behavior of $F'(x)$

The function:

$$G(x) = 45x^2 e^{3x^3}$$

maps $x$ to the slope $A$ of the tangent line at $x$. Let's analyze its behavior:

- At $x=0$: $G(0) = 0$
- As $x \to \infty$: $G(x) \to \infty$, since $e^{3x^3}$ dominates growth.
- As $x \to -\infty$: $e^{3x^3} \to 0$, so $G(x) \to 0$.

The function $G(x)$ is non-negative everywhere and tends to infinity for large positive $x$. Its minimal value is at $x=0$, where $G(0) = 0$. The function is strictly increasing for $x > 0$.

Finding the Maximum of $G(x)$

Since $G(x)$ tends to infinity as $x \to \infty$, for positive $x$, it increases without bound. For negative $x$, $G(x) \to 0$, reaching a minimum at $x=0$.

Thus, the range of $G(x)$ for $x \geq 0$ is $[0, \infty)$. The positive values of $A$ correspond to the values $G(x)$ takes for some $x \geq 0$.

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Determining the Critical Point and Maximum Slope

Is there a maximum of $G(x)$?

Given the behavior, $G(x)$ is increasing for $x > 0$, so it doesn't have a maximum in that domain—it extends to infinity.

However, to understand the range thoroughly, especially if the goal involves the maximum slope or a specific point, we analyze the derivative of $G(x)$ to check for extrema.

Differentiating $G(x)$

Let:

$$G(x) = 45x^2 e^{3x^3}$$

Use the product rule:

$$G'(x) = 45 \left( 2x e^{3x^3} + x^2 \cdot e^{3x^3} \cdot 9x^2 \right)$$

Simplify:

$$G'(x) = 45 e^{3x^3} \left( 2x + 9x^4 \right)$$

Factor:

$$G'(x) = 45 e^{3x^3} x (2 + 9x^3)$$

Set $G'(x) = 0$:

$$45 e^{3x^3} x (2 + 9x^3) = 0$$

Since $e^{3x^3} \neq 0$, solutions occur when:

1. $x=0$
2. $2 + 9x^3=0 \Rightarrow x^3 = -\frac{2}{9} \Rightarrow x = - \left( \frac{2}{9} \right)^{1/3}$

This critical point at $x = - (2/9)^{1/3}$ indicates a potential extremum.

Behavior at Critical Points

- At $x=0$, $G(0)=0$, and $G'(0)=0$.
- At $x= - (2/9)^{1/3}$, check whether this is a

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