Let G Be A Differentiable Function Such That G(4)=0.325 And G(x)=1xex(cos(x100)) . What Is The Value

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Understanding the behavior of differentiable functions and their derivatives is fundamental in calculus. When given a specific functional form and a point value, such as G(4) = 0.325 and G(x) = 1xex(cos(x100)), it prompts us to analyze the function's properties, compute derivatives, and interpret the results within the context of calculus concepts. This article explores the detailed steps involved in understanding this function, calculating its derivative, and interpreting its value at a specific point.

Analyzing the Given Function G(x)

The function provided is:

G(x) = 1xex(cos(x100))

At first glance, this expression appears somewhat ambiguous due to the notation. To proceed effectively, it's essential to interpret the notation correctly.

Interpreting the Function Notation

The expression contains several components:

- The term "1x" suggests a product involving x, possibly indicating x multiplied by 1, which simplifies to x.
- The "ex" likely refers to the exponential function e^x.
- The "cos(x100)" indicates the cosine of x multiplied by 100.

Given this, the most reasonable interpretation of G(x) is:

G(x) = x e^{cos(100x)}

This interpretation aligns with common mathematical notation and makes the function well-defined and differentiable across real numbers.

Restated Function

Therefore, the function simplifies to:

G(x) = x e^{cos(100x)}

This function is differentiable because it involves elementary functions (polynomial, exponential, cosine) that are differentiable everywhere.

Understanding the Function's Behavior

Before calculating derivatives or specific values, it's helpful to analyze the behavior of G(x).

Components of G(x)

- The linear component: x
- The exponential component: e^{cos(100x)}

Since cos(100x) oscillates between -1 and 1, the exponential term varies between e^{-1} and e^{1}.

- Minimum of e^{cos(100x)}: e^{-1} ≈ 0.3679
- Maximum of e^{cos(100x)}: e^{1} ≈ 2.7183

Thus, G(x) oscillates with amplitude influenced by x and the exponential of an oscillating cosine.

Range and Behavior at Specific Points

- At x=0: G(0) = 0 e^{cos(0)} = 0 e^{1} = 0
- As x increases, G(x) can take on a wide range of values depending on the oscillations of cos(100x).

Calculating the Derivative G'(x)

To understand the rate of change of G(x), we differentiate it with respect to x.

Applying the Product Rule

$G(x) = x e^{\cos(100x)}$

Let’s identify:

- $u(x) = x$
- $v(x) = e^{\cos(100x)}$

Then,

$G'(x) = u'(x) v(x) + u(x) v'(x)$

Calculating derivatives:

- $u'(x) = 1$
- $v(x) = e^{\cos(100x)}$

Next, differentiate v(x):

$v'(x) = e^{\cos(100x)}$ derivative of cos(100x)

Recall that:

$d/dx [\cos(100x)] = -100 \sin(100x)$

Therefore,

$v'(x) = e^{\cos(100x)} (-100 \sin(100x)) = -100 \sin(100x) e^{\cos(100x)}$

Putting it all together:

$G'(x) = 1 e^{\cos(100x)} + x (-100 \sin(100x) e^{\cos(100x)})$

Simplify:

$G'(x) = e^{\cos(100x)} - 100x \sin(100x) e^{\cos(100x)}$

Factor $e^{\cos(100x)}$:

$G'(x) = e^{\cos(100x)} [1 - 100x \sin(100x)]$

Evaluating G(4) and G'(4)

Given:

G(4) = 0.325

and the function:

G(x) = x e^{cos(100x)}

we can verify that the value aligns with the formula.

Calculating G(4)

Compute:

G(4) = 4 e^{cos(400)}

Given G(4) = 0.325, this allows us to verify the consistency of the function's form.

Calculate cos(400):

cos(400) radians

Since cosine is periodic with period 2π (~6.283), reduce 400 modulo 2π:

400 / (2π) ≈ 400 / 6.283 ≈ 63.66

Calculate the fractional part:

63 6.283 ≈ 396.0

Remaining:

400 - 396 ≈ 4

So, cos(400) ≈ cos(4 radians)

cos(4 radians) ≈ -0.6536

Now, compute:

G(4) = 4 e^{ -0.6536 } ≈ 4 e^{ -0.6536 } ≈ 4 0.520

≈ 2.08

But since the given G(4) is 0.325, there's a discrepancy, which suggests a mismatch or that the initial assumption about the function's form is incorrect or that the notation in the original problem is different.

Alternatively, perhaps the notation in the problem is intended differently, or the function is more complex. Nonetheless, for the purpose of this article, we proceed with the interpretation G(x) = x e^{cos(100x)}.

What Is The Value at x = 4?

Using the interpreted function:

G(4) = 4 e^{cos(400)}

From earlier calculations:

cos(400) ≈ -0.6536

So,

G(4) ≈ 4 e^{ -0.6536 } ≈ 4 0.520 ≈ 2.08

This is significantly different from the provided value of 0.325, indicating potential inconsistency.

Possible explanations:

- The notation "G(x)=1xex(cos(x100))" might be misinterpreted.
- The function might involve different constants or factors.
- The value 0.325 might refer to a different point or a scaled version.

Conclusion and Insights

Despite the discrepancies, the core mathematical approach involves:

- Correctly interpreting the function notation
- Applying the product rule for differentiation
- Understanding the oscillatory behavior of cosine and its exponential
- Evaluating the function at specific points using known trigonometric values

Key takeaways:

1. The function G(x) = x e^{cos(100x)} combines polynomial and exponential

oscillatory components, making its behavior rich and complex.
2. Its derivative involves both the exponential term and the sine function, reflecting the oscillatory nature.
3. Evaluating the function at particular points requires reducing angles modulo 2π to find cosine values.
4. Discrepancies in expected and computed values highlight the importance of precise notation and interpretation.

Further Exploration

- Investigate how the amplitude of G(x) varies as x increases.
- Study the critical points where G'(x) = 0 to understand maxima and minima.
- Explore the integral of G(x) over specific intervals to analyze the accumulated behavior.

In summary, understanding the derivative of functions like G(x) = x e^{cos(100x)} is crucial in calculus, providing insights into the function's increasing/decreasing behavior, oscillations, and extrema, which are valuable across physics, engineering, and applied mathematics.

Note: For precise calculations and specific values, always verify the function's exact form and the context provided.

Frequently Asked Questions

What is the value of the function G at x = 4 given G(4) = 0.325?

The value of G at x = 4 is 0.325, as given in the problem statement.

What is the expression for the differentiable function G(x)?

G(x) = 1 x e^x cos(x 100).

How can we verify that G(x) is differentiable?

Since G(x) is composed of elementary differentiable functions (polynomials, exponentials, and trigonometric functions), and their composition is differentiable, G(x) is differentiable.

What is the significance of G(4) = 0.325 in analyzing G(x)?

It provides a specific point on G(x), which can be used for approximation or verification of calculations involving G(x) or its derivatives.

How do we interpret the term cos(x 100) in G(x)?

It introduces high-frequency oscillations into G(x), as cos(x 100) oscillates rapidly as x varies.

What is the derivative G'(x) of the function G(x)?

Using the product rule, G'(x) = d/dx [x e^x cos(x 100)] which involves differentiating each component accordingly.

Can we compute G(4) directly from the expression G(x) = x e^x cos(100x)?

Yes, by substituting x = 4 into the expression: G(4) = 4 e^4 cos(400). However, the given value is 0.325, indicating an approximation or specific context.

What challenges might arise when calculating G(4) explicitly?

Calculating G(4) explicitly involves evaluating e^4 and cos(400), which may require high-precision computation due to the oscillatory nature of the cosine term.

Is the value of G(4) = 0.325 consistent with the expression G(x) = x e^x cos(100x)?

Given the oscillatory behavior of cos(400) and the exponential growth e^4, the value 0.325 suggests an approximation or specific context, as the exact value from the expression could vary significantly.

Additional Resources

Let G Be A Differentiable Function Such That G(4)=0.325 And G(x)=1xex(cos(x100)). What Is The Value is a compelling mathematical inquiry that combines aspects of calculus, function analysis, and computational techniques. This problem prompts us to explore the nature of the function G, understand its differentiability, evaluate its value at a specific point, and interpret the implications of its structure. In this review, we will delve into the components of the function, analyze its properties, and discuss methods to determine G(4), providing a comprehensive understanding of this interesting mathematical challenge.

Understanding the Function G(x)

Breaking Down the Definition

The function under consideration is defined as:

$$ G(x) = 1x \cdot e^{x} \cdot \cos(x \times 100) $$

At first glance, the notation appears to be a bit ambiguous, but a common interpretation is that the function is:

$$ G(x) = x \cdot e^{x} \cdot \cos(100x) $$

This interpretation aligns with standard mathematical notation, where "1x" is simply "x," and the multiplication signs are explicit.

Features of the Function:

- Polynomial component: The x term introduces a linear factor.
- Exponential component: e^{x} grows rapidly for large x, influencing the overall amplitude.
- Oscillatory component: $\cos(100x)$ introduces high-frequency oscillations, creating rapid fluctuations in the function's value.

Implications:

- The function is smooth and differentiable everywhere because it is composed of elementary differentiable functions.
- The high frequency of the cosine term implies that G(x) oscillates rapidly, especially as x increases.

Differentiability and Mathematical Properties

Is G(x) Differentiable?

Since $G(x)$ is constructed from elementary functions—polynomial, exponential, and cosine—it is differentiable for all real x. Each component is infinitely differentiable over the real line, and their product remains differentiable.

Derivative of G(x):

Applying the product rule multiple times and chain rule as needed, the derivative $G'(x)$ is:

$$
G'(x) = \frac{d}{dx} \left(x e^{x} \cos(100x) \right)
$$

Expanding this using the product rule:

$$
G'(x) = \frac{d}{dx}(x) \cdot e^{x} \cos(100x) + x \cdot \frac{d}{dx}(e^{x}) \cos(100x) + x e^{x} \cdot \frac{d}{dx}(\cos(100x))
$$

Calculating each term:

- $\frac{d}{dx}(x) = 1$
- $\frac{d}{dx}(e^{x}) = e^{x}$
- $\frac{d}{dx}(\cos(100x)) = -100 \sin(100x)$

Thus,

$$
G'(x) = e^{x} \cos(100x) + x e^{x} \cos(100x) + x e^{x} (-100 \sin(100x))
$$

Simplifying:

$$
G'(x) = e^{x} \cos(100x) (1 + x) - 100 x e^{x} \sin(100x)
$$

This derivative exists everywhere, confirming G(x) is differentiable on $\mathbb{R}$.

Pros and Cons:

- Pros:
- Smoothness guarantees well-behaved calculus properties.
- Differentiability allows for analysis via techniques like Taylor series, optimization, etc.

- Cons:
- The high-frequency oscillations can make analytical evaluation challenging.
- Rapid oscillations might complicate numerical methods due to issues like

aliasing or numerical instability.

Calculating G(4): The Core Problem

Given Data and Objective

The problem states:

- $G(4) = 0.325$
- $G(x) = x e^{x} \cos(100x)$

and asks, "What is the value?"—which likely refers to confirming or evaluating $G(4)$.

Understanding the given data:

- The explicit function suggests that:

$$
G(4) = 4 \times e^{4} \times \cos(400)
$$

- The value of e^{4} is approximately:

$$
e^{4} \approx 54.5981
$$

- Therefore:

$$
G(4) \approx 4 \times 54.5981 \times \cos(400)
$$

Now, the critical piece is evaluating $\cos(400)$.

Evaluating $\cos(400)$

Since $\cos(\theta)$ is periodic with period 2π, we can reduce the angle modulo 2π:

$$
400 \text{ radians} \mod 2\pi
$$

Calculate the equivalent angle:

$$
\text{Number of periods in } 400: \quad N = \frac{400}{2\pi} \approx \frac{400}{6.2832} \approx 63.66
$$

The fractional part:

$$
0.66 \text{ periods}
$$

Thus, the equivalent angle within one period:

$$
\theta_{\text{mod}} = 400 - 2\pi \times 63 \approx 400 - 6.2832 \times 63 = 400 - 395.8416 \approx 4.1584
$$

Now, evaluate $\cos(4.1584)$:

Using a calculator or approximation:

$$
\cos(4.1584) \approx -0.533
$$

Putting it all together:

$$
G(4) \approx 4 \times 54.5981 \times (-0.533) \approx 218.3924 \times (-0.533) \approx -116.3
$$

This contrasts with the given $G(4) = 0.325$, suggesting that either:

- The actual function might include additional factors or modifications not captured in the initial interpretation, or
- The problem is asking for a theoretical or approximate value rather than a precise calculation.

Interpreting the Discrepancy and Final Results

Analysis of the Discrepancy

Given the calculations:

- The theoretical value based on $G(x) = x e^{x} \cos(100x)$ at $x=4$ is approximately -116.3.
- The problem states $G(4) = 0.325$, which is vastly different.

Possible explanations:

- The notation might be misinterpreted; perhaps the function is:

$$
G(x) = \frac{1}{x} e^{x} \cos(100x)
$$

which would drastically change the evaluation.

- Alternatively, the problem might involve a normalized or scaled version of the function.

Testing the alternative:

$$
G(4) = \frac{1}{4} \times e^{4} \times \cos(400) \approx 0.25 \times 54.5981 \times (-0.533) \approx 13.6495 \times (-0.533) \approx -7.27
$$

Still not close to 0.325.

Further, perhaps the problem states that the function is:

$$
G(x) = \text{some scaled or shifted version}
$$

or perhaps the question is about estimating the value of $G(4)$ under certain conditions.

Methods to Approximate or Confirm the Value

Numerical Evaluation

Given the high oscillation of $\cos(100x)$, numerical methods such as:

- Monte Carlo integration for average behaviors.
- Numerical differentiation to analyze the slope.
- Graphical plotting to visualize oscillations and approximate the function's value at specific points.

are valuable tools.

Analytical Approaches

- Fourier analysis: to understand the oscillatory behavior.
- Asymptotic analysis: for large x, though at $x=4$, the oscillations are significant but manageable.

Pros and Cons of Approaches

Pros:

- Numerical methods provide approximate but practical solutions when analytical evaluation is complex.
- Analytical techniques deepen understanding of the function's behavior.

Cons:

- High-frequency oscillations make numerical evaluation computationally intensive.
- Analytical solutions may be intractable due to the oscillatory term.

Summary and Final Remarks

The problem of evaluating $G(4)$ for the function $G(x) = x e^{x} \cos(100x)$ illustrates the challenges and intricacies of analyzing functions with combined polynomial, exponential, and oscillatory components. While the straightforward substitution suggests a value around \(-116.3

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