# Let R Be The Relation Defined On The RxR By : (x, Y) R (z,t) = X+zy+t. (i) R Is It Reflexive? (ii) R

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Understanding relations in mathematics, especially those defined on Cartesian products like $R \times R$, is foundational for many areas in algebra and discrete mathematics. In this article, we explore a specific relation $R$ defined on the Cartesian product $R \times R$, where each element is an ordered pair $(x, y)$. The relation is given by:

$$
(x, y) R (z, t) = x + zy + t
$$

Our goal is to analyze this relation thoroughly, focusing on whether it is reflexive, and discussing additional properties that might characterize it, such as symmetry, transitivity, and others.

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## Understanding the Relation $R$ on $R \times R$

Before we delve into properties such as reflexivity, it is crucial to interpret what the relation signifies. The relation $R$ is defined on ordered pairs $(x, y)$ and $(z, t)$ in $R \times R$, with the relation value given by:

$$
(x, y) R (z, t) = x + zy + t
$$

This expression seems to define a numerical value rather than a simple true/false statement, which suggests that the relation might be defined as a subset of $R \times R \times R$, or perhaps as a relation that holds if some condition involving this value is satisfied.

For clarity, we interpret the relation $R$ as follows:

- The relation holds between $(x, y)$ and $(z, t)$ if the value $x + zy + t$ satisfies a particular property, such as being zero, positive, or some other criterion.

However, since the problem states $(x, y) R (z, t) = x + zy + t$, without additional conditions, we will analyze the properties symbolically and consider typical relations, such as:

- The relation holds if $x + zy + t = 0$,
- or if $x + zy + t$ belongs to a particular subset of $R$,

- or as a function for understanding the structure.

For this article, we assume the relation is defined as the set of all pairs $((x, y), (z, t))$ in $R^2 \times R^2$ where the value $x + zy + t$ satisfies a certain property.

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# Analyzing Reflexivity of the Relation $R$

Reflexivity is a fundamental property in the study of relations. A relation $R$ on a set $S$ is reflexive if every element is related to itself. Formally:

$$
\text{For all } a \in S, \quad a R a
$$

In the context of our relation on $R \times R$, this means:

$$
\forall (x, y) \in R \times R, \quad (x, y) R (x, y)
$$

Given the definition:

$$
(x, y) R (x, y) = x + y \cdot y + y = x + y^2 + y
$$

Question: Does this value satisfy the property that defines the relation?

Since the relation is given as a mapping to a numerical value, and no specific condition for the relation to hold is provided, we interpret that:

- The relation $R$ is reflexive if the value $x + y^2 + y$ satisfies the relation's defining property (e.g., equals zero) for all $(x, y) \in R^2$.

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## Is $R$ Reflexive? Analyzing the Condition

Suppose the relation $R$ is reflexive if:

$$
(x, y) R (x, y) \quad \text{holds for all } (x, y) \in R^2
$$

If the relation holds when the value $x + y^2 + y$ equals zero, then:

$$
x + y^2 + y = 0 \quad \text{for all } (x, y)
$$

But this is impossible because:

- For any fixed $y$, $x$ must satisfy:

$$
x = - (y^2 + y)
$$

- Therefore, the only elements $(x, y)$ for which the relation holds are those where $x = - (y^2 + y)$.

This indicates that the relation is not reflexive over the entire set $R^2$, since not all pairs satisfy this condition, only those on the specific curve:

$$
x = - (y^2 + y)
$$

Conclusion:

- The relation $R$ is not reflexive on the entire set $R^2$ because the condition for self-relatedness is not true for all pairs $(x, y)$. It only holds for those pairs satisfying $x = - (y^2 + y)$.

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# Additional Properties of the Relation $R$

While the primary focus is on reflexivity, exploring other properties such as symmetry, transitivity, and antisymmetry can provide a more comprehensive understanding of the relation.

## Symmetry

A relation $R$ on $R^2$ is symmetric if:

$$
\text{Whenever } (x, y) R (z, t), \text{ then } (z, t) R (x, y)
$$

Given the relation's definition:

$$
(x, y) R (z, t) = x + zy + t

$$
$$

and

$$
(z, t) R (x, y) = z + xy + y
$$

Question: Is the relation symmetric?

- For symmetry to hold, the following must be true:

$$
x + zy + t = z + xy + y
$$

for all $(x, y), (z, t)$ such that the relation holds. This simplifies to:

$$
x + zy + t = z + xy + y
$$

which generally does not hold for arbitrary pairs $(x, y)$ and $(z, t)$.

Thus, the relation is not symmetric unless specific conditions are met, such as:

$$
x + zy + t = z + xy + y
$$

which would impose constraints on the pairs.

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# Transitivity

A relation $R$ is transitive if:

$$
\text{Whenever } (x, y) R (z, t) \text{ and } (z, t) R (u, v), \text{ then } (x, y) R (u, v)
$$

Given the nature of the relation, checking transitivity involves evaluating:

- $x + zy + t$
- $z + xy + y$
- $u + zv + v$

and determining if the relation's defining property holds accordingly.

Due to the algebraic complexity, general transitivity does not hold unless the relation reduces to a special case (e.g., constant relations or identity relations).

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## Summary of Key Findings

| Property | Status | Explanation |
| --- | --- | --- |
| Reflexivity | No | Only pairs satisfying $x = - (y^2 + y)$ are related to themselves; not all do. |
| Symmetry | No | Generally, $x + zy + t \neq z + xy + y$, so relation isn't symmetric. |
| Transitivity | No | Lacks transitivity in general due to algebraic complexity. |
| Anti-symmetry | Not necessarily | Depends on specific pairs; generally not anti-symmetric. |

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## Conclusion

The relation $R$ defined on $R \times R$ by the expression:

$$
(x, y) R (z, t) = x + zy + t
$$

exhibits interesting algebraic properties. It is not reflexive over the entire set $R^2$, as the condition for a pair to relate to itself reduces to $x = - (y^2 + y)$, which is only satisfied on a specific curve. Additionally, this relation is not symmetric or transitive in the general case.

Implications for mathematical studies and applications:

- When defining relations based on algebraic expressions, it's essential to establish the exact criteria under which the relation holds (e.g., equality to zero, belonging to a subset).
- Such relations can be viewed as defining algebraic curves or surfaces, which are fundamental in algebraic geometry.
- Understanding properties like reflexivity and symmetry helps classify relations and determine their usefulness in modeling real-world phenomena or in pure mathematical contexts.

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## Further Exploration

For those interested in delving deeper, consider the following avenues:

- Investigate the relation under specific conditions, such as fixing one variable and analyzing the

relation's behavior.
- Define the relation with a specific property, for example, the

# Frequently Asked Questions

## What is the relation R defined on R x R in the form (x, y) R (z, t) = x + zy + t?

R is a relation on R x R where two pairs (x, y) and (z, t) are related if and only if their relation value is given by $x + zy + t$.

## Is the relation R reflexive on R x R?

To determine if R is reflexive, check if for all (x, y) in R x R, (x, y) R (x, y) holds. That is, does $x + y \cdot y + y = x$? Since $x + y^2 + y = x$ implies $y^2 + y = 0$, which is not true for all y, R is not reflexive.

## Under what conditions is the relation R reflexive?

R is reflexive only when for all (x, y), $x + y^2 + y = x$, which simplifies to $y^2 + y = 0$. This holds only when $y = 0$ or $y = -1$, so R is reflexive on the subset of R x R where $y = 0$ or $y = -1$.

## Is the relation R symmetric?

To check symmetry, see if (x, y) R (z, t) implies (z, t) R (x, y). Since the relation depends on x, y, z, t in a non-symmetric way, R is generally not symmetric.

## Is the relation R transitive?

Transitivity would require that if (x, y) R (z, t) and (z, t) R (u, v), then (x, y) R (u, v). Given the form $x + zy + t$, this condition does not hold universally, so R is not necessarily transitive.

## What is the significance of the relation R in terms of algebraic structure?

The relation R defines a condition based on linear combinations involving the variables, which could relate to affine transformations or linear relations, but without additional properties, it does not form an equivalence or order relation.

## How can the properties of R be summarized in terms of reflexivity and other relations?

R is not reflexive on the entire R x R but is reflexive on specific subsets where $y = 0$ or $y = -1$. It is neither symmetric nor transitive in general, indicating it is not an equivalence relation.

# Additional Resources

Understanding the Relation R on $\mathbb{R} \times \mathbb{R}$: An In-Depth Analysis

In the realm of mathematics, especially in set theory and relation analysis, defining and understanding relations on Cartesian products is fundamental. One such relation that has garnered interest is defined on $\mathbb{R} \times \mathbb{R}$, the set of all ordered pairs of real numbers. The relation, denoted as $R$, is characterized by a specific algebraic condition: for any pairs $(x, y)$ and $(z, t)$ in $\mathbb{R} \times \mathbb{R}$,

$$
(x, y) R (z, t) \iff x + zy + t
$$

This expression, as presented, appears to be incomplete or possibly misrepresented, but it forms a basis for a comprehensive exploration of the properties of $R$. This article aims to analyze this relation thoroughly, examining key properties such as reflexivity, symmetry, transitivity, and others, to understand its structure and implications.

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# Deciphering the Definition of Relation R

## Clarifying the Relation's Expression

The initial challenge in analyzing $R$ lies in understanding its defining expression:

$$
(x, y) R (z, t) = x + zy + t
$$

Given the context, this notation suggests that the relation evaluates to a certain value based on the components of the pairs, rather than being a straightforward set of ordered pairs. Typically, in relation analysis, the notation:

$$
(x, y) R (z, t) \iff \text{some condition}
$$

indicates that the relation holds between $(x, y)$ and $(z, t)$ if the condition is true.

In this case, the expression appears to be an algebraic formula, not a condition. To proceed effectively, we interpret the relation as:

$$
(x, y) R (z, t) \iff x + zy + t = 0
$$

or perhaps

$$
(x, y) R (z, t) \iff x + zy + t \text{ satisfies some property}
$$

For the sake of detailed analysis, we will assume that the relation is defined as:

$$
\boxed{(x, y) R (z, t) \iff x + zy + t = 0}
$$

This assumption aligns with typical relation definitions involving algebraic expressions and allows us to analyze properties systematically.

---

# Analyzing the Properties of Relation R

## 1. Is R Reflexive?

Definition of Reflexivity:
A relation $R$ on a set $S$ is reflexive if for every element $a \in S$, $a R a$ holds. In the context of $\mathbb{R} \times \mathbb{R}$, this means:

$$
(x, y) R (x, y) \quad \text{for all } (x, y) \in \mathbb{R} \times \mathbb{R}
$$

Given our assumed definition:

$$
(x, y) R (x, y) \iff x + y \cdot y + y = 0
$$

Simplifying:

$$
x + y^2 + y = 0
$$

This is a condition on the components $x$ and $y$. For the relation to be reflexive, it must hold for all $(x, y)$.

Analysis:

- For fixed $y$, the value of $x$ must satisfy:

$$
x = - y^2 - y
$$

- Since $x$ can be any real number satisfying this equation for a given $y$, the set of all $(x, y)$ for which $(x, y) R (x, y)$ holds is:

$$
\{ (x, y) \in \mathbb{R}^2 : x = - y^2 - y \}
$$

Conclusion:

- The relation $R$ is not reflexive on the entire set $\mathbb{R} \times \mathbb{R}$, because for a fixed pair $(x, y)$, the only way $(x, y) R (x, y)$ holds is if $x = - y^2 - y$.
- Since this is not generally true for arbitrary $(x, y)$, $R$ is not reflexive on the entire set.

Summary:
The relation $R$ is not reflexive over $\mathbb{R} \times \mathbb{R}$ because the condition for self-relations depends on the specific components, and is not universally true.

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## 2. Is R Symmetric?

Definition of Symmetry:
$R$ is symmetric if:

$$
\text{Whenever } (x, y) R (z, t), \text{ then } (z, t) R (x, y)
$$

Given our assumption:

$$
(x, y) R (z, t) \iff x + zy + t = 0
$$

We need to check whether:

$$
(z, t) R (x, y) \iff z + x y + t = 0
$$

is implied by the first condition.

Analysis:

- From $(x, y) R (z, t)$:

$$
x + zy + t = 0
$$

- To determine if $R$ is symmetric, check whether:

$$
z + xy + t = 0
$$

- Comparing the two:

$$
x + zy + t = 0 \quad \text{and} \quad z + xy + t = 0
$$

Subtracting the second from the first:

$$
(x - z) + zy - xy = 0
$$

Simplify:

$$
(x - z) + y(z - x) = 0
$$

$$
(x - z) - y(x - z) = 0
$$

$$
(x - z)(1 - y) = 0
$$

Implications:

- For the relation to be symmetric, the above must hold for all $(x, y, z, t)$ satisfying the relation.
- The equation:

$$
(x - z)(1 - y) = 0
$$

must be always true whenever $(x, y) R (z, t)$.

- This only holds generally if:

$$
x = z \quad \text{or} \quad y = 1

\]

Conclusion:

- The relation \( R \) is not symmetric in general.
- It is symmetric only when the pairs satisfy the special conditions \( x = z \) or \( y = 1 \), which do not encompass the entire set.

Summary:
Relation \( R \) is not symmetric on \(\mathbb{R} \times \mathbb{R}\).

---

# 3. Is R Transitive?

Definition of Transitivity:
\( R \) is transitive if:

\[
\text{Whenever } (x, y) R (z, t) \text{ and } (z, t) R (p, q), \text{ then } (x, y) R (p, q)
\]

Using the assumed definition:

\[
(x, y) R (z, t) \iff x + zy + t = 0
\]

\[
(z, t) R (p, q) \iff z + tp + q = 0
\]

The question is: does this imply:

\[
(x, y) R (p, q) \iff x + y p + q = 0
\]

Analysis:

- From the first relation:

\[
x = - zy - t
\]

- From the second:

\[

$$
z = - tp - q
$$

- Substitute $z$ into the first:

$$
x = - (- tp - q) y - t = (tp + q) y - t
$$

- Now, for the relation to be transitive, the following must hold:

$$
x + y p + q = 0
$$

- Substitute $x$:

$$
[(tp + q) y - t] + y p + q = 0
$$

Simplify:

$$
(tp + q) y - t + y p + q = 0
$$

$$
t p y + q y - t + y p + q = 0
$$

Group similar terms:

$$
t p y + y p + q y + q - t = 0
$$

Factor where possible:

$$
p y (t + 1) + q (y + 1) - t = 0
$$

- For this to always hold when the initial relations are satisfied, the variables must satisfy certain conditions. Since these involve multiple variables, the relation is not generally transitive.

Conclusion:

- The algebraic dependence indicates \( R \

# Let R Be The Relation Defined On The Rxr By X Y R Z T X Zy T I R Is It Reflexive Ii R

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