

Let The Discrete Random Variables Y1 And Y2 Have The Joint Probability Function: $p(y_1, y_2) = 1/3$, For

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Understanding the behavior of discrete random variables is essential in probability theory and statistics. When analyzing the joint distribution of two discrete variables, Y1 and Y2, the joint probability function provides critical insights into how these variables interact and the likelihood of various outcomes. In particular, a joint probability function such as $p(y_1, y_2) = 1/3$, for specific (y_1, y_2) pairs, indicates uniform probability across the feasible outcomes. This article explores the properties, calculations, and implications of such a joint probability function, offering a comprehensive guide for students, researchers, and practitioners.

Introduction to Discrete Random Variables and Joint Probability Functions

Before delving into specific cases, it is vital to understand the fundamental concepts of discrete random variables and their joint probability functions.

What Are Discrete Random Variables?

- Discrete random variables are variables that take on a countable number of distinct values.
- Examples include the number of heads in coin flips, the number of cars passing through a toll, or the number of defective items in a batch.
- They are characterized by probability mass functions (pmfs), which assign probabilities to individual outcomes.

Joint Probability Functions

- The joint probability function, $p(y_1, y_2)$, describes the probability that Y1 takes on a specific value y_1 and Y2 takes on a specific value y_2 simultaneously.
- It must satisfy two key properties:
 1. Non-negativity: $p(y_1, y_2) \geq 0$ for all y_1, y_2 .
 2. Summation to 1: The sum over all possible pairs (y_1, y_2) of $p(y_1, y_2)$ equals 1.

Analyzing a Uniform Joint Probability Function: $p(y_1, y_2) = 1/3$

When the joint probability function is constant, such as $p(y_1, y_2) = 1/3$ for certain pairs, it indicates a uniform distribution over those pairs. This scenario simplifies many calculations and provides insights into the structure of the joint distribution.

Conditions for $p(y_1, y_2) = 1/3$

- The constant probability value, $1/3$, must be assigned across a set of possible outcome pairs (y_1, y_2) .
- The total sum of probabilities across all these pairs must equal 1, which constrains the number of pairs involved.

Determining the Support of Y_1 and Y_2

- The support refers to the set of all (y_1, y_2) pairs with non-zero probability.
- If $p(y_1, y_2) = 1/3$ for each pair, then the number of such pairs, N , must satisfy:
 - $N \times (1/3) = 1$
 - Which implies $N = 3$

Therefore, only three outcome pairs have non-zero probability, each equal to $1/3$, ensuring the total probability sums to 1.

Possible Outcome Pairs and Their Probabilities

Given the above, the joint distribution is uniform over three specific pairs. These pairs can vary depending on the context, but typically, they are explicitly defined.

Example Scenario

Suppose Y_1 and Y_2 are discrete variables representing outcomes of some experiment with three possible outcomes:

- Y_1 can take values $y_{1_1}, y_{1_2}, y_{1_3}$
- Y_2 can take values $y_{2_1}, y_{2_2}, y_{2_3}$

Given the joint probability function $p(y_1, y_2) = 1/3$ for three specific pairs, the pairs could be, for example:

- (y_{1_1}, y_{2_1})

- (y_{12}, y_{22})
- (y_{13}, y_{23})

Each of these pairs has a probability of $1/3$, and other pairs have zero probability.

Calculating Marginal and Conditional Probabilities

Understanding the joint distribution enables us to compute marginal and conditional probabilities, which are essential for analysis.

Marginal Probability Functions

- The marginal probability of Y_1 , $p(y_1)$, is obtained by summing over all possible y_2 :

$$p(y_1) = \sum p(y_1, y_2)$$

- Similarly, for Y_2 :

$$p(y_2) = \sum p(y_1, y_2)$$

- For the pairs with non-zero probability, the marginals are:

For each y_1 :

$$p(y_1) = \text{sum of probabilities where } Y_1 = y_1$$

For the pairs with $p(y_1, y_2) = 1/3$, the marginal probabilities depend on how many pairs include each y_1 and y_2 .

Calculations in the Example

- Given three pairs, suppose:
- (a, b) , (c, d) , (e, f)
- Marginal for Y_1 :

$$p(y_1=a) = 1/3$$

$$p(y_1=c) = 1/3$$

$$p(y_1=e) = 1/3$$

- Marginal for Y_2 :

$$p(y_2=b) = 1/3$$

$$p(y_2=d) = 1/3$$

$$p(y_2=f) = 1/3$$

- These values depend on the specific pairs, but the principle remains.

Independence and Dependence of Y1 and Y2

A key question in joint distributions is whether the variables are independent or dependent.

Testing for Independence

- Y1 and Y2 are independent if and only if:

$$p(y_1, y_2) = p(y_1) \times p(y_2) \text{ for all pairs.}$$

- With the uniform distribution over three pairs, the marginals are uniform, each summing to 1/3 over their respective values.

Implication for the Example

- If $p(y_1, y_2) = 1/3$ for each of the three pairs, and the marginals are uniform (each 1/3), then:

$$p(y_1) \times p(y_2) = (1/3) \times (1/3) = 1/9$$

- Since this does not equal 1/3, the variables are not independent in this case.

Applications and Practical Significance

Understanding such joint probability distributions has multiple practical applications across fields.

In Data Analysis and Statistics

- Modeling categorical data where outcomes are equally likely.
- Simplifying calculations in hypothesis testing involving joint distributions.

In Engineering and Computer Science

- Designing randomized algorithms where outcomes are uniformly distributed.
- Analyzing the behavior of systems with finite state spaces.

In Economics and Social Sciences

- Modeling scenarios where two discrete variables are linked with uniform joint probabilities.
- Analyzing decision-making processes with equal likelihood outcomes.

Conclusion: Key Takeaways

- A joint probability function $p(y_1, y_2) = 1/3$ for specific outcome pairs indicates a uniform distribution over those pairs.
- The total number of such pairs must be three to satisfy the total probability rule.
- Marginal probabilities can be derived by summing over the relevant variable's values.
- The independence of Y_1 and Y_2 depends on whether the joint probabilities factor into the product of marginals.
- Such distributions are powerful tools for modeling discrete phenomena with a finite set of outcomes and are widely applicable across various disciplines.

Summary of Important Concepts

- **Discrete Random Variables:** Variables taking countable values.
- **Joint Probability Function:** Defines the likelihood of simultaneous outcomes.
- **Uniform Distribution:** Equal probability assigned to certain outcome pairs.
- **Support:** The set of outcome pairs with non-zero probability.
- **Marginal Distributions:** Probabilities of individual variables, derived from the joint.
- **Independence:** When joint probability factors into the product of marginals.

Final Remarks

An understanding of joint probability functions like $p(y_1, y_2) = 1/3$ is fundamental in probability theory. Recognizing the structure of such distributions helps in modeling real-world scenarios, performing statistical inference, and designing algorithms. Whether in theoretical research or practical applications, mastering these concepts ensures a robust foundation in the analysis of discrete random variables.

Remember: Always verify the total probability sums to 1 and check the support set of the joint distribution to ensure accurate modeling and analysis.

Frequently Asked Questions

What is the joint probability function $p(y_1, y_2)$ given for the discrete random variables Y_1 and Y_2 ?

The joint probability function is $p(y_1, y_2) = 1/3$ for the specified values of y_1 and y_2 .

What are the possible values of Y_1 and Y_2 in the given joint probability distribution?

The problem statement does not specify the exact values, but typically, Y_1 and Y_2 are discrete variables taking values where $p(y_1, y_2) = 1/3$, indicating a finite set of possible pairs.

How do you verify that $p(y_1, y_2) = 1/3$ is a valid joint probability function?

To verify, sum $p(y_1, y_2)$ over all possible pairs (y_1, y_2) ; if the total sum equals 1, then $p(y_1, y_2)$ is valid. For example, if there are three pairs, $3(1/3) = 1$, confirming validity.

What is the marginal probability function of Y_1 based on the given joint distribution?

The marginal probability $p_{Y_1}(y_1)$ is obtained by summing $p(y_1, y_2)$ over all y_2 . If the distribution is uniform over three pairs, then each y_1 's marginal probability depends on how many y_2 values correspond to it.

How can we determine the independence of Y_1 and Y_2 from the joint probability function?

Y_1 and Y_2 are independent if $p(y_1, y_2) = p_{Y_1}(y_1) p_{Y_2}(y_2)$ for all pairs. Given the uniform joint distribution, check if the product of marginals equals the joint probability.

If $p(y_1, y_2) = 1/3$ for certain pairs, what does this imply about the total probability over all pairs?

It implies that the total probability over all possible pairs sums to 1. For example, if there are three such pairs, $3 (1/3) = 1$, satisfying the total probability axiom.

How do you compute the expected values $E[Y_1]$ and $E[Y_2]$ given the joint distribution?

Calculate $E[Y_1]$ by summing $y_1 p_{Y_1}(y_1)$, and similarly for $E[Y_2]$, using the marginal probabilities derived from the joint distribution.

What are the implications of the joint probability function being uniform ($p(y_1, y_2) = 1/3$)?

It suggests that all the possible pairs (y_1, y_2) are equally likely, indicating a uniform distribution over the set of pairs.

How can the joint distribution $p(y_1, y_2) = 1/3$ help in calculating covariance or correlation between Y_1 and Y_2 ?

Using the joint distribution, first find the marginal distributions, then compute $E[Y_1]$, $E[Y_2]$, $E[Y_1 Y_2]$, and finally apply covariance and correlation formulas to measure their dependence.

Additional Resources

Let the Discrete Random Variables Y_1 and Y_2 Have the Joint Probability Function: $p(y_1, y_2) = 1/3$, For

Introduction

In the realm of probability theory and statistical analysis, understanding the joint behavior of discrete random variables is fundamental. The joint probability function encapsulates the likelihood of different combinations of

outcomes for multiple variables, providing critical insight into their dependence structure and overall distribution. In this comprehensive investigation, we delve into the intriguing case where two discrete random variables, Y_1 and Y_2 , possess a joint probability function characterized by a constant value of $p(y_1, y_2) = 1/3$, for specific values of y_1 and y_2 .

This scenario, while seemingly straightforward, unravels complex questions surrounding the nature of independence, marginal distributions, and the broader implications for statistical modeling. Our exploration aims to clarify these aspects, offering a detailed analysis suitable for researchers, statisticians, and students interested in the nuances of joint distributions.

Defining the Scenario: The Joint Probability Function $p(y_1, y_2) = 1/3$

The statement "Let the discrete random variables Y_1 and Y_2 have the joint probability function $p(y_1, y_2) = 1/3$ " prompts immediate considerations:

- Domain Specification: The function is defined over a set of pairs (y_1, y_2) .
- Constant Value: The joint probability is uniform and equal to $1/3$ for the specified pairs.
- Implication of Uniformity: Such a uniform joint distribution suggests a particular structure of dependence and marginal distributions.

Before proceeding, it is crucial to clarify the precise domain over which $p(y_1, y_2) = 1/3$ holds. Typically, in discrete joint distributions, the function is non-zero over a finite set of pairs, summing up to 1 across all such pairs. The assumption that $p(y_1, y_2) = 1/3$ for each pair indicates that the total number of such pairs is three, thereby satisfying the probability axioms.

Fundamental Assumptions and Domain Specification

To analyze this distribution thoroughly, we need to define the domain explicitly:

- Suppose the set of possible values for Y_1 is $\{a, b, c\}$.
- Suppose the set of possible values for Y_2 is $\{x, y, z\}$.

Given that $p(y_1, y_2) = 1/3$ for each pair, then the total probability over all pairs must be 1:

$$\sum_{(y_1, y_2)} p(y_1, y_2) = 1$$

If there are 3 such pairs, each with probability $1/3$, the total sums up to 1:

$$\sum_{i=1}^3 \frac{1}{3} = 1$$

Hence, the joint distribution is uniform over these three pairs, and zero elsewhere.

Thus, the joint probability function can be summarized as:

y_1	y_2	$p(y_1, y_2)$
a	x	1/3
b	y	1/3
c	z	1/3

Alternatively, the specific pairs might vary, but the key idea remains: the joint distribution assigns probability 1/3 to each of three distinct pairs.

Key Properties of the Distribution

Marginal Distributions

From the joint probability function, we can derive the marginal distributions for Y_1 and Y_2 .

Marginal distribution of Y_1 :

$$p_{Y_1}(y_1) = \sum_{y_2} p(y_1, y_2)$$

Given the pairs:

- $p_{Y_1}(a) = p(a, x) = 1/3$
- $p_{Y_1}(b) = p(b, y) = 1/3$
- $p_{Y_1}(c) = p(c, z) = 1/3$

Similarly, for Y_2 :

$$p_{Y_2}(y_2) = \sum_{y_1} p(y_1, y_2)$$

- $p_{Y_2}(x) = p(a, x) = 1/3$
- $p_{Y_2}(y) = p(b, y) = 1/3$
- $p_{Y_2}(z) = p(c, z) = 1/3$

Observation: Both marginals are uniform across their respective domains.

Independence Analysis

Two variables are independent if:

$$p(y_1, y_2) = p_{\{Y_1\}}(y_1) \times p_{\{Y_2\}}(y_2)$$

Given the marginals are uniform:

$$p_{\{Y_1\}}(a) = p_{\{Y_1\}}(b) = p_{\{Y_1\}}(c) = 1/3$$

$$p_{\{Y_2\}}(x) = p_{\{Y_2\}}(y) = p_{\{Y_2\}}(z) = 1/3$$

Multiplying:

$$p_{\{Y_1\}}(a) \times p_{\{Y_2\}}(x) = 1/3 \times 1/3 = 1/9$$

But the joint probability for the pair (a, x) is 1/3, which is markedly different from 1/9. Therefore:

$$p(a, x) \neq p_{\{Y_1\}}(a) \times p_{\{Y_2\}}(x)$$

This indicates that Y_1 and Y_2 are not independent.

Investigating the Dependence Structure

Given the joint distribution assigns probability 1/3 to three specific pairs, and zero elsewhere, the variables are perfectly dependent in a particular way. This structure suggests a deterministic relationship restricted to these pairs.

Visualizing the Distribution

Imagine a 3x3 grid representing all possible pairs:

	x	y	z
a	1/3	0	0
b	0	1/3	0
c	0	0	1/3

This sparse matrix indicates that each value of Y_1 corresponds to exactly one value of Y_2 with a probability of $1/3$, and vice versa.

Implication: The distribution embodies a perfect matching structure, where each y_1 value is associated with exactly one y_2 value, and vice versa.

Conditional Distributions

- Conditional on $Y_1 = a$:

$$\begin{aligned} & \backslash [\\ p(Y_2 = x \mid Y_1 = a) &= \frac{p(a, x)}{p_{Y_1}(a)} = \frac{1/3}{1/3} = 1 \\ & \backslash] \end{aligned}$$

- Conditional on $Y_1 \neq a$:

$$\begin{aligned} & \backslash [\\ p(Y_2 \neq x \mid Y_1 = a) &= 0 \\ & \backslash] \end{aligned}$$

Similarly, for other pairs, the conditional distributions are deterministic.

This indicates a functional dependence where Y_2 is a deterministic function of Y_1 in the support set.

Broader Implications and Applications

Modeling Dependence

This distribution exemplifies a case where the joint probability is concentrated on a small subset of outcomes, illustrating perfect dependence within a constrained set. Such structures are useful in modeling scenarios where outcomes are tightly coupled, such as in:

- Matching problems: Assignments where each element in one set is paired with exactly one element in another.
- Conditional deterministic relationships: Situations where knowing one variable completely determines the other.

Limitations and Extensions

While this specific distribution demonstrates interesting dependence characteristics, it is limited to a small support. Extending this to larger, more complex distributions involves considering probabilistic mixtures and relaxing the perfect matching condition.

Significance in Statistical Theory

Understanding distributions like the one described enhances comprehension of:

- Dependence and independence: Recognizing when variables are not independent, especially in the presence of deterministic relationships.
- Marginal and joint distributions: Clarifying how uniform joint distributions influence marginals.
- Conditional probabilities: Demonstrating cases with degenerate or deterministic conditional distributions.

These insights are foundational for advanced topics such as Bayesian networks, Markov processes, and multivariate analysis.

Conclusion

The exploration of discrete random variables Y_1 and Y_2 with a joint probability function $p(y_1, y_2) = 1/3$ over a set of three pairs reveals a nuanced dependence structure. Despite uniform marginals, the variables are not independent, exhibiting a perfect matching relationship that underscores the importance of analyzing joint distributions beyond surface-level assumptions.

This case serves as a pedagogical example illustrating key concepts in probability theory, including marginalization, dependence, and the interpretation of joint distributions. Recognizing such structures aids in designing accurate statistical models and understanding the underlying relationships in data.

In sum, the distribution $p(y_1, y_2) = 1/3$, for specified pairs, exemplifies the rich tapestry of relationships possible within the framework of discrete probability, emphasizing the need for careful analysis when modeling dependent variables.

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