Let W Be The Subspace Of R 4 Spanned By Vectors W1 = (2, 1, 3, 3), W2 = (1, 2, 3, 0) And W3 = (3, 1,

Let W Be The Subspace Of R^4 Spanned By Vectors W1 = (2, 1, 3, 3), W2 = (1, 2, 3, 0), And W3 = (3, 1,

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Understanding the Subspace W in R^4

When working with vector spaces in linear algebra, one fundamental concept is the idea of a subspace. In this context, a subspace is a subset of a vector space that itself satisfies the properties of a vector space. Specifically, it must be closed under vector addition and scalar multiplication, and contain the zero vector.

In this article, we focus on the subspace W of $\mathbb{R}^4$ that is spanned by three vectors:

$$
W_1 = (2, 1, 3, 3), \quad W_2 = (1, 2, 3, 0), \quad W_3 = (3, 1, \ldots)
$$

(It's important to note that the vector W3 appears incomplete in the initial statement. For the purpose of this discussion, we will assume W3 is a complete vector, such as $W_3 = (3, 1, 2, 4)$. If the original vector W3 is different, please specify. For now, we proceed with a complete W3 to illustrate the process.)

Defining the Subspace W

The subspace W is defined as:

$$
W = \operatorname{span}\{W_1, W_2, W_3\}
$$

which means W consists of all linear combinations of W1, W2, and W3:

$$
W = \left\{ \alpha W_1 + \beta W_2 + \gamma W_3 \mid \alpha, \beta, \gamma \in \mathbb{R} \right\}
$$

Understanding the structure of W involves answering critical questions:

- What is the dimension of W?
- Are the vectors W_1, W_2, W_3 linearly independent?
- What is a basis for W?
- How can we represent W explicitly?

Step 1: Analyzing the Vectors

Let's assume the vectors are:

$$
W_1 = (2, 1, 3, 3)
$$
$$
W_2 = (1, 2, 3, 0)
$$
$$
W_3 = (3, 1, 2, 4)
$$

(Adjust as needed if the actual W3 differs.)

Step 2: Checking for Linear Independence

To determine the dimension of W, we need to verify whether these vectors are linearly independent. The standard method is to set up a matrix with these vectors as rows or columns and perform row operations to find its rank.

Construct the matrix:

$$
M = \begin{bmatrix}
2 & 1 & 3 & 3 \\
1 & 2 & 3 & 0 \\
3 & 1 & 2 & 4
\end{bmatrix}
$$

Perform row operations:

1. Swap R1 and R2 for simplicity:

\[
\begin{bmatrix}
1 & 2 & 3 & 0 \\
2 & 1 & 3 & 3 \\
3 & 1 & 2 & 4
\end{bmatrix}
\]

2. Eliminate the first element in R2 and R3:

- R2 -> R2 - 2R1:

\[
R2: (2 - 21, 1 - 22, 3 - 23, 3 - 20) = (0, -3, -3, 3)
\]

- R3 -> R3 - 3R1:

\[
R3: (3 - 31, 1 - 32, 2 - 33, 4 - 30) = (0, -5, -7, 4)
\]

Updated matrix:

\[
\begin{bmatrix}
1 & 2 & 3 & 0 \\
0 & -3 & -3 & 3 \\
0 & -5 & -7 & 4
\end{bmatrix}
\]

3. Make the leading coefficient in R2 positive:

- R2 -> -1/3 R2:

\[
R2: (0, 1, 1, -1)
\]

4. Eliminate the second variable in R3:

- R3 -> R3 - (-5)R2 = R3 + 5R2:

\[
R3: (0, -5 + 51, -7 + 51, 4 + (-5)(-1)) = (0, 0, -2, 9)
\]

Now, the matrix is:

\[

\begin{bmatrix}
1 & 2 & 3 & 0 \\
0 & 1 & 1 & -1 \\
0 & 0 & -2 & 9
\end{bmatrix}
\]

5. Back substitution to find pivots:

- R3 -> R3 / -2:

\[
R3: (0, 0, 1, -4.5)
\]

- Eliminate the third variable in R2:

R2 -> R2 - R3:

\[
R2: (0, 1 - 1, 1 - (-4.5), -1 - (-4.5)) = (0, 0, 0, 3.5)
\]

- Eliminate the third variable in R1:

R1 -> R1 - 3R3:

\[
R1: (1, 2 - 30, 3 - 31, 0 - 3(-4.5)) = (1, 2, 0, 13.5)
\]

- Eliminate the second variable in R1:

R1 -> R1 - 2R2:

\[
R1: (1, 2 - 20, 0 - 20, 13.5 - 23.5) = (1, 2, 0, 13.5 - 7) = (1, 2, 0, 6.5)
\]

The row echelon form indicates three pivots, so the vectors are linearly independent, and the dimension of W is 3.

Step 3: Basis for W

Since all three vectors are linearly independent, the set \(\{W_1, W_2, W_3\}\) itself forms a basis for W:

$$
\text{Basis of } W = \{ (2, 1, 3, 3), \ (1, 2, 3, 0), \ (3, 1, 2, 4) \}
$$

This basis allows us to describe W explicitly, and any vector in W can be written as a linear combination of these basis vectors.

Step 4: Expressing W as a Span

Any vector $v \in W$ can be written as:

$$
v = a W_1 + b W_2 + c W_3
$$

where $a, b, c \in \mathbb{R}$.

Explicitly:

$$
v = a (2, 1, 3, 3) + b (1, 2, 3, 0) + c (3, 1, 2, 4)
$$

which expands to:

$$
v = (2a + b + 3c, \ a + 2b + c, \ 3a + 3b + 2c, \ 3a + 0b + 4c)
$$

Component-wise:

$$
v = (v_1, v_2, v_3, v_4)
$$

with the relations:

$$
v_1 = 2a + b + 3c
$$
$$
v_2 = a + 2b + c
$$
$$
v_3 = 3a + 3b + 2c
$$
\[

v_4 = 3a + 4c
\]

Step 5: Finding the Equation of W

To describe W explicitly, it’s often useful to find the equations that characterize the subspace. Since W is 3-dimensional in \(\mathbb{R}^4\), it can be represented as the solution set to one linear equation (or more).

Using the relations, we can eliminate parameters to find these equations.

From the above:

- From \(v_4 = 3a + 4c\), express \(a\):

\[
a = \frac{v_4 - 4c}{3}
\]

- From \(v_2 = a + 2b + c\), substitute \(a\):

\[
v_2 = \

Frequently Asked Questions

How do you find the dimension of the subspace W spanned by vectors W1, W2, and W3 in R^4?

To find the dimension of W, form a matrix with W1, W2, and W3 as rows or columns and perform row reduction to determine the number of linearly independent vectors. The number of pivot positions after row reduction gives the dimension of W.

How can we determine if vectors W1, W2, and W3 are linearly independent in R^4?

Arrange the vectors into a matrix and perform row operations to find its rank. If the rank equals the number of vectors (3), then they are linearly independent; otherwise, they are dependent.

What is the process to find a basis for the subspace W spanned

by W1, W2, and W3?

Construct a matrix with the vectors as rows or columns, perform row reduction to identify the linearly independent vectors, and those vectors form a basis for W.

How do you compute the dimension of the subspace W in R^4 spanned by the given vectors?

By checking the rank of the matrix formed by W1, W2, and W3 after row reduction, the resulting number of leading pivots indicates the dimension of W.

If W3 is incomplete or contains a typo, how can we correctly analyze the span of the vectors W1, W2, and W3?

First, clarify or obtain the complete vector W3. Once all three vectors are properly defined, proceed with forming a matrix and performing row operations to analyze linear independence and span.

Additional Resources

Let W be the subspace of $\mathbb{R}^4$ spanned by vectors $W_1 = (2, 1, 3, 3)$, $W_2 = (1, 2, 3, 0)$, and $W_3 = (3, 1, \ldots)$

Understanding the structure and properties of subspaces in $\mathbb{R}^4$ is fundamental in linear algebra, especially when analyzing how specific vectors span these subspaces. In this guide, we will thoroughly explore the subspace W of $\mathbb{R}^4$ generated by the vectors W_1, W_2, and W_3, examining their linear independence, the dimension of W, and the basis that constitutes W. This comprehensive breakdown aims to clarify the process of determining the span of given vectors and the underlying principles involved.

Introduction to Subspaces and Spanning Sets

Before delving into the specifics of the vectors W_1, W_2, and W_3, it's essential to understand some foundational concepts:

- Subspace: A subset of a vector space that is itself a vector space under the same operations of addition and scalar multiplication.
- Span: The set of all linear combinations of a given set of vectors.
- Basis: A set of linearly independent vectors that span a subspace.
- Dimension: The number of vectors in a basis for the subspace.

In our case, the subspace W is defined as:

\[W = \text{span}\{ W_1, W_2, W_3 \} \]

Our goal is to determine the structure of W, including its dimension and a basis for it.

Step 1: Understanding the Given Vectors

Let's revisit the vectors provided:

- $W_1 = (2, 1, 3, 3)$
- $W_2 = (1, 2, 3, 0)$
- $W_3 = (3, 1, \ldots)$ — Note: the third component is incomplete, which is critical for the analysis.

Note: Since the third component of W_3 is incomplete, for a comprehensive analysis, we'll assume it is specified as $W_3 = (3, 1, c, d)$, where c and d are known real numbers. If these are not provided, the analysis cannot be completed precisely. For illustration, let's assume:

$$W_3 = (3, 1, 4, 2)$$

This hypothetical choice allows us to demonstrate the process. If actual values differ, adjust accordingly.

Step 2: Formulating the Problem

Our main task is to determine:

- Are the vectors W_1, W_2, W_3 linearly independent?
- What is the dimension of the subspace W?
- Find a basis for W.

To answer these, we'll analyze the vectors' linear relationships, typically by setting up a matrix and performing row operations.

Step 3: Constructing the Matrix and Row Reduction

Create a matrix A with the vectors as rows:

$$
A = \begin{bmatrix}
2 & 1 & 3 & 3 \\
1 & 2 & 3 & 0 \\
3 & 1 & 4 & 2
\end{bmatrix}
$$

Our goal is to row reduce A to echelon form to identify the rank, which corresponds to the dimension of the span.

Step 3.1: Row Operations

Step 1: Use R_1 as the pivot row.

- Make zeros below the pivot in the first column:

$$
R_2 \leftarrow R_2 - \frac{1}{2} R_1
$$
$$
R_3 \leftarrow R_3 - \frac{3}{2} R_1
$$

Calculations:

- $R_2: (1, 2, 3, 0) - 0.5 \times (2, 1, 3, 3) = (1 - 1, 2 - 0.5, 3 - 1.5, 0 - 1.5) = (0, 1.5, 1.5, -1.5)$

- $R_3: (3, 1, 4, 2) - 1.5 \times (2, 1, 3, 3) = (3 - 3, 1 - 1.5, 4 - 4.5, 2 - 4.5) = (0, -0.5, -0.5, -2.5)$

Updated matrix:

$$
\begin{bmatrix}
2 & 1 & 3 & 3 \\
0 & 1.5 & 1.5 & -1.5 \\
0 & -0.5 & -0.5 & -2.5
\end{bmatrix}
$$

Step 2: Normalize R_2:

$$
R_2 \leftarrow \frac{2}{3} R_2
$$

$$
\left(0, 1.5, 1.5, -1.5\right) \times \frac{2}{3} = (0, 1, 1, -1)
$$

Updated matrix:

$$
\begin{bmatrix}
2 & 1 & 3 & 3 \\
0 & 1 & 1 & -1 \\
0 & -0.5 & -0.5 & -2.5
\end{bmatrix}
$$

Step 3: Use R_2 to eliminate the second component in R_3:

\[
R_3 \leftarrow R_3 + 0.5 R_2
\]

Calculations:

\[
(0, -0.5, -0.5, -2.5) + 0.5 \times (0, 1, 1, -1) = (0, -0.5 + 0.5, -0.5 + 0.5, -2.5 - 0.5) = (0, 0, 0, -3)
\]

Updated matrix:

\[
\begin{bmatrix}
2 & 1 & 3 & 3 \\
0 & 1 & 1 & -1 \\
0 & 0 & 0 & -3
\end{bmatrix}
\]

Step 4: Normalize \(R_3 \):

\[
R_3 \leftarrow -\frac{1}{3} R_3 = (0, 0, 0, 1)
\]

Step 5: Back substitution to eliminate above:

- Eliminate the last component in \(R_2 \):

\[
R_2 \leftarrow R_2 + R_3
\]

\[
(0, 1, 1, -1) + (0, 0, 0, 1) = (0, 1, 1, 0)
\]

- Eliminate the last component in \(R_1 \):

\[
R_1 \leftarrow R_1 - 3 R_3
\]

\[
(2, 1, 3, 3) - 3 \times (0, 0, 0, 1) = (2, 1, 3, 0)
\]

- Now, eliminate the third component in \(R_1 \):

\[

$$
R_1 \leftarrow R_1 - R_2
$$

$$
(2, 1, 3, 0) - (0, 1, 1, 0) = (2, 0, 2, 0)
$$

- Normalize R_1:

$$
R_1 \leftarrow \frac{1}{2} R_1 = (1, 0, 1, 0)
$$

Step 4: Interpreting the Row-Reduced Matrix

The row echelon form is:

$$
\begin{bmatrix}
1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

This form reveals:

- Rank = 3 (since there are three non-zero rows).
- The vectors are linearly independent in the span of the original vectors.
- The subspace W has dimension 3.

Step 5: Identifying a Basis for W

From the row-reduced form, the basis vectors correspond to the original vectors associated with the leading ones. Extracting these, the basis for W consists of:

$$
\boxed{
\begin{cases}
V_1 = (2, 1, 3, 3) \\
V_2 = (1, 2, 3, 0) \\
V_3 = (3, 1, 4, 2)
\end{cases}
}
$$

Alternatively, these vectors can be expressed as the original spanning set, confirming the basis.

Step 6: Expressing

Let W Be The Subspace Of R 4 Spanned By Vectors W1 2 1 3 3 W2 1 2 3 0 And W3 3 1

Let W Be The Subspace Of R 4 Spanned By Vectors W1 2 1 3 3 W2 1 2 3 0 And W3 3 1

Related Articles

- Let B = {b1, B2} And C = {c1, C2} Be Bases For R2. Find The Change-of-coordinates Matrix From B To C
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