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Let Z= F (x, Y) = 3 X + 6x Y –5 Y . Define Az = F(x+dx, Y +dy) F(x, Y) And Dz= F'(x, Y)dx + F'(x, Y

Introduction to Multivariable Calculus and Function Analysis

Understanding the behavior of multivariable functions is essential in various fields such as physics, engineering, economics, and data science. These functions depend on multiple variables, and analyzing their changes requires specific tools like differentials, partial derivatives, and total derivatives. In this context, we focus on the function:

$$Z = F(x, Y) = 3x + 6xy - 5Y$$

and explore how small changes in x and Y influence the value of Z. We will define the incremental change A_z and the differential D_z, examining their significance, computation, and applications.

Understanding the Function $Z = F(x, Y)$

Function Composition and Variables

The function $Z = F(x, Y)$ is composed of:

- A linear term in x: $3x$
- A bilinear term involving both x and Y: $6xy$
- A linear term in Y: $-5Y$

This function models a surface in three-dimensional space, where each point (x, Y) maps to a value Z.

Graphical Interpretation

Visualizing Z helps in understanding how it varies with x and Y:

- The surface is a plane with a tilt depending on the coefficients.
- The term $6xy$ introduces curvature, making the surface nonlinear in terms of x and Y.
- Changes in x and Y can drastically alter Z.

Defining Variations: A_z and D_z

Incremental Change A_z

The quantity A_z captures the change in Z when both x and Y are incremented by small amounts dx and dy, respectively:

$$
A_z = F(x + dx, Y + dy) - F(x, Y)
$$

This represents the actual change in the function's value as the variables change.

Total Differential D_z

The differential D_z provides an approximation to A_z, especially useful when dx and dy are very small:

$$
D_z = F_x(x, Y) \, dx + F_Y(x, Y) \, dy
$$

where F_x and F_Y are the partial derivatives of F with respect to x and Y, respectively.

Calculating Partial Derivatives for $F(x, Y)$

Partial Derivative with Respect to x: F_x

Differentiate $F(x, Y) = 3x + 6xy - 5Y$ with respect to x, treating Y as constant:

$$
F_x = \frac{\partial}{\partial x} (3x + 6xy - 5Y) = 3 + 6Y
$$

- The derivative of $3x$ is 3.
- The derivative of $6xy$ with respect to x is $6Y$.
- The term $-5Y$ is constant with respect to x, so its derivative is 0.

Partial Derivative with Respect to Y: F_Y

Differentiate $F(x, Y)$ with respect to Y, treating x as constant:

$$
F_Y = \frac{\partial}{\partial Y} (3x + 6xy - 5Y) = 6x - 5
$$

- The terms $3x$ and $6xy$ with respect to Y are 0 and $6x$, respectively.
- The derivative of $-5Y$ is -5.

Expressing A_z and D_z in Terms of dx and dy

Increment A_z Calculation

Expanding $F(x + dx, Y + dy)$:

$$
F(x + dx, Y + dy) = 3(x + dx) + 6(x + dx)(Y + dy) - 5(Y + dy)
$$

Breaking it down:

$$
= 3x + 3dx + 6(xY + xdy + ydx + dx \, dy) - 5Y - 5dy
$$

Simplify:

$$
= 3x + 6xY + 3dx + 6xdy + 6ydx + 6dx \, dy - 5Y - 5dy
$$

Subtract $F(x, Y) = 3x + 6xy - 5Y$:

$$
A_z = (3x + 6xY + 3dx + 6xdy + 6ydx + 6dx \, dy - 5Y - 5dy) - (3x + 6xy - 5Y)
$$

Simplify:

$$
A_z = 3dx + 6xdy + 6ydx + 6dx \, dy - 5dy
$$

Since dx and dy are small, the term $6dx \, dy$ is of second order and can often be neglected for approximation purposes:

$$
A_z \approx 3dx + 6xdy + 6ydx - 5dy
$$

Total Differential D_z

Using the partial derivatives:

$$
D_z = (3 + 6Y)dx + (6x - 5)dy
$$

This provides a linear approximation:

$$
A_z \approx D_z
$$

valid when dx and dy are small.

Practical Applications of A_z and D_z

Approximate Change in Function Values

- A_z gives the actual change in Z for finite dx and dy.
- D_z provides a quick estimate, especially valuable in optimization and sensitivity analysis.

Optimization and Sensitivity Analysis

- Understanding how small changes affect Z is crucial in optimizing functions.
- Engineers and scientists use differentials to analyze the impact of variables and to make informed decisions.

Error Estimation in Numerical Methods

- When approximating functions, D_z helps estimate errors.
- It guides in choosing appropriate step sizes dx and dy to balance accuracy and computational effort.

Conclusion and Summary

Understanding the function $Z = 3x + 6xy - 5Y$ and the concepts of A_z and D_z is fundamental in multivariable calculus. The incremental change A_z captures the actual variation in Z when variables change, while the total differential D_z offers a linear approximation, simplifying complex calculations. By calculating partial derivatives F_x and F_Y, one can efficiently estimate how Z responds to small changes in x and Y, enabling applications across various scientific and engineering disciplines.

This comprehensive analysis emphasizes the importance of differentials in understanding the nuances of multivariable functions and provides essential tools for practical problem-solving in advanced mathematics and applied sciences.

Frequently Asked Questions

What is the function Z defined as in terms of x and Y?

Z is defined as $Z = F(x, Y) = 3X + 6XY - 5Y$.

How do you express the change Az in terms of F and small increments dx and dy?

Az is defined as $Az = F(x+dx, Y+dy) - F(x, Y)$, representing the total change in F when x and Y are incremented by dx and dy respectively.

What is the meaning of Dz in terms of derivatives of F?

$Dz = F'_x(x, Y) dx + F'_Y(x, Y) dy$, which approximates the change in F using its partial derivatives at (x, Y) multiplied by the small changes dx and dy.

How do you compute the partial derivatives F'_x and F'_Y of Z?

Given $Z = 3x + 6xy - 5Y$, the partial derivatives are $F'_x = 3 + 6Y$ and $F'_Y = 6x - 5$.

What is the practical significance of calculating Az and Dz?

Calculating Az and Dz helps approximate the actual change in the function Z when the variables x and Y undergo small changes, which is useful in differential calculus and sensitivity analysis.

How can the approximation Dz be used to estimate Az?

Since Dz approximates the change in Z for small dx and dy, it can be used as an estimate for Az, especially when dx and dy are very small, by assuming $Az \approx Dz$.

Additional Resources

Understanding the Differential of a Function: A Comprehensive Guide to Let $Z = F(x, Y) = 3X + 6XY - 5Y$

In the realm of multivariable calculus, the concept of differentials plays a crucial role in understanding how functions change with respect to their variables. When analyzing a function such as $Z = F(x, Y) = 3X + 6XY - 5Y$, it’s essential to grasp the notions of Az and Dz—the differential approximations that capture infinitesimal changes in the function's value as the variables x and Y vary slightly. This guide aims to unpack these concepts step-by-step, providing a clear understanding of how to compute and interpret Az and Dz for the given function.

Introduction to the Function and Its Context

The Function Z = F(x, Y)

Let us consider the function:

$$ Z = F(x, Y) = 3x + 6xy - 5Y $$

- Variables involved: x and Y
- Nature of the function: It combines linear and bilinear terms, making it a typical example to study differentials in multivariable calculus.

Why Are Differentials Important?

Differentials help us understand the behavior of functions when their input variables change slightly. They are fundamental in:

- Approximating the change in the function's value (linear approximation).
- Analyzing sensitivities and rates of change.
- Facilitating the computation of derivatives in multivariable contexts.

Breaking Down the Differential Concepts

1. The Incremental Change: A_z

Definition:
$$ A_z = F(x + dx, Y + dy) - F(x, Y) $$

This expression measures the actual change in the function F when the variables x and Y are incremented by small amounts dx and dy.

2. The Differential Approximation: D_z

Definition:
$$ D_z = F_x(x, Y) \, dx + F_Y(x, Y) \, dy $$

Here, F_x and F_Y represent the partial derivatives of F with respect to x and Y, respectively. The differential D_z provides a linear approximation to A_z, valid for infinitesimally small dx and dy.

Step-by-Step Computation of the Differentials

Step 1: Find Partial Derivatives F_x and F_Y

Given:

$$ F(x, Y) = 3x + 6xy - 5Y $$

- Partial derivative with respect to x:

$$
F_x = \frac{\partial}{\partial x} (3x + 6xy - 5Y) = 3 + 6Y
$$

- Partial derivative with respect to Y:

$$
F_Y = \frac{\partial}{\partial Y} (3x + 6xy - 5Y) = 6x - 5
$$

Step 2: Express A_z

Calculate the actual change A_z:

$$
A_z = F(x + dx, Y + dy) - F(x, Y)
$$

Substitute into F:

$$
F(x + dx, Y + dy) = 3(x + dx) + 6(x + dx)(Y + dy) - 5(Y + dy)
$$

Expand each term:

$$
= 3x + 3 dx + 6(x Y + x dy + dx Y + dx dy) - 5Y - 5 dy
$$

Distribute:

$$
= 3x + 3 dx + 6 x Y + 6 x dy + 6 dx Y + 6 dx dy - 5Y - 5 dy
$$

Subtract the original $F(x, Y)$:

$$
A_z = [3x + 3 dx + 6 x Y + 6 x dy + 6 dx Y + 6 dx dy - 5Y - 5 dy] - [3x + 6 x Y - 5Y]
$$

Cancel common terms:

$$
A_z = 3 dx + 6 x dy + 6 dx Y + 6 dx dy - 5 dy
$$

Since dx, dy are infinitesimally small, terms involving $dx dy$ are of second order and typically neglected in the linear approximation.

Step 3: Express D_z

Using the derivatives:

$$
D_z = F_x(x, Y) \, dx + F_Y(x, Y) \, dy = (3 + 6Y) dx + (6x - 5) dy
$$

This is the linear approximation of A_z, valid when dx and dy are very small.

Interpreting the Results

The Actual Change A_z

$$
A_z \approx 3 dx + 6 x dy + 6 dx Y - 5 dy
$$

- Contains first-order terms (dx, dy) and second-order terms (like $dx dy$), which are often ignored in first-order approximations.
- Represents the exact change in the function for finite increments.

The Differential D_z

$$
D_z = (3 + 6Y) dx + (6x - 5) dy
$$

- Provides a linear approximation to A_z, valid for very small dx and dy.
- Easier to compute and interpret, especially when analyzing sensitivities.

Practical Application: Approximating Changes in the Function

Suppose you have specific small changes:

- $dx = 0.01$
- $dy = 0.02$
- Values at the point: $x = 2$, $Y = 3$

Compute D_z:

$$
F_x(2, 3) = 3 + 6 \times 3 = 3 + 18 = 21
$$
$$
F_Y(2, 3) = 6 \times 2 - 5 = 12 - 5 = 7
$$

$$
D_z = 21 \times 0.01 + 7 \times 0.02 = 0.21 + 0.14 = 0.35
$$

Approximate A_z:

Calculate the actual change:

\[

$$
A_z \approx 3 \times 0.01 + 6 \times 2 \times 0.02 + 6 \times 0.01 \times 3 - 5 \times 0.02
$$

$$
= 0.03 + 0.24 + 0.18 - 0.10 = 0.35
$$

The linear approximation D_z provides an accurate estimate for the actual change A_z, illustrating the utility of differentials.

Summary and Key Takeaways

- The function $Z = F(x, Y) = 3x + 6xy - 5Y$ involves both linear and bilinear terms, serving as a representative example for differential analysis.
- Incremental change A_z reflects the actual change in the function when variables change by finite amounts.
- Differential D_z offers a linear, first-order approximation, computed via the partial derivatives:

$$
D_z = F_x(x, Y) \, dx + F_Y(x, Y) \, dy
$$

- Calculating derivatives:

$$
F_x = 3 + 6Y \quad \text{and} \quad F_Y = 6x - 5
$$

- In practical scenarios, for small dx and dy, $A_z \approx D_z$, enabling quick estimations of how the function responds to variable changes.

Final Remarks

Understanding the concepts of Az and Dz is fundamental in multivariable calculus, optimization, and modeling real-world phenomena where multiple variables influence a quantity. By mastering the computation and interpretation of differentials, you gain insight into the sensitivity and behavior of complex functions, paving the way for advanced analysis and problem-solving in mathematics, engineering, and economics.

Let Z F X Y 3 X 6x Y 5 Y Define Az F X Dx Y Dy F X Y And Dz F X Y Dx F X Y

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