Line L Passes Through Point (2,37) And Line P Is The Graph Of Y+3=5(x+4) .If L Is Parallel To P What

Line L Passes Through Point (2,37) And Line P Is The Graph Of Y+3=5(x+4). If L Is Parallel To P What

Understanding the properties of lines in coordinate geometry is fundamental in solving various geometric problems. In this article, we will explore a particular scenario involving two lines: Line L, which passes through a specific point, and Line P, defined by an equation. Our goal is to analyze the conditions under which Line L is parallel to Line P, and to determine the equation of Line L accordingly.

Understanding the Given Lines and Their Equations

Before delving into the problem, it is essential to comprehend the given information about the lines involved.

Line P: The Equation of the Line

The equation provided for Line P is:

$$y + 3 = 5(x + 4)$$

This is in point-slope form, which makes it straightforward to identify its slope and intercepts.

Converting to slope-intercept form:

$$
\begin{aligned}
y + 3 &= 5(x + 4) \\
y + 3 &= 5x + 20 \\
y &= 5x + 20 - 3 \\
y &= 5x + 17
\end{aligned}
$$

Key features of Line P:

- Slope (m_P): 5
- Y-intercept: 17 (the point where the line crosses the y-axis)

Understanding the slope is crucial because it determines the line's orientation and whether another line is parallel or perpendicular to it.

Line L: Passing Through a Specific Point

Line L passes through the point:

$$ (2, 37) $$

However, the equation of Line L is not directly given. To analyze Line L, especially in relation to Line P, we need to find the possible equations of Line L that satisfy the given conditions.

Conditions for Parallel Lines in Coordinate Geometry

The key property of parallel lines is that they have the same slope but different intercepts.

In brief:

- Two lines are parallel if and only if their slopes are equal.
- If lines are parallel, their equations share the same slope but differ in their y-intercepts.

Therefore:

$$
\text{If Line L is parallel to Line P, then } m_L = m_P = 5
$$

where m_L is the slope of Line L.

Finding the Equation of Line L

Given that Line L passes through the point (2, 37) and is parallel to Line P, which has a slope of 5, we can write the equation of Line L using point-slope form.

Using Point-Slope Form:

$$
y - y_1 = m(x - x_1)
$$

Plugging in the point (2, 37):

$$
y - 37 = 5(x - 2)
$$

Simplify:

$$
y - 37 = 5x - 10
$$

$$
y = 5x - 10 + 37
$$

$$
y = 5x + 27
$$

Result:

$$
\boxed{
\text{Equation of Line L: } y = 5x + 27
}
$$

This line passes through (2, 37) and is parallel to Line P.

Analyzing the Relationship Between Lines L and P

Having established the equations:

- Line P: $y = 5x + 17$
- Line L: $y = 5x + 27$

We observe:

- Both lines have the same slope (5), confirming they are parallel.
- The y-intercept of Line L (27) differs from that of Line P (17), which confirms they are distinct lines and do not coincide.

Implications:

- The lines are parallel in the plane.
- The perpendicular distance between the lines can be calculated if needed.

- They do not intersect, which is characteristic of parallel lines.

Additional Considerations and Applications

Understanding the relationship between these lines can be useful in various real-world scenarios, such as:

1. Design and Engineering: Ensuring components are aligned parallel to each other.
2. Navigation and Mapping: Determining parallel routes or pathways.
3. Physics: Analyzing parallel trajectories or force lines.

Calculating the Distance Between Parallel Lines:

If needed, the perpendicular distance between the two lines $y = 5x + 17$ and $y = 5x + 27$ can be calculated using the formula:

$$
d = \frac{|c_2 - c_1|}{\sqrt{1 + m^2}}
$$

where c_1 and c_2 are the y-intercepts, and m is the slope.

Applying values:

$$
d = \frac{|27 - 17|}{\sqrt{1 + 5^2}} = \frac{10}{\sqrt{1 + 25}} = \frac{10}{\sqrt{26}} \approx \frac{10}{5.10} \approx 1.96
$$

This indicates that the lines are approximately 1.96 units apart in the coordinate plane.

Summary and Key Takeaways

- The line $y + 3 = 5(x + 4)$ simplifies to $y = 5x + 17$, with a slope of 5.
- Line L, passing through (2, 37), and parallel to Line P, has an equation $y = 5x + 27$.
- Parallel lines share the same slope but have different y-intercepts.
- The distance between the two parallel lines can be calculated and is approximately 1.96 units.

Practical applications of this analysis include designing parallel pathways, analyzing geometric constraints, and solving related algebraic problems.

Conclusion

In conclusion, when a line passes through a given point and is parallel to another line, its equation can be systematically determined by maintaining the same slope. For the specific problem where Line L passes through (2,37) and is parallel to Line P, which is given by $y + 3 = 5(x + 4)$, the equation of Line L is:

$$
\boxed{
y = 5x + 27
}
$$

This comprehensive analysis not only provides the equation of the required line but also emphasizes the fundamental concepts of slope, parallelism, and line equations in coordinate geometry. Mastery of these principles is essential for solving diverse mathematical and real-world problems involving lines and their relationships.

Frequently Asked Questions

What is the slope of line P given the equation y + 3 = 5(x + 4)?

The slope of line P is 5.

How do you determine the equation of line L passing through point (2, 37) and parallel to line P?

Since line L is parallel to P, it has the same slope (5). Using point-slope form with point (2, 37), the equation is $y - 37 = 5(x - 2)$.

What is the equation of line L that passes through (2, 37) and is parallel to line P?

The equation of line L is $y = 5x + 27$.

Does the point (2, 37) lie on line L? How can we verify?

Yes, substituting $x=2$ into $y=5x+27$ gives $y=52+27=37$, confirming the point lies on line L.

What is the significance of lines being parallel in this context?

Parallel lines have the same slope but different y-intercepts, meaning they never intersect and maintain a constant distance apart.

If line P's equation is y = 5x - 17, what is the equation of line L passing through (2, 37) and parallel to P?

Line L will have the same slope 5, so its equation is y = 5x + 27, passing through (2, 37).

Additional Resources

Line L Passes Through Point (2,37) And Line P Is The Graph Of Y+3=5(x+4). If L Is Parallel To P What

Understanding the relationships between lines in coordinate geometry is fundamental to mastering algebraic concepts and their geometric interpretations. When analyzing problems involving lines passing through specific points and being parallel to given lines, it’s crucial to comprehend how slopes, point-slope forms, and equations interact. In this guide, we will explore the problem: Line L passes through the point (2, 37), and line P is given by the equation Y + 3 = 5(x + 4). If line L is parallel to line P, what is the equation of line L?

This problem exemplifies the importance of understanding slope-intercept forms, point-slope forms, and the properties of parallel lines in the coordinate plane. Let’s delve into the details, step by step, and develop a clear, comprehensive understanding.

Understanding the Problem

At its core, the problem involves two key components:

- Line P, which is already given in a form that allows us to determine its slope.
- Line L, which passes through a specific point (2, 37), and is parallel to line P, meaning it shares the same slope.

The ultimate goal is to find the equation of line L based on these conditions.

Breaking Down the Components

1. Understanding Line P

Given the equation:

Y + 3 = 5(x + 4)

This is written in point-slope form, which generally looks like:

$Y - Y_1 = m(x - X_1)$

or, in some cases, can be rearranged into slope-intercept form (Y = mX + b).

In our case, the equation is:

$Y + 3 = 5(x + 4)$

To understand this line better, we need to:

- Simplify it to slope-intercept form.
- Identify its slope (m).

2. The Concept of Parallel Lines

Parallel lines in the coordinate plane share the same slope but have different y-intercepts. This is a key property that allows us to find the equation of line L once we determine the slope of line P.

3. The Point (2, 37)

Line L passes through this point, so our final equation must satisfy this coordinate.

Step-by-Step Solution

Step 1: Convert Line P to Slope-Intercept Form

Starting with the given equation:

$Y + 3 = 5(x + 4)$

Distribute the 5:

$Y + 3 = 5x + 20$

Subtract 3 from both sides:

$Y = 5x + 20 - 3$

$Y = 5x + 17$

Conclusion: The slope of line P (m_p) is 5.

Step 2: Determine the Slope of Line L

Since line L is parallel to line P, it must have the same slope:

$m_l = m_p = 5$

Step 3: Find the Equation of Line L

Line L passes through (2, 37), and its slope is 5.

Using point-slope form:

$Y - Y_1 = m(X - X_1)$

Plugging in the point (2, 37):

$Y - 37 = 5(X - 2)$

Simplify:

$Y - 37 = 5X - 10$

Add 37 to both sides:

$Y = 5X - 10 + 37$

$Y = 5X + 27$

Final Equation of Line L:

$Y = 5X + 27$

Additional Insights and Applications

Understanding the Significance of Slope

The slope, $m = 5$, indicates that for every increase of 1 in x, y increases by 5. This steepness is consistent between both lines, a hallmark of parallelism.

Verifying the Solution

- The point (2, 37) satisfies the line L equation:

$Y = 5(2) + 27 = 10 + 27 = 37$ ✓

- The slope matches that of line P, confirming the parallel condition.

Common Variations and Related Problems

1. What if the point changes?

If line L passes through a different point, substitute that point into the point-slope form to find the new line.

2. What if the slope of line P changes?

The method remains the same: find the slope of the given line, then write the parallel line's equation passing through the given point.

3. How do different forms affect the process?

- Standard form ($Ax + By + C = 0$): can be converted to slope-intercept form for easy slope identification.
- Two-point form: used when two points are known; helpful if the problem involves two points rather than a slope.

Summary and Key Takeaways

- To find the equation of a line passing through a point and parallel to another line, identify the slope of the given line.
- Convert the given line to slope-intercept form to find its slope.
- Use the point-slope form with the known point and slope to find the new line's equation.
- Confirm the solution by substituting the point into the equation.

Final Remarks

Mastering the process of translating between different line forms, understanding the properties of parallel lines, and applying algebraic techniques will significantly enhance your problem-solving skills in coordinate geometry. This particular problem underscores the importance of careful algebraic manipulation and geometric understanding, which are fundamental to higher-level mathematics, physics, engineering, and related fields.

Remember, practice with various types of line equations and points will solidify your grasp of these concepts and enable you to approach similar problems with confidence and clarity.

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