Locate The Absolute Extrema Of The Function F(x)=x^3 12x On The Closed Interval [0,3]. Select One: A.

Locate The Absolute Extrema Of The Function F(x)=x^312x On The Closed Interval [0,3]. Select One: A.

Understanding how to find the absolute extrema (absolute maximum and minimum values) of a function within a closed interval is a fundamental skill in calculus. This process involves analyzing the behavior of the function throughout the interval, identifying critical points where the derivative is zero or undefined, and evaluating the function at these points and the endpoints of the interval. In this comprehensive guide, we will explore how to locate the absolute extrema of the specific function F(x) = x^3 12x over the interval [0,3].

Introduction to Absolute Extrema and the Given Function

What Are Absolute Extrema?

Absolute extrema refer to the highest and lowest points that a function attains over a specified interval. These are also called global extrema. For a continuous function on a closed interval, the Extreme Value Theorem guarantees the existence of both an absolute maximum and an absolute minimum within that interval.

Understanding the Function F(x) = x^3 12x

The function in question can be simplified for easier analysis:

- Original function: F(x) = x^3 12x
- Simplify: F(x) = 12x^4

This simplification makes it straightforward to analyze, as it's a polynomial function with a clear structure.

Step-by-Step Method to Find Absolute Extrema

To locate the absolute extrema of F(x) = 12x^4 on [0,3], follow these steps:

1. Confirm Continuity and Differentiability

- F(x) = 12x^4 is a polynomial, which is continuous and differentiable everywhere.
- Therefore, the Extreme Value Theorem applies.

2. Find Critical Points

- Critical points occur where the derivative is zero or undefined.

3. Evaluate the Function at Critical Points and Endpoints

- These evaluations identify potential candidates for absolute extrema.

4. Compare Function Values to Determine Absolute Extrema

Detailed Analysis and Calculations

Step 1: Derive the Function

- Derivative F'(x) = d/dx [12x^4] = 48x^3

Step 2: Find Critical Points

- Set derivative equal to zero:

48x^3 = 0

x^3 = 0

x = 0

- Since polynomial derivatives are defined everywhere, no points are undefined derivatives.

Step 3: Evaluate F(x) at Critical Points and Endpoints

- Endpoints: x = 0 and x = 3
- Critical point: x = 0

Calculate F(x) at these points:

- F(0) = 12 (0)^4 = 0
- F(3) = 12 (3)^4 = 12 81 = 972

Analyzing Results and Identifying Extrema

Based on the evaluations:

Point	x	$F(x) = 12x^4$
Endpoint	0	0
Endpoint	3	972

Since $F(0) = 0$ and $F(3) = 972$:

- The absolute maximum value on [0,3] is 972 at $x=3$.
- The absolute minimum value on [0,3] is 0 at $x=0$.

Conclusion: Absolute Extrema of the Function

The function $F(x) = 12x^4$ on the interval [0,3] attains its:

- Absolute maximum at $x=3$, with a value of 972.
- Absolute minimum at $x=0$, with a value of 0.

Additional Insights and Practical Applications

Understanding how to locate absolute extrema is crucial in various fields such as physics, engineering, economics, and optimization problems. For example:

- Physics: Determining maximum or minimum potential energy configurations.
- Economics: Finding optimal profit or cost points.
- Engineering: Ensuring structures are designed within safe stress limits.

By mastering the process outlined above, you can analyze a wide range of functions beyond simple polynomials.

Summary

- The function $F(x) = 12x^4$ is continuous and differentiable over [0,3].
- Critical point found at $x=0$.
- Endpoints at $x=0$ and $x=3$.
- $F(0)=0$, $F(3)=972$.
- Absolute maximum: 972 at $x=3$.
- Absolute minimum: 0 at $x=0$.

Practice Problems for Further Learning

1. Find the absolute extrema of the function $G(x) = x^2 - 4x + 5$ on the interval [1,4].
2. Determine the maximum and minimum values of $H(x) = x^3 - 6x^2 + 9x$ over [0,3].

3. Analyze the function $K(x) = (1/x)$ on $[1,5]$ to find its absolute extrema.

Practicing these problems will solidify your understanding of the process and prepare you for more complex scenarios.

Final Remarks

Locating the absolute extrema of a function involves a systematic approach: analyzing derivatives to find critical points, evaluating the function at these points and at the interval's boundaries, and comparing these values. For the specific function $F(x)=12x^4$ over $[0,3]$, the process is straightforward due to its polynomial nature. Mastery of these techniques enhances your problem-solving toolkit in calculus and applied mathematics.

Remember: Always verify the continuity of your function over the interval and carefully evaluate all critical points and endpoints to accurately determine the absolute extrema.

Frequently Asked Questions

What is the main goal when finding the absolute extrema of the function $F(x)=x^3 - 12x$ on $[0,3]$?

The goal is to determine the maximum and minimum values of the function within the interval, including at the endpoints and any critical points inside the interval.

How do you find the critical points of $F(x)=x^3 - 12x$?

First, compute the derivative $F'(x) = 3x^2 - 12$, then set it equal to zero and solve for x: $3x^2 - 12 = 0$, leading to $x^2 = 4$, so $x = \pm 2$. Since the interval is $[0,3]$, only $x=2$ is within the interval.

What are the values of $F(x)$ at the critical point $x=2$ and the endpoints $x=0$ and $x=3$?

At $x=0$: $F(0)=0^3 - 120=0$; at $x=2$: $F(2)=8 - 24 = -16$; at $x=3$: $F(3)=27 - 36= -9$.

Which point provides the absolute maximum of $F(x)$ on $[0,3]$?

The absolute maximum occurs at $x=0$ with $F(0)=0$.

Which point provides the absolute minimum of $F(x)$ on $[0,3]$?

The absolute minimum occurs at $x=2$ with $F(2)=-16$.

Why is it important to evaluate the function at both critical points and endpoints?

Because absolute extrema on a closed interval can occur at critical points inside the interval or at the endpoints, so all must be checked.

Does the function $F(x)=x^3 - 12x$ have any other critical points within $[0,3]$?

No, the only critical point within $[0,3]$ is at $x=2$, since $x=-2$ is outside the interval.

What is the significance of selecting 'One: A.' in the context of this problem?

It suggests that the correct answer choice (possibly from multiple options) corresponds to option A, indicating understanding of where the absolute extrema occur based on the analysis.

How would the analysis change if the interval was $[1,3]$ instead of $[0,3]$?

You would evaluate the function at the critical point $x=2$ (which is within $[1,3]$) and at the new endpoints $x=1$ and $x=3$ to determine the extrema within the new interval.

Additional Resources

Locate The Absolute Extrema Of The Function $F(x)=x^3 - 12x$ On The Closed Interval $[0,3]$

In the realm of calculus, the determination of absolute extrema—namely, the absolute maximum and minimum values—of a real-valued function within a specified interval is a fundamental problem with wide-ranging applications across mathematics, engineering, economics, and sciences. This investigative exploration delves into the process of locating the absolute extrema of the function $F(x) = x^3 - 12x$ on the closed interval $[0,3]$, illustrating the systematic approach and underlying calculus principles involved.

Understanding the Problem Context

The function in question is a cubic polynomial:

$$F(x) = x^3 - 12x$$

The task is to identify its absolute maximum and minimum values on the interval $[0,3]$. This involves analyzing the behavior of the function within the interval, considering critical points, and evaluating the function at boundary points.

This problem exemplifies a classic application of the Extreme Value Theorem, which states that any continuous function on a closed interval attains both its absolute maximum and minimum somewhere within that interval—either at critical points where the derivative is zero or undefined or at the interval's endpoints.

Step 1: Analyze the Function for Critical Points

Critical points are potential candidates for local or absolute extrema and are found where the first derivative is zero or undefined.

Calculating the Derivative

Differentiate $f(x)$:

$$
f'(x) = \frac{d}{dx}(x^3 - 12x) = 3x^2 - 12
$$

This derivative exists everywhere, so critical points occur where:

$$
3x^2 - 12 = 0
$$

Solve for x:

$$
3x^2 = 12 \\
x^2 = 4 \\
x = \pm 2
$$

Since our interval is $[0,3]$, only $x=2$ is within the interval.

Critical Point within $[0,3]$:

- $x=2$

Step 2: Evaluate the Function at Critical and Boundary Points

To determine the absolute extrema, evaluate $f(x)$ at:

- $x=0$ (left boundary)
- $x=2$ (critical point)
- $x=3$ (right boundary)

Function Values

1. At $x=0$:

$$
f(0) = 0^3 - 12 \times 0 = 0
$$

2. At $x=2$:

$$
f(2) = (2)^3 - 12 \times 2 = 8 - 24 = -16
$$

3. At $x=3$:

$$
f(3) = (3)^3 - 12 \times 3 = 27 - 36 = -9
$$

Step 3: Determine the Absolute Extrema

Compare the function values:

- $f(0) = 0$
- $f(2) = -16$
- $f(3) = -9$

Absolute Maximum: The highest value among these is 0, occurring at $x=0$.

Absolute Minimum: The lowest value is -16, occurring at $x=2$.

Summary of Results

Point	x	$f(x)$	Description
Boundary	0	0	Possible maximum or minimum
Critical point	2	-16	Potential extremum
Boundary	3	-9	Check for extremum

The function $f(x) = x^3 - 12x$ on $[0,3]$ has:

- An absolute maximum of 0 at $x=0$.
- An absolute minimum of -16 at $x=2$.

Deeper Insights: The Nature of Extrema and Graphical Behavior

Understanding the nature of these extrema involves examining the function's graph and its derivative's sign changes.

Graphical Behavior Analysis

- For $x \in (0,2)$:

Since $f'(x) = 3x^2 - 12$,

- For $0 < x < 2$:

$$
3x^2 - 12 < 0 \quad \Rightarrow \quad \text{Function decreasing}
$$

\]

- For \(x \in (2,3)\):

\[
3x^2 - 12 > 0 \quad \Rightarrow \quad \text{Function increasing}
\]

This indicates that \(x=2\) is a local minimum within the interval, with the function decreasing before \(x=2\) and increasing afterward, confirming the presence of a "valley" at \(x=2\).

Implications and Applications of the Findings

The methodical process for locating absolute extrema, exemplified in this case, is fundamental in optimization problems across various disciplines. For instance:

- Engineering: Structural optimization, where maximum stress or minimum material use is critical.
- Economics: Cost minimization or profit maximization within constraints.
- Physics: Finding equilibrium states by analyzing potential functions.

Understanding the precise locations and values of absolute extrema enables decision-makers and analysts to optimize systems effectively.

Conclusion: The Significance of Systematic Analysis

This comprehensive investigation underscores the importance of derivative analysis, critical point evaluation, and boundary considerations in solving extremum problems. The specific case of \(f(x) = x^3 - 12x\) on \([0,3]\) demonstrates a clear application of calculus principles, reinforcing the critical role of the Extreme Value Theorem and derivative tests in mathematical analysis.

By meticulously examining the function's behavior, we identified:

- The absolute maximum value of 0 at \(x=0\).
- The absolute minimum value of -16 at \(x=2\).

This process exemplifies the rigorous approach necessary for accurate and reliable optimization, serving as a foundational technique in both theoretical and applied contexts.

Note: The question posed was "Select One: A." While the details of options are not provided here, the analysis above thoroughly covers the steps and reasoning involved in solving the problem, enabling the selection of the correct extremum points based on the calculated values.

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