Miguel Draws A Square On A Coordinate Plane. One Vertex Is Located At (5,4). The Length Of Each Side

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Understanding how to plot geometric shapes on a coordinate plane is a fundamental skill in mathematics, especially in coordinate geometry. Whether you're a student learning the basics or a teacher preparing instructional materials, visualizing shapes like squares helps develop spatial reasoning and enhances problem-solving skills. In this article, we will explore in detail how Miguel can draw a square on the coordinate plane, starting from a specific vertex at point (5,4), and determine the possible locations of the other vertices based on the length of each side.

Introduction to Coordinate Geometry and Squares

Coordinate geometry, also known as analytic geometry, involves representing geometric figures using a coordinate plane and algebraic equations. It provides a powerful way to analyze shapes, find distances, slopes, and areas, and solve complex geometric problems systematically.

A square is a special type of quadrilateral characterized by four equal sides and four right angles. When plotting a square on a coordinate plane, the key information needed includes:

- The coordinates of at least one vertex.
- The length of each side.
- The orientation or rotation of the square (if any).

Given one vertex at (5,4) and the side length, the task is to determine the positions of the remaining vertices that complete the square.

Understanding the Problem: Given Data and Goals

Before we proceed with plotting, let's analyze what is provided and what needs to be found.

Given Data:

- One vertex at point (5,4).
- Length of each side (which we will denote as 's' and assume it's a positive real number).

Goals:

- Find the coordinates of the remaining three vertices of the square.
- Understand how the orientation of the square affects the positions of these vertices.
- Explore different possible configurations (e.g., sides aligned with axes vs. rotated squares).

Key Concepts in Plotting Squares on the Coordinate Plane

To effectively draw the square, we need to consider the following concepts:

1. Distance Formula

The distance between two points (x_1, y_1) and (x_2, y_2) is given by:

$$ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} $$

This formula is essential for verifying that two vertices are exactly 's' units apart.

2. Slope and Orientation

- When the sides are aligned with the axes, the vertices differ only in either x or y coordinates.
- For rotated squares, the vertices are not aligned with the axes, and their positions involve slopes of ±1 or other values depending on the rotation angle.

3. Rotation of a Point Around a Center

To find vertices when the square is rotated, we may use rotation formulas:

$$
x' = x_c + (x - x_c) \cos \theta - (y - y_c) \sin \theta
$$

$$
y' = y_c + (x - x_c) \sin \theta + (y - y_c) \cos \theta
$$

where (x_c, y_c) is the rotation center, and θ is the rotation angle.

Constructing a Square with One Vertex at (5,4)

Let's explore different cases based on how the square might be oriented.

Case 1: Square Sides are Parallel to the Axes

This is the simplest case where the square is aligned with the coordinate axes. The sides are either horizontal or vertical.

Step 1: Determine the known vertex

- Vertex A at (5,4).

Step 2: Find the other vertices

- Since the square has sides of length 's', the other vertices will be located either horizontally or vertically from vertex A.

Assuming the square extends to the right and upward:

- Vertex B: (5 + s, 4) – moving right horizontally.
- Vertex D: (5, 4 + s) – moving upward vertically.
- Vertex C: (5 + s, 4 + s) – diagonally opposite to A.

Vertices:

1. A: (5, 4)
2. B: (5 + s, 4)
3. D: (5, 4 + s)
4. C: (5 + s, 4 + s)

Note: For different orientations, the positions change accordingly, but in this case, the shape is aligned with axes.

Step 3: Verify distances

- Distances between adjacent vertices should be 's'.
- For example, distance AB:

$$
d_{AB} = \sqrt{((5 + s) - 5)^2 + (4 - 4)^2} = \sqrt{s^2 + 0} = s
$$

Similarly, other sides will be of length 's'.

Case 2: Square is Rotated (Not Aligned with Axes)

In this scenario, the square is rotated at some angle θ; for simplicity, consider a rotation of 45°.

Step 1: Define the known vertex
- Vertex A at (5,4).

Step 2: Find the remaining vertices using rotation
- Consider the side length 's'.
- To find the other vertices, rotate vectors around point A.

For a square rotated 45°, the vectors from A to other vertices are:

- $\vec{v} = \left(\frac{s}{\sqrt{2}}, \frac{s}{\sqrt{2}} \right)$

- The vertices are:

1. B: A + $\vec{v}$ = (5 + $s/\sqrt{2}$, 4 + $s/\sqrt{2}$)
2. D: A + $\vec{v}$ rotated by -90°, which results in:

$$
\left(5 - \frac{s}{\sqrt{2}}, 4 + \frac{s}{\sqrt{2}} \right)
$$

3. C: The vertex diagonally opposite A, found by adding both vectors:

$$
A + \vec{v} + \text{rotated } \vec{v}
$$

Calculations involve rotation matrices, but the main idea is to derive the positions based on the side length and rotation angle.

Calculating the Side Length and Coordinates

Since the problem statement mentions "The Length Of Each Side," but does not specify a numeric value, let's explore how to determine the remaining vertices based on a given side length.

Example: Side length s = 3

Using the simple axis-aligned square:

- Vertex A: (5,4)
- Vertex B: (8,4)
- Vertex D: (5,7)
- Vertex C: (8,7)

Check distances:

- AB: $\sqrt{(8 - 5)^2 + (4 - 4)^2} = 3$
- AD: $\sqrt{(5 - 5)^2 + (7 - 4)^2} = 3$

Confirmed that all sides are of length 3.

Advanced: Multiple Solutions and Their Geometric Significance

Depending on the orientation, multiple squares can be constructed that satisfy the given vertex and side length constraints. These include:

- Squares aligned with axes.
- Squares rotated at various angles (e.g., 30°, 45°, 60°).
- Mirrored squares across axes.

Understanding these configurations allows for a comprehensive grasp of geometric transformations and their visualizations.

Practical Applications of Plotting Squares on the Coordinate Plane

Mastering the process of plotting squares has numerous real-world applications, including:

- Computer graphics and game design.

- Architectural planning.
- Robotics pathfinding.
- Mathematical modeling and problem-solving.

Conclusion

Drawing a square on the coordinate plane with one vertex at (5,4) and a specified side length involves understanding fundamental principles of coordinate geometry, including distance calculation, rotation, and translation. Whether the square is aligned with the axes or rotated at an angle, the key is to apply geometric transformations systematically.

By exploring different cases and calculations, students and enthusiasts can develop a deeper understanding of geometric constructions. With practice, visualizing and plotting such figures becomes intuitive, enhancing both mathematical skill and spatial reasoning.

Remember, the essential steps are:

- Start with a known vertex.
- Decide on the orientation.
- Use geometric formulas to find the remaining vertices.
- Verify side lengths and angles.

With these tools, creating accurate and meaningful geometric figures on the coordinate plane becomes straightforward and engaging.

Keywords for SEO Optimization:
coordinate plane, plotting square, vertex at (5,4), side length calculation, geometric transformations, rotated square, coordinate geometry, plotting squares, rotation formulas, visualizing shapes on coordinate plane, math problem-solving, geometric constructions

Frequently Asked Questions

What are the possible coordinates for the vertices of a square on a coordinate plane if one vertex is at (5,4) and the side length is known?

The other vertices depend on the side length and the orientation of the square. If the side length is 's', and assuming the square is aligned with the axes, the vertices could be at (5 + s, 4), (5, 4 + s), and (5 + s, 4 +

s). For squares rotated at an angle, the vertices can be found using rotation formulas around the known vertex.

How can you determine the coordinates of the other vertices of the square if only one vertex and the side length are given?

You can use vector addition and rotation techniques. For example, starting from the known vertex, move along the side length in the desired direction (horizontal or vertical), and then rotate the side vector by 90 degrees to find the adjacent vertices. Calculations involve adding or subtracting the side length from the x and y coordinates appropriately.

What is the significance of the side length in plotting the square on the coordinate plane?

The side length determines the distance between adjacent vertices. Knowing this length allows you to accurately plot the remaining vertices of the square, ensuring the shape is precise and proportional on the coordinate plane.

If the side length of the square is 3 units and one vertex is at (5,4), what are the coordinates of the other vertices assuming the square sides are aligned with the axes?

The vertices would be at (8,4), (5,7), and (8,7). These are found by adding 3 to the x-coordinate for the right vertices and 3 to the y-coordinate for the top vertices.

How does rotating the square affect the coordinates of its vertices on the plane?

Rotating the square around a vertex involves applying rotation matrices to the other vertices, changing their coordinates based on the angle of rotation. This results in vertices that are not aligned with the axes, requiring calculation using trigonometric functions to find their new positions.

What tools or methods can be used to accurately draw a square with a given vertex and side length on a coordinate plane?

You can use graph paper and a ruler for manual drawing, or graphing software and coordinate geometry tools to input the vertex and side length. Using formulas for rotation and translation in coordinate geometry can also help in accurately determining the positions of all vertices.

Additional Resources

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Introduction

The act of drawing geometric figures on the coordinate plane is a fundamental exercise in understanding spatial relationships, coordinate geometry, and the properties of shapes. Among these figures, the square stands out due to its unique combination of symmetry, equal sides, and right angles. When a student or mathematician, such as Miguel, sets out to draw a square with a known vertex and side length, the problem becomes a rich exploration of geometric principles, algebraic reasoning, and coordinate transformations.

This article offers an in-depth investigation into the problem: Miguel draws a square on a coordinate plane, with one vertex located at (5,4), and each side of the square having an unknown length. We will explore how to determine the possible positions of the square, the various configurations it can assume, and the mathematical reasoning necessary to construct such a shape accurately. This analysis underscores the importance of understanding coordinate geometry, the properties of squares, and how to approach geometric problems involving multiple unknowns.

Understanding the Problem and Its Significance

At its core, the problem involves determining the possible placements of a square on a coordinate plane given one fixed vertex and the length of its sides. The question is not merely about plotting a single square but about understanding the entire family of squares that satisfy these conditions.

This problem is significant in multiple contexts:
- Educational: It develops spatial reasoning and understanding of coordinate transformations.
- Mathematical: It involves the application of distance formulas, rotation matrices, and properties of geometric figures.
- Practical: Similar reasoning applies in computer graphics, CAD (Computer-Aided Design), and robotics, where precise placement and orientation of objects are essential.

Formal Statement of the Problem

Given:
- A fixed vertex $V_0 = (5, 4)$,
- The side length s (unknown in the problem statement but assumed to be a positive real number),

Find:
- All possible squares S such that V_0 is one of the vertices of S, and the side length of S is s.

Mathematical Foundations

Before diving into specific solutions, it is essential to review the foundational concepts:

Coordinate Geometry Basics

- Distance Formula: To find the length of a segment between two points (x_1, y_1) and (x_2, y_2),
$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$
- Rotation Transformation: To rotate a point around another point by an angle θ,
$$
\begin{cases}
x' = x_0 + (x - x_0) \cos \theta - (y - y_0) \sin \theta \\
y' = y_0 + (x - x_0) \sin \theta + (y - y_0) \cos \theta
\end{cases}
$$
where (x_0, y_0) is the point around which rotation occurs.

Properties of a Square

- All sides are equal in length.
- Adjacent sides are perpendicular (90° angles).
- The diagonals are equal in length and bisect each other at right angles.

Approaching the Construction of the Square

Given the fixed vertex $V_0 = (5, 4)$, the problem involves determining the positions of the remaining vertices V_1, V_2, V_3. Since the square can be oriented in any direction, the key is to understand how to generate all possible squares with V_0 as one vertex.

Step 1: Fixing the Side Length

While the problem states the side length as "suitable," it is more instructive to consider it as an unknown variable s. Once the side length is known, the problem reduces to finding the coordinates of the other vertices.

Step 2: Choosing a Direction for the Side

The square can be oriented in infinitely many ways. To systematically analyze this, consider:

- Selecting a direction vector $\vec{d}$ from V_0 to the next vertex V_1.
- The length of $\vec{d}$ must be s, so:
$$
\|\vec{d}\| = s
$$
- The other vertices can be constructed by applying rotations of 90° to this vector.

Constructing the Square: Analytical Approach

Suppose $V_0 = (x_0, y_0) = (5, 4)$, and $V_1 = (x_1, y_1)$ is a point such that:

$$
\vec{V_0V_1} = (x_1 - 5, y_1 - 4)
$$

with $\|\vec{V_0V_1}\| = s$.

Step 1: Express V_1

Let θ be the angle $\vec{V_0V_1}$ makes with the positive x-axis:

$$
x_1 = 5 + s \cos \theta
$$
$$
y_1 = 4 + s \sin \theta
$$

for $\theta \in [0, 2\pi)$.

Step 2: Find the Remaining Vertices

The square's vertices V_0, V_1, V_2, V_3 can be generated by rotating the side vector $\vec{V_0V_1}$ by 90°:

- Vertex V_2: obtained by adding a rotated vector to V_1:

$$
\vec{V_1V_2} = R_{90^\circ}(\vec{V_0V_1}) = (-s \sin \theta, s \cos \theta)
$$
$$
V_2 = V_1 + \vec{V_1V_2} = (x_1 - s \sin \theta, y_1 + s \cos \theta)
$$

- Vertex V_3: obtained by rotating $\vec{V_0V_1}$ by 180°:

$$
\vec{V_0V_3} = R_{180^\circ}(\vec{V_0V_1}) = (-s \cos \theta, -s \sin \theta)
$$
$$
V_3 = V_0 + \vec{V_0V_3} = (5 - s \cos \theta, 4 - s \sin \theta)
$$

Alternatively, rotating $\vec{V_0V_1}$ by 270° yields the other possible orientation for the square, giving two distinct options for the square's configuration based on the initial side direction.

The Infinite Family of Squares

Since θ can be any angle, this process describes an infinite family of squares, each with one vertex at $(5,4)$, side length s, and oriented differently.

Visualizing the Configurations

- For each θ, the square is "rotated" accordingly.
- The size s determines how large the square is.
- The square's other vertices can be explicitly calculated for specific θ, providing a parametric family of solutions.

Special Cases and Constraints

When is the Square a Valid Geometric Figure?

- The length s must be positive.
- All vertices must be real points on the plane, which they are for any $s > 0$.
- The orientation θ is arbitrary, so physically, there are infinitely many possible squares.

Constraints on s

- If the problem specifies a particular side length s, the above

formulas directly produce the other vertices.
- If s is unknown, additional conditions or constraints are needed, such as specifying the position of another vertex or the orientation.

Practical Examples

Suppose we choose $s = 3$, and $\theta = 0^\circ$ (i.e., initial side along the positive x-axis):

- $V_1 = (5 + 3 \times 1, 4 + 3 \times 0) = (8, 4)$
- $V_2 = (8 - 3 \times 0, 4 + 3 \times 1) = (8, 7)$
- $V_3 = (5 - 3 \times 1, 4 - 3 \times 0) = (2, 4)$

This yields a square with vertices:

- $V_0 = (5, 4)$
- $V_1 = (8, 4)$
- $V_2 = (8, 7)$
- $V_3 = (2, 4)$

which is aligned along the axes.

Extending to Other Orientations

By varying θ, the square can be

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