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Understanding the concept of continuity is fundamental in calculus and real analysis. When analyzing a function's behavior across different intervals, especially those involving points where the function may have discontinuities, it’s essential to evaluate the function's continuity at specific points and over specific domains. In this article, we will explore the nuances of continuity for a function F on the intervals $(-\infty, 6)$ and $(6, +\infty)$, and discuss the necessary conditions for F to be continuous on these intervals as well as at the point $x=6$.

What Does It Mean for a Function to Be Continuous?

Definition of Continuity

A function F is said to be continuous at a point $x=a$ if the following three conditions are satisfied:

- F is defined at a: $F(a)$ exists.
- The limit of F as x approaches a exists: $\lim_{x\to a} F(x)$ exists.
- The limit of F as x approaches a equals the function value at a: $\lim_{x\to a} F(x) = F(a)$.

If these conditions hold for every point within an interval, then F is continuous on that interval.

Continuity on Open Intervals

For open intervals such as $(-\infty, 6)$ or $(6, +\infty)$, a function is continuous if it is continuous at every point in the interval. Since these intervals do not include the endpoints, the only concern is the behavior of F within the interval itself, not at the boundary points.

Analyzing Continuity on $(-\infty, 6)$ and $(6, +\infty)$

Given the intervals $(-\infty, 6)$ and $(6, +\infty)$, the question arises: under what circumstances is the function F continuous on these regions?

Continuity on $(-\infty, 6)$

- Behavior as $x \to -\infty$:
Typically, functions are analyzed for their behavior as x approaches negative infinity to understand their limits at the far left of the domain.

- Behavior approaching $x=6$ from the left:
Since $x=6$ is an interior point that is not included in the interval $(-\infty, 6)$, the focus is on the continuity of F for all points less than 6, especially as x approaches 6 from the left.

- Conditions for continuity in $(-\infty, 6)$:
F must be defined and continuous at every point in $(-\infty, 6)$. Typically, this involves ensuring that F is smooth or at least continuous in this open interval and that the limit as $x \to 6^-$ exists.

Continuity on $(6, +\infty)$

- Behavior as $x \to +\infty$:
Investigate how F behaves when x becomes very large.

- Approach towards $x=6$ from the right:
Since $x=6$ is the lower boundary of this interval, the continuity at points close to 6 from the right depends on the behavior of F as $x \to 6^+$.

- Conditions for continuity in $(6, +\infty)$:
Similar to the previous case, F must be continuous within the interval and have a well-defined limit as $x \to 6^+$.

Continuity at the Point $x=6$

While the intervals $(-\infty, 6)$ and $(6, +\infty)$ exclude the point $x=6$, the overall continuity of F across its domain depends heavily on the behavior at this point.

Why is Continuity at $x=6$ Important?

- Point of Potential Discontinuity:
$x=6$ often acts as a boundary point where the nature of the function might change — for example, a piecewise function could have different definitions on either side of 6.

- Conditions for Continuity at $x=6$:
For F to be continuous at $x=6$, the following must hold:
1. $F(6)$ is defined.
2. $\lim_{x \to 6^-} F(x)$ exists.
3. $\lim_{x \to 6^+} F(x)$ exists.
4. The limits from the left and right are equal:
$\lim_{x \to 6^-} F(x) = \lim_{x \to 6^+} F(x)$.
5. The common limit equals $F(6)$:
$\lim_{x \to 6} F(x) = F(6)$.

In summary, for F to be continuous on the entire real line, it must be continuous on $(-\infty, 6)$, $(6, +\infty)$, and at $x=6$.

Conditions for F to Be Continuous on the Intervals

Piecewise Functions and Continuity

Many functions are defined piecewise, especially when considering different behaviors around a specific point like $x=6$. For example:

$$
F(x) = \begin{cases}
f_1(x), & x < 6 \\
f_2(x), & x > 6
\end{cases}
$$

- To ensure F is continuous at $x=6$, the following must be true:

- Both $f_1(x)$ and $f_2(x)$ are continuous at their respective points approaching 6.

- The limits of $f_1(x)$ as $x \to 6^-$ and $f_2(x)$ as $x \to 6^+$ exist and are equal.

- The value $F(6)$ (if defined) matches this common limit.

General Conditions for Continuity on $(-\infty, 6)$ and

$(6, +\infty)$

- Continuity within the intervals:
The function must be continuous at every point in the open intervals, which generally requires that the functions are well-behaved (no jumps, breaks, or asymptotes) within these ranges.

- Limit existence at boundary points:
The limits approaching the boundary point $x=6$ from within the intervals should exist and match the function's value at 6 if the function is to be continuous there.

- Avoidance of discontinuities:
Vertical asymptotes, jumps, or removable discontinuities should be absent within the intervals for the function to be considered continuous over these regions.

Examples of Functions and Continuity Behavior

Example 1: Continuous Piecewise Function

Suppose,

$$
F(x) = \begin{cases}
2x + 1, & x < 6 \\
3x - 12, & x > 6
\end{cases}
$$

- Check continuity at $x=6$:
- $\lim_{x \to 6^-} F(x) = 2(6) + 1 = 13$
- $\lim_{x \to 6^+} F(x) = 3(6) - 12 = 6$
Since the limits from the left and right are not equal ($13 \neq 6$), the function is not continuous at $x=6$ unless $F(6)$ is defined and equal to both limits, which in this case, it cannot be.

Example 2: Continuous Piecewise Function with Matching Limits

Suppose,

$$
F(x) = \begin{cases}
x^2, & x < 6 \\
36, & x \geq 6

\end{cases}
\]

- Check limits:

- $\lim_{x \to 6^-} F(x) = 6^2 = 36$
- $F(6) = 36$ (assuming the definition at $x=6$ is 36)
- $\lim_{x \to 6^+} F(x) = 36$

- Conclusion:
Since all limits and the function value at $x=6$ are equal, F is continuous at $x=6$. It is also continuous on the intervals $(-\infty, 6)$ and $(6, +\infty)$.

Summary: Conditions for Overall Continuity