PLEASE HELP ME!!let G Be The Function Given By G (x) = 3x^4 - 8x^3. At What Value Of X On The Closed

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Understanding how to analyze and interpret functions is fundamental in calculus and algebra. In particular, determining the maximum and minimum values of a function within a specific interval, known as the closed interval, is a crucial skill. This article provides a comprehensive guide to solving a problem involving the function $G(x) = 3x^4 - 8x^3$, focusing on identifying the critical points and evaluating the function at these points to find its extrema on a closed interval.

Understanding the Problem

Before diving into the calculations, let’s clarify what the problem entails:

- The function in question is $G(x) = 3x^4 - 8x^3$.
- The task is to find the value(s) of x within a specified closed interval where the function attains its maximum and minimum.
- Typically, the interval is given, such as $[a, b]$. If the interval is not specified, it is common to analyze the function over a standard interval, like $[0, 3]$, or to determine the extrema over the entire real line if no bounds are specified.

Important notes:

- When dealing with extrema on a closed interval, both the critical points within the interval and the endpoints must be evaluated.
- Critical points occur where the derivative is zero or undefined.
- The absolute maximum or minimum on the interval will be among these critical points and the endpoints.

Step-by-Step Approach to Finding the Extrema

The process for solving this type of problem involves several key steps:

1. Find the Derivative $G'(x)$

Calculating the derivative helps identify critical points where the slope of the function is zero or undefined.

Given $G(x) = 3x^4 - 8x^3$:

$$
G'(x) = \frac{d}{dx}(3x^4) - \frac{d}{dx}(8x^3) = 12x^3 - 24x^2
$$

2. Set the Derivative Equal to Zero to Find Critical Points

Solve:

$$
12x^3 - 24x^2 = 0
$$

Factor out common terms:

$$
12x^2 (x - 2) = 0
$$

This yields potential critical points:

$$
x = 0 \quad \text{or} \quad x = 2
$$

---

3. Determine the Nature of Critical Points (Max, Min, or Saddle)

Use the second derivative or test intervals to classify the critical points.

Calculate the second derivative:

$$
G''(x) = \frac{d}{dx}(12x^3 - 24x^2) = 36x^2 - 48x
$$

Evaluate $G''(x)$ at critical points:

- At $x=0$:

$$
G''(0) = 36(0)^2 - 48(0) = 0
$$

Second derivative test is inconclusive when it equals zero; hence, consider the first derivative test or analyze the sign of $G'(x)$ around $x=0$.

- At $x=2$:

$$
G''(2) = 36(2)^2 - 48(2) = 36 \times 4 - 96 = 144 - 96 = 48
$$

Since $G''(2) > 0$, $x=2$ corresponds to a local minimum.

---

4. Evaluate the Function at Critical Points and Endpoints

To find the absolute extrema, evaluate $G(x)$ at:

- Critical points: $x=0$ and $x=2$
- Endpoints of the interval $[a, b]$ (if specified)

Assuming the interval is [0, 3], evaluate:

- $G(0)$:

$$
G(0) = 3(0)^4 - 8(0)^3 = 0
$$

- $G(2)$:

$$
G(2) = 3(2)^4 - 8(2)^3 = 3 \times 16 - 8 \times 8 = 48 - 64 = -16
$$

- $G(3)$:

$$
G(3) = 3(3)^4 - 8(3)^3 = 3 \times 81 - 8 \times 27 = 243 - 216 = 27
$$

Interpreting the Results

From the function evaluations:

x-value	G(x)	Interpretation
0	0	Candidate for maximum/minimum
2	-16	Candidate for minimum
3	27	Candidate for maximum

Conclusion:

- The absolute maximum of $G(x)$ on $[0, 3]$ is 27 at $x=3$.
- The absolute minimum is -16 at $x=2$.

Generalizing the Approach to Other Intervals

This process can be applied to any closed interval $[a, b]$. The steps include:

1. Find the derivative $G'(x)$.
2. Solve $G'(x) = 0$ to find critical points within the interval.
3. Evaluate $G(x)$ at critical points and endpoints a and b.
4. Compare the function values to identify the maximum and minimum.

Additional Tips for Analyzing Polynomial Functions

- Always check the domain of the function before solving.
- Use the first or second derivative test to classify critical points.
- When the derivative is zero or undefined within the interval, these points are critical candidates.
- Evaluate the function at the interval endpoints, as extrema can occur there.

Real-World Applications of Finding Extrema

Determining maxima and minima of functions like $G(x) = 3x^4 - 8x^3$ is essential in various fields:

- Physics: To find maximum or minimum values of a physical quantity, such as velocity or potential

energy.
- Economics: To identify optimal profit or cost points.
- Engineering: To optimize design parameters for maximum efficiency or safety.
- Biology: To determine optimal conditions for biological processes.

Summary

In this comprehensive guide, we explored how to find the value(s) of x where the function $G(x) = 3x^4 - 8x^3$ attains its maximum and minimum on a closed interval. The key steps involved:

- Calculating the derivative $G'(x)$,
- Solving for critical points,
- Classifying these points using the second derivative or interval tests,
- Evaluating the function at critical points and endpoints,
- Comparing these values to determine the extrema.

By mastering this process, you can confidently analyze a wide range of polynomial functions and determine their behavior within specified domains.

Remember: Always analyze the function over the given interval, consider critical points, and evaluate at the endpoints to accurately identify the absolute maximum and minimum values.

Frequently Asked Questions

How do I find the critical points of the function $G(x) = 3x^4 - 8x^3$?

To find critical points, first compute the derivative $G'(x) = 12x^3 - 24x^2$. Then, set $G'(x) = 0$ and solve for x: $12x^3 - 24x^2 = 0$, which simplifies to $12x^2(x - 2) = 0$. Therefore, the critical points are at $x = 0$ and $x = 2$.

How do I determine the maximum and minimum values of G(x) on a closed interval?

Use the Extreme Value Theorem: evaluate G(x) at the critical points within the interval and at the endpoints. Compare these values to identify the absolute maximum and minimum on the interval.

What is the significance of the endpoints when finding the

extrema of G(x) on a closed interval?

Since the interval is closed, the maximum and minimum can occur at critical points or at the endpoints. Therefore, always evaluate G(x) at both the critical points inside the interval and the endpoints to find the absolute extrema.

Can you explain how to determine if a critical point corresponds to a maximum or minimum for G(x)?

Yes. Use the second derivative test: compute G''(x) = 36x^2 - 48x. Evaluate G''(x) at each critical point. If G''(x) > 0, it's a local minimum; if G''(x) < 0, it's a local maximum. If G''(x) = 0, the test is inconclusive, and further analysis is needed.

How do I evaluate G(x) at the endpoints of a closed interval to find extrema?

Simply substitute the endpoint values into G(x). For example, if the interval is [a, b], calculate G(a) and G(b). These values, along with those at critical points, will help you determine the absolute extrema on the interval.

Additional Resources

Understanding the Function $G(x) = 3x^4 - 8x^3$: A Deep Dive into Its Behavior and Critical Points

When approaching a mathematical problem involving functions, especially polynomial functions like $G(x) = 3x^4 - 8x^3$, it's essential to analyze multiple aspects to understand the function's behavior thoroughly. This includes identifying its domain, range, critical points, and the specific values of x where certain properties (like maxima, minima, or inflection points) occur, particularly on a closed interval. In this detailed review, we will explore these components step-by-step, ultimately answering the question: "At what value of x on the closed interval does G attain its maximum or minimum?"

Breaking Down the Function $G(x) = 3x^4 - 8x^3$

Before diving into critical point analysis, it’s crucial to understand the nature of G(x):

- Type of Function: Polynomial of degree 4, which is a quartic function.
- Leading Coefficient: 3 (positive), indicating that as x approaches $\pm\infty$, G(x) tends to $+\infty$.
- Behavior at Extremes: Since the leading term is positive ($3x^4$), the function rises to infinity as |x| becomes large.

Understanding the Domain and Range

- Domain: All real numbers $(-\infty, \infty)$, because polynomial functions are defined everywhere on the real line.
- Range: The set of all real numbers that $G(x)$ can take. Since $G(x)$ tends to $+\infty$ as $|x|$ increases and has potential local minima and maxima, the range is [minimum value, $+\infty$).

Finding Critical Points of G(x)

Critical points occur where the first derivative $G'(x)$ equals zero or is undefined. For polynomial functions like $G(x)$, the derivative exists everywhere, so we only need to find where $G'(x) = 0$.

Calculating the Derivative G'(x)

Given $G(x) = 3x^4 - 8x^3$,

- $G'(x) = d/dx\ [3x^4 - 8x^3] = 12x^3 - 24x^2$.

Factoring G'(x)

To find critical points:

- $G'(x) = 12x^3 - 24x^2 = 12x^2(x - 2)$.

This factors neatly into:

- $G'(x) = 12x^2(x - 2)$.

Solving G'(x) = 0 for Critical Points

Set $G'(x) = 0$:

- $12x^2(x - 2) = 0$.

The solutions are:

1. $x^2 = 0 \Rightarrow x = 0$,
2. $x - 2 = 0 \Rightarrow x = 2$.

Critical points are therefore at $x = 0$ and $x = 2$.

Classifying Critical Points: Maxima, Minima, or Inflection Points

To classify whether these critical points are maxima, minima, or points of inflection, we analyze the second derivative, $G''(x)$:

Calculating G''(x)

- $G''(x) = d/dx [G'(x)] = d/dx [12x^3 - 24x^2] = 36x^2 - 48x$.

Evaluating G''(x) at Critical Points

- At $x = 0$:

$G''(0) = 36(0)^2 - 48(0) = 0$.

- At $x = 2$:

$G''(2) = 36(2)^2 - 48(2) = 36(4) - 96 = 144 - 96 = 48$.

Interpretation:

- At $x = 0$: The second derivative is zero, indicating the test is inconclusive; further analysis (like the third derivative or the first derivative test) is needed.
- At $x = 2$: $G''(2) > 0$, indicating a local minimum at $x = 2$.

Deepening the Critical Point Analysis

Given that the second derivative at $x = 0$ is zero, we need to analyze the behavior around this point:

- First derivative test: Check the sign of $G'(x)$ around $x = 0$.
- For x just less than 0 (say, $x = -0.1$):

$G'(-0.1) = 12(-0.1)^3 - 24(-0.1)^2 = 12(-0.001) - 24(0.01) = -0.012 - 0.24 = -0.252$ (negative).

- For x just greater than 0 (say, $x = 0.1$):

$G'(0.1) = 12(0.001) - 24(0.01) = 0.012 - 0.24 = -0.228$ (negative).

Since G' is negative on both sides of $x = 0$, the function is decreasing through $x = 0$, suggesting a point of inflection rather than a maximum or minimum at $x = 0$.

Summary of Critical Points and Their Nature

Critical Point	G''(x)	Nature	Explanation
x = 0	0	Inflection point (saddle)	G' negative on both sides; no maximum or minimum.
x = 2	48	Local minimum	G'' > 0 indicates a minimum at x = 2.

Evaluating G(x) at Critical Points and Endpoints

The question specifies "on the closed interval," but the interval hasn't been explicitly provided in the prompt. Typically, in such problems, the interval is given, for example, [a, b]. For demonstration, let's assume the interval is [0, 3].

Note: If the actual interval differs, similar steps can be adapted.

Calculating G(x) at Critical Points and Endpoints

- $G(0) = 3(0)^4 - 8(0)^3 = 0$.
- $G(2) = 3(2)^4 - 8(2)^3 = 3(16) - 8(8) = 48 - 64 = -16$.
- $G(3) = 3(3)^4 - 8(3)^3 = 3(81) - 8(27) = 243 - 216 = 27$.

Determining the Absolute Maximum and Minimum on [0, 3]

- At $x = 0$: $G(0) = 0$.
- At $x = 2$: $G(2) = -16$.
- At $x = 3$: $G(3) = 27$.

Conclusion:

- Absolute maximum on [0, 3] is at $x = 3$ with $G(3) = 27$.
- Absolute minimum is at $x = 2$ with $G(2) = -16$.

General Approach to Finding Extrema on a Closed Interval

When dealing with a function on a closed interval [a, b], the general steps are:

1. Find critical points within the interval by solving $G'(x) = 0$.
2. Evaluate the function at all critical points inside [a, b].
3. Evaluate G at the endpoints $x = a$ and $x = b$.
4. Compare these values to identify the maximum and minimum.

This process ensures all potential extreme values are considered, including endpoints and critical points.

Applications and Practical Significance

Understanding the behavior of functions like $G(x) = 3x^4 - 8x^3$ has numerous applications:

- Optimization Problems: In fields like engineering, economics, and physics, determining maximum and minimum values helps optimize processes, costs, or designs.
- Curve Sketching: Critical points and inflection points inform the shape of the graph, aiding visualization.
- Real-world Modelling: Polynomial functions can model physical phenomena, such as potential energy, profit functions, or population dynamics.

Summary and Final Remarks

To recap:

- The critical points of $G(x)$ are at $x = 0$ and $x = 2$.
- $x = 2$ corresponds to a local minimum, with $G(2) = -16$.
- $x = 0$ is an inflection point, not a maximum or a minimum.
- The function's behavior at infinity suggests it tends toward $+\infty$ as $|x|$ increases.
- For a specific interval (e.g., [0, 3]), the maximum is at the endpoint $x = 3$ with $G(3) = 27$, and the minimum at $x = 2$ with $G(2) = -16$.

Therefore, if your problem involves finding the value of x on a closed interval that maximizes or minimizes G(x), follow this structured approach: find critical points, evaluate at endpoints, and compare the values.

Additional Tips for Students and Pract

[Please Help Me Let G Be The Function Given By G X 3x 4 8x 3 At What Value Of X On The Closed]

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