Please Help:Question 10Find The Equation Of The Parabola With Its Focus At (3,4) And Its Directrix Y

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Understanding how to find the equation of a parabola given its focus and directrix is a fundamental concept in conic sections. Parabolas are U-shaped curves that can be described mathematically by their focus point and directrix line. In this article, we will delve into the process of deriving the equation of a parabola with the focus at (3, 4) and the directrix y, which is a horizontal line. Whether you're a student studying conic sections or someone preparing for examinations, this comprehensive guide will clarify the steps involved and deepen your understanding of parabola equations.

What Is a Parabola?

A parabola is a symmetric, U-shaped curve that can open upward, downward, left, or right depending on its orientation. It is a conic section formed by the intersection of a right circular conical surface and a plane parallel to its side.

Key features of a parabola include:

- Focus: A fixed point where all the parabola's rays seem to reflect or focus.
- Directrix: A fixed line perpendicular or parallel to the parabola's axis of symmetry, serving as a reference for the parabola's definition.
- Vertex: The point where the parabola turns or its lowest/highest point, located midway between the focus and the directrix.
- Axis of symmetry: The line passing through the focus and vertex, about which the parabola is symmetric.

Understanding the Focus and Directrix in Parabola Equations

The parabola can be defined as the locus of points equidistant from the focus and the directrix. This definition is central to deriving the equation of the parabola given its focus and directrix.

Mathematically:

- For any point (x, y) on the parabola, the distance to the focus (x_f, y_f) equals the perpendicular distance to the directrix line.

Expressed as:

$$
\text{Distance to focus} = \text{Distance to directrix}
$$

Given Data: Focus at (3, 4) and Directrix y

In this problem, the focus is at the point $(3, 4)$, and the directrix is a line parallel to the x-axis, represented as $y = k$. Because the directrix is given as just 'Y', we need to determine its position relative to the focus.

Assumption:

- The directrix line is horizontal, i.e., of the form $y = c$.

Key points to identify:

- The position of the directrix relative to the focus.
- The parabola's orientation (opening upwards or downwards).

Step-by-Step Process to Find the Equation of the Parabola

1. Determine the Directrix Line

Since the focus is at $(3, 4)$, and the directrix is a line with a constant y-value, the parabola opens either upward or downward depending on the directrix's position relative to the focus.

- If the directrix is below the focus (i.e., $y < 4$), the parabola opens upward.
- If the directrix is above the focus (i.e., $y > 4$), the parabola opens downward.

In most typical problems, unless specified, the directrix is below the focus for an upward-opening parabola.

Suppose the directrix is at $y = c$, with $c < 4$.

2. Find the Distance Between Focus and Directrix

The distance from the focus to the directrix line is:

$$
| y_f - c | = | 4 - c |
$$

This distance is also the distance from the vertex to the focus or directrix, since the vertex lies midway between them.

3. Find the Vertex Coordinates

The vertex (x_v, y_v) lies midway between the focus and the directrix:

$$
y_v = \frac{ y_f + c }{2}
$$
$$
x_v = x_f = 3
$$

Because the focus and directrix are horizontally aligned (parallel to x-axis), the vertex shares the same x-coordinate as the focus.

4. Write the Parabola Equation

When the parabola opens upward or downward, and its vertex is at (x_v, y_v), the standard form of the parabola's equation is:

$$
(y - y_v) = \frac{1}{4p} (x - x_v)^2
$$

where:

- p is the distance from the vertex to the focus (or directrix).
- The sign of p determines the opening direction:
- If $p > 0$, parabola opens upward.
- If $p < 0$, parabola opens downward.

Calculating the Parameters for the Parabola

1. Determine p

Given:

$$
p = y_f - y_v = \text{distance from vertex to focus}
$$

Since the vertex is at:

$$
y_v = \frac{4 + c}{2}
$$

and the focus is at $y = 4$, then:

$$
p = 4 - y_v = 4 - \frac{4 + c}{2} = \frac{8 - (4 + c)}{2} = \frac{8 - 4 - c}{2} = \frac{4 - c}{2}
$$

2. Final equation of the parabola:

$$
(y - y_v) = \frac{1}{4p} (x - 3)^2
$$

which simplifies to:

$$
(y - y_v) = \frac{1}{4 \times \frac{4 - c}{2}} (x - 3)^2 = \frac{1}{2 (4 - c)} (x - 3)^2
$$

Complete Example: Find the Equation with Focus at (3, 4) and Directrix y = 2

Let's choose a specific directrix $y = 2$ to illustrate the process.

1. Find c:

$$
c = 2
$$

2. Calculate y_v:

$$
y_v = \frac{4 + 2}{2} = \ \frac{6}{2} = 3
$$

3. Calculate p:

$$
p = 4 - 3 = 1
$$

Since $p > 0$, the parabola opens upward.

4. Write the equation:

$$
(y - 3) = \frac{1}{4 \times 1} (x - 3)^2 = \frac{1}{4} (x - 3)^2
$$

Final equation:

$$
\boxed{(y - 3) = \frac{1}{4} (x - 3)^2}
$$

This parabola has focus at $(3, 4)$, directrix at $y = 2$, and vertex at $(3, 3)$.

General Form of the Equation of a Parabola with Given Focus and Directrix

Based on the steps above, the general form for the parabola with focus (x_f, y_f) and directrix $y = c$ (assuming the directrix is below the focus) is:

$$
\boxed{
(y - y_v) = \frac{1}{4p} (x - x_f)^2
}
$$

where:

- $y_v = \frac{ y_f + c }{2}$
- $p = y_f - y_v = \frac{ y_f - c }{2}$

Note: If the directrix is above the focus, the parabola opens downward, and p is negative.

Special Cases and Variations

1. Vertical Parabolas (Opening Up or Down)

- Focus and directrix aligned vertically.
- Equation form: $(x - h)^2 = 4p(y - k)$.

2. Horizontal Parabolas (Opening Left or Right)

- Focus and directrix aligned horizontally.
- Equation form: $(y - k)^2 = 4p(x - h)$.

3. Focus at Different Coordinates

- Adjust the derivation by considering the focus's position relative to the directrix.
- Follow similar steps to find the vertex, p, and write the standard form.

Practical Applications of Parabola Equations

Understanding how to derive the equation of a parabola from focus and directrix has numerous applications in various fields, including:

- Physics: Reflection and focus in satellite dishes and headlights.
- Engineering: Designing parabolic arches and bridges.
- Astronomy: Trajectories of projectiles under gravity.
- Mathematics Education: Enhancing comprehension of conic sections and quadratic functions.

Summary and Key

Frequently Asked Questions

How do you find the equation of a parabola given its focus at (3, 4) and a horizontal directrix Y?

To find the equation, first determine the directrix line's equation (Y = k). Then, use the definition of a parabola as the set of points equidistant from the focus and directrix. Set up the distance formula for a generic point (x, y) to the focus and directrix, and derive the parabola's equation accordingly.

What is the general method to derive the parabola's equation when given focus and directrix?

The method involves setting the distance from any point (x, y) on the parabola to the focus equal to its distance to the directrix. This results in an equation that can be simplified to the standard or vertex form of a parabola.

If the focus is at (3, 4) and the directrix is y = d, how do you determine the value of d?

The directrix's position depends on the parabola's orientation and the focus's location. If the parabola opens upward or downward, the directrix is a horizontal line at a distance equal to the focal length from the focus. To find d, use the fact that the focus is midway between the vertex and the directrix, and relate this to the parabola's parameters.

Can you provide the explicit equation of the parabola with focus at (3, 4) and directrix y = c?

Yes. If the directrix is y = c and the focus is at (3, 4), then the parabola's vertex is midway between the focus and directrix, at $y = (4 + c)/2$. The parabola opens upward if $c < 4$, downward if $c > 4$. The standard form can be derived as $(x - h)^2 = 4p(y - k)$, where (h, k) is the vertex, and p is the distance from the vertex to the focus.

How do you verify that the derived equation correctly represents the parabola with the given focus and directrix?

You can verify by selecting points on the parabola's equation and confirming that their distances to the focus and directrix are equal. Alternatively, substitute the focus coordinates into the equation to ensure it satisfies the parabola's definition.

Additional Resources

Question 10: Find the Equation of the Parabola With Its Focus at (3, 4) and Its Directrix y = ?

Understanding the geometric and algebraic properties of parabolas is fundamental in both pure mathematics and applied sciences. The problem of deriving the equation of a parabola given its focus and directrix is a classic exercise that combines coordinate geometry, algebra, and conic section theory. In this article, we will delve deeply into this problem, exploring the principles underpinning the parabola's structure, the step-by-step process for deriving its equation, and the broader implications of such an analysis.

Introduction to Parabolas: Focus and Directrix

A parabola is a symmetric, open curve that can be defined as the locus of points equidistant from a fixed point called the focus and a fixed line called the directrix. This geometric definition is foundational because it provides a direct link between the parabola's shape and its key components.

Focus:
The focus is a fixed point located inside the parabola. For a parabola opening upward or

downward, the focus lies along the axis of symmetry, which is a vertical line passing through the vertex.

Directrix:
The directrix is a fixed line located outside the parabola, serving as a reference line. The parabola's defining property is that any point on the parabola is equidistant from the focus and the directrix.

Key Relationship:
For any point $P(x, y)$ on the parabola, the distance to the focus $F(x_f, y_f)$ equals the perpendicular distance to the directrix line $y = y_d$.

Understanding the Given Data: Focus at (3, 4) and Directrix y = ?

The problem specifies that the focus of the parabola is at $(3, 4)$, but the directrix's equation is not provided—it is represented as $y = ?$. Typically, in standard problems, the directrix is either above or below the focus, depending on the parabola's orientation.

Given the focus at $(3, 4)$, common scenarios include:

- The parabola opens upward if the directrix is below the focus, i.e., $y = k$ with $y < 4$.

- The parabola opens downward if the directrix is above the focus, i.e., $y = k$ with $y > 4$.

Since the problem asks to find the equation given the focus and an unspecified directrix, the most logical assumption is that the directrix is a horizontal line, possibly $y = d$, where d is to be determined or given.

Assumption:
Let’s assume the directrix is a line $y = d$, with $d \neq 4$. The goal is to find the equation of the parabola with focus at $(3, 4)$ and directrix at $y = d$.

Mathematical Foundations: Deriving the Equation of the Parabola

The core property of a parabola is that for any point $P(x, y)$ on it, the distance to the focus equals the perpendicular distance to the directrix:

$$
\text{Distance to focus} = \text{Distance to directrix}
$$

Expressed mathematically:

$$
\sqrt{(x - x_f)^2 + (y - y_f)^2} = \text{distance from } (x, y) \text{ to } y = d
$$

Since the focus is at $(3, 4)$, and the directrix is at $y = d$, the distances are:

- To focus: $\sqrt{(x - 3)^2 + (y - 4)^2}$

- To directrix: the vertical distance $|y - d|$

The defining property becomes:

$$
\sqrt{(x - 3)^2 + (y - 4)^2} = |y - d|
$$

Step-by-Step Derivation of the Equation

Step 1: Remove the absolute value by considering the orientation

Since the parabola opens upward or downward depending on the relative position of the focus and directrix, we analyze the case for the parabola opening upward (which is the most common assumption in such problems).

Assuming $y > d$, then:

$$
\sqrt{(x - 3)^2 + (y - 4)^2} = y - d
$$

Step 2: Square both sides to eliminate the square root

$$
(x - 3)^2 + (y - 4)^2 = (y - d)^2
$$

Step 3: Expand both sides

Left side:

$$
(x - 3)^2 + y^2 - 8y + 16
$$

Right side:

$$
y^2 - 2dy + d^2
$$

Step 4: Simplify the equation

Subtract y^2 from both sides:

$$
(x - 3)^2 - 8y + 16 = -2dy + d^2
$$

Bring all terms to one side:

$$
(x - 3)^2 - 8y + 16 + 2dy - d^2 = 0
$$

Or:

\[

$$
(x - 3)^2 + (-8y + 2dy) + (16 - d^2) = 0
$$

Factor y terms:

$$
(x - 3)^2 + y(-8 + 2d) + (16 - d^2) = 0
$$

Step 5: Express y explicitly

Rearranged:

$$
y(-8 + 2d) = - (x - 3)^2 - (16 - d^2)
$$

$$
y = \frac{ - (x - 3)^2 - (16 - d^2) }{ -8 + 2d }
$$

Simplify numerator:

$$
y = \frac{ - (x - 3)^2 - 16 + d^2 }{ -8 + 2d }
$$

Divide numerator and denominator by 2 to simplify:

$$
y = \frac{ -\frac{1}{2}(x - 3)^2 - 8 + \frac{d^2}{2} }{ -4 + d }
$$

Determining the Directrix: Additional Constraints

To fully specify the parabola, the position of the directrix must be known or deduced. Commonly, the directrix is chosen symmetrically relative to the focus.

Given the focus at $(3, 4)$, the parabola's vertex lies midway between the focus and the directrix, along the axis of symmetry. The key is to identify the value of d—the directrix's line equation.

Possible scenarios:

- If the parabola opens upward, the directrix is below the focus: $y = d$, with $d < 4$.

- If the parabola opens downward, $y = d$, with $d > 4$.

Standard form:

The distance between focus and directrix is $2a$, where a is the focal length. The vertex is located at a point halfway between focus and directrix along the axis.

Choosing a Specific Directrix and Final Equation

To illustrate, assume the directrix is at $y = 2$, which is below the focus at $y=4$. The distance between focus and directrix:

$$
|4 - 2| = 2
$$

The vertex is midway at $y = 3$, with $x = 3$ (since the focus's x-coordinate is 3). This suggests the parabola opens upward with vertex at $(3, 3)$.

Standard form of a parabola opening upward:

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$$
(y - y_v) = \frac{1}{4a} (x - x_v)^2
$$

where (x_v, y_v) is the vertex, and a is the focal length (distance from vertex to focus).

Since the focus is at $(3, 4)$ and vertex at $(3, 3)$:

$$
a = \text{distance from vertex to focus} = 1
$$

Therefore, the parabola's equation is:

$$
y - 3 = \frac{1}{4} (x - 3)^2
$$

or equivalently:

$$
y = \frac{1}{4} (x - 3)^2 + 3
$$

General Form and Final Equation

This equation describes a parabola with focus at \((3,4) \) and directrix at \(y=2 \). Its key features include:

- Vertex: \((3, 3) \)

- Focus: \((3, 4) \)

- Directrix: \(y=2 \)

- Focal length \(a \): 1

- Equation:

\[
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