

Plot The Point Whose Cylindrical Coordinates Are Given. Then Find The Rectangular Coordinates Of The

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Understanding how to convert between different coordinate systems is fundamental in mathematics, physics, engineering, and computer graphics. Cylindrical coordinates offer a convenient way to describe points in three-dimensional space, especially when dealing with objects that have symmetry about a central axis, such as pipes, cylinders, and other rotationally symmetric shapes. In this article, we will explore how to plot a point given its cylindrical coordinates and then find its corresponding rectangular (Cartesian) coordinates. We will also delve into the concepts behind these coordinate systems, the formulas involved, and practical examples to solidify understanding.

Understanding Coordinate Systems

Before diving into conversions and plotting, it's essential to understand the two primary coordinate systems involved: rectangular (Cartesian) coordinates and cylindrical coordinates.

Rectangular (Cartesian) Coordinates

- Defined by three axes: x , y , and z .
- Any point in space is represented as (x, y, z) .
- Suitable for many applications but less effective in dealing with rotational symmetry.

Cylindrical Coordinates

- Combines a radial distance, an angular component, and a height.
- Defined as (r, θ, z) , where:
 - r is the radial distance from the z -axis.
 - θ (theta) is the angle measured from the positive x -axis to the projection of the point onto the xy -plane.
 - z is the same as in Cartesian coordinates, representing height along the z -axis.
- Useful when dealing with objects or phenomena with circular symmetry.

Plotting a Point in Cylindrical Coordinates

Given a set of cylindrical coordinates (r, θ, z) , plotting the point involves understanding these components:

Step-by-Step Process

1. Identify the parameters:

- Radial distance (r): How far the point is from the z -axis.
- Angle (θ): The direction of the point's projection onto the xy -plane.
- Height (z): Vertical position along the z -axis.

2. Locate the point's projection onto the xy -plane:

- Use polar coordinates to find the (x, y) components:
- $x = r \cos(\theta)$
- $y = r \sin(\theta)$

3. Plot the projection:

- Mark the point (x, y) in the xy -plane.

4. Move vertically along the z -axis:

- From the projection, move up or down along z to reach the point's height.

5. Mark the point in 3D space:

- The combination of (x, y, z) marks the location.

Visualization Tips

- Visualize the radial distance as the length of a line from the z -axis.
- Think of θ as the angle around the z -axis, similar to standard polar coordinates.
- The height z indicates elevation, making the point three-dimensional.

Converting Cylindrical Coordinates to Rectangular Coordinates

The key to this conversion lies in the formulas that relate (r, θ, z) to (x, y, z) .

Conversion Formulas

Given a point with cylindrical coordinates (r, θ, z) :

- $x = r \cos(\theta)$
- $y = r \sin(\theta)$
- $z = z$ (remains unchanged)

These formulas allow for straightforward conversion and are fundamental in many applications.

Example Calculation

Suppose a point has cylindrical coordinates:

- $r = 5$
- $\theta = 30^\circ$

- $z = 10$

Step 1: Convert θ to radians (if necessary):

- $\theta = 30^\circ = \pi/6$ radians

Step 2: Calculate x and y :

- $x = 5 \cos(\pi/6) = 5 (\sqrt{3}/2) \approx 5 \cdot 0.8660 \approx 4.33$

- $y = 5 \sin(\pi/6) = 5 (1/2) = 2.5$

Step 3: z remains the same:

- $z = 10$

Result: The rectangular coordinates are approximately (4.33, 2.5, 10).

Practical Applications of Coordinate Conversion

Converting between cylindrical and rectangular coordinates is essential in various fields:

Engineering and Design

- Designing cylindrical components like pipes or tanks often involves specifying points in cylindrical coordinates, which then need to be converted for manufacturing or computer modeling.

Physics and Electromagnetism

- Magnetic fields around wires or solenoids are naturally described in cylindrical coordinates, making conversions necessary for calculations and simulations.

Computer Graphics and Visualization

- Rendering 3D objects with rotational symmetry often requires coordinate conversions to accurately depict scenes.

Robotics and Navigation

- Robots operating in environments with circular paths may use cylindrical coordinates for path planning, converting to rectangular coordinates for precise movement commands.

Additional Considerations in Coordinate Conversion

While the basic formulas are straightforward, some practical considerations include:

- **Angle measurement:** Ensure θ is in radians when applying trigonometric functions in most programming languages.
- **Negative r values:** In standard cylindrical coordinates, r is usually non-negative. If given a negative r , adjust θ by π radians and take r as positive.
- **Multiple representations:** Since $\cos(\theta)$ and $\sin(\theta)$ are periodic, the same point can have multiple (r, θ, z) representations, differing by multiples of 2π in θ .

Common Mistakes to Avoid

- Using degrees instead of radians: Many calculators and programming languages require angles in radians.
- Misinterpreting θ : Remember that θ is measured from the positive x-axis in the xy-plane.
- Ignoring the sign of r : Positive r values are standard; negative r can complicate interpretation.
- Forgetting z remains the same: The height coordinate z is unaffected by the (r, θ) conversion.

Additional Examples and Practice Problems

To reinforce understanding, here are some practice problems:

1. Given cylindrical coordinates $(r=3, \theta=120^\circ, z=4)$, find the rectangular coordinates.
2. Plot the point with rectangular coordinates $(x=6, y=6, z=2)$. Convert to cylindrical coordinates.
3. Determine the cylindrical coordinates of a point at $(x=0, y=10, z=5)$.

Solutions:

1. Convert θ to radians: $120^\circ = 2\pi/3$ radians
 - $r = 3$
 - $x = 3 \cos(2\pi/3) = 3 (-1/2) = -1.5$
 - $y = 3 \sin(2\pi/3) = 3 (\sqrt{3}/2) \approx 3 \cdot 0.8660 \approx 2.598$
 - $z = 4$
 - Cylindrical coordinates: $(r=3, \theta=120^\circ, z=4)$
2. Given $(x=6, y=6, z=2)$:
 - $r = \sqrt{6^2 + 6^2} = \sqrt{36 + 36} = \sqrt{72} \approx 8.485$
 - $\theta = \arctangent(y/x) = \arctangent(6/6) = \arctangent(1) = 45^\circ = \pi/4$ radians
 - $z = 2$
 - Cylindrical coordinates: approximately $(8.485, 45^\circ, 2)$

3. For $(x=0, y=10, z=5)$:

- $r = \sqrt{0^2 + 10^2} = 10$

- $\theta = \arctangent(10/0) \rightarrow \theta = 90^\circ$ (or $\pi/2$ radians)

- $z = 5$

- Cylindrical coordinates: $(10, 90^\circ, 5)$

Conclusion

Mastering the conversion between cylindrical and rectangular coordinates is a vital skill that enhances spatial understanding and problem-solving in mathematics and applied sciences. Plotting a point in cylindrical coordinates involves understanding the radial distance, angular component, and height, which collectively describe the point's position in three-dimensional space. Converting to rectangular coordinates requires applying the fundamental formulas, enabling integration with various analytical and computational tools. Whether designing engineering components, analyzing physical phenomena, or creating visualizations, proficiency in these conversions facilitates accurate and efficient work across disciplines.

By practicing these concepts and understanding their applications, you can confidently navigate between coordinate systems and leverage their respective advantages for your specific needs.

Frequently Asked Questions

What are cylindrical coordinates and how are they represented?

Cylindrical coordinates are a 3D coordinate system represented by (r, θ, z) , where r is the radius from the z -axis, θ is the angle in the xy -plane from the positive x -axis, and z is the height along the z -axis.

How do you convert cylindrical coordinates to rectangular coordinates?

To convert cylindrical coordinates (r, θ, z) to rectangular coordinates (x, y, z) , use the formulas: $x = r \cos \theta$, $y = r \sin \theta$, and $z = z$.

Given cylindrical coordinates (r, θ, z) , how can I plot the point in 3D space?

First, convert the cylindrical coordinates to rectangular coordinates using $x = r \cos \theta$, $y = r \sin \theta$, and $z = z$. Then, plot the point at (x, y, z) in 3D space.

What is the importance of the angle θ in cylindrical coordinates?

The angle θ determines the direction of the point from the positive x -axis in the xy -plane, effectively

locating the point around the z-axis.

Can you provide an example of plotting a point given cylindrical coordinates?

Yes. For example, given $(r=3, \theta=60^\circ, z=4)$, convert to rectangular: $x=3 \cos 60^\circ=1.5$, $y=3 \sin 60^\circ \approx 2.598$, $z=4$. So, the point is at $(1.5, 2.598, 4)$.

What are common mistakes when converting cylindrical coordinates to rectangular coordinates?

Common mistakes include mixing up the angles in degrees and radians, using incorrect formulas for conversion, or miscalculating the trigonometric functions.

How can I verify that my rectangular coordinates are correct after conversion?

You can verify by converting your rectangular coordinates back to cylindrical coordinates and checking if they match the original given values.

How does changing the value of r or θ affect the position of the point in space?

Increasing r moves the point further from the z-axis, while changing θ rotates the point around the z-axis. Changing z moves the point vertically along the z-axis.

Why is understanding the relationship between cylindrical and rectangular coordinates useful?

Understanding this relationship helps in visualizing points in 3D space, solving problems in cylindrical symmetry, and simplifying calculations in various fields like engineering and physics.

Additional Resources

Plot The Point Whose Cylindrical Coordinates Are Given. Then Find The Rectangular Coordinates Of The

Understanding how to plot the point whose cylindrical coordinates are given and then find the rectangular coordinates of the same point is a fundamental skill in multivariable calculus, physics, and engineering. Cylindrical coordinates provide an alternative way of describing points in three-dimensional space, especially useful when dealing with problems exhibiting cylindrical symmetry. Transitioning between cylindrical and rectangular (Cartesian) coordinates allows for greater flexibility in problem-solving and visualization.

In this comprehensive guide, we'll explore the concepts behind cylindrical coordinates, step through a detailed example, and provide a step-by-step process to convert and plot points between these two

coordinate systems effectively.

Understanding Cylindrical Coordinates

What Are Cylindrical Coordinates?

Cylindrical coordinates are a three-dimensional coordinate system described by three values:

- r (radius): The distance from the z -axis to the point projected onto the xy -plane.
- θ (theta): The angle measured counterclockwise from the positive x -axis to the line connecting the origin to the projection of the point onto the xy -plane.
- z (height): The same as the Cartesian z -coordinate, representing the height above the xy -plane.

This system is particularly advantageous when dealing with problems involving circular or cylindrical symmetry, such as in electromagnetism, fluid flow, and mechanical systems.

Relationship Between Cylindrical and Rectangular Coordinates

The conversion formulas linking the two systems are:

- From cylindrical to rectangular:

- $x = r \cos(\theta)$
- $y = r \sin(\theta)$
- $z = z$

- From rectangular to cylindrical:

- $r = \sqrt{x^2 + y^2}$
- $\theta = \arctan(y / x)$ (considering the quadrant)
- $z = z$

Plotting a Point in Cylindrical Coordinates

Step 1: Understand the Given Coordinates

Suppose you are given a point in cylindrical coordinates, such as:

- $r = 5$
- $\theta = 45^\circ$
- $z = 3$

Your first task is to interpret these values geometrically.

Step 2: Convert to Radians (If Necessary)

Since most mathematical functions use radians, convert θ from degrees to radians:

- $\theta = 45^\circ = \pi/4$ radians ≈ 0.7854 radians

Step 3: Find the Cartesian Coordinates

Using the conversion formulas:

- $x = r \cos(\theta) = 5 \cos(\pi/4) = 5 (\sqrt{2} / 2) \approx 3.5355$

- $y = r \sin(\theta) = 5 (\sqrt{2} / 2) \approx 3.5355$

- $z = 3$

Resulting rectangular coordinates: $(x, y, z) \approx (3.5355, 3.5355, 3)$

Step 4: Plot the Point

- Begin by plotting the projection of the point onto the xy-plane at (3.5355, 3.5355).

- From the origin, draw a line at an angle θ (45°) and mark a distance r (5 units).

- From this point, move vertically upward to $z = 3$.

- Connect these points to visualize the 3D position.

Finding the Rectangular Coordinates From Cylindrical Coordinates

Step 1: Write Down the Formulae

Recall the conversion formulae:

- $x = r \cos(\theta)$

- $y = r \sin(\theta)$

- $z = z$

Step 2: Plug in the Values

For the given example:

- $r = 5$

- $\theta = \pi/4$ radians

- $z = 3$

Calculate:

- $x = 5 \cos(\pi/4) \approx 3.5355$

- $y = 5 \sin(\pi/4) \approx 3.5355$

- $z = 3$

Step 3: Verify the Coordinates

Check that the coordinates make sense geometrically:

- The distance from the origin in the xy-plane: $\sqrt{(3.5355)^2 + (3.5355)^2} \approx 5$, matching r .

- The angle θ is consistent with the coordinates.

- The height z remains unchanged.

Step-by-Step Guide: Plotting and Conversion

Step 1: Identify the Given Coordinates

- Confirm the values of r , θ (degree or radians), and z .
- Convert θ to radians if necessary.

Step 2: Find the Rectangular Coordinates

- Use the formulas:
 - $x = r \cos(\theta)$
 - $y = r \sin(\theta)$
 - $z = z$
- Calculate each coordinate accurately.

Step 3: Plot in 3D Space

- Plot the projection on the xy -plane at (x, y) .
- Draw a line from the origin to (x, y) at the angle θ .
- Extend vertically to z .
- Connect the points to visualize the location in 3D space.

Step 4: Confirm the Conversion

- Ensure the conversion matches the original cylindrical coordinates.
- Use the Pythagorean theorem in the xy -plane as a consistency check.

Practical Tips for Working with Cylindrical Coordinates

- Always keep track of units and angle measurement modes (degrees vs. radians).
- Use a calculator or software that handles inverse trigonometric functions carefully, paying attention to quadrants.
- Visualize the point in 3D to develop spatial intuition.
- Remember that the z -coordinate remains unchanged when converting between systems.

Examples for Practice

Example 1:

Given cylindrical coordinates: $r = 3$, $\theta = 120^\circ$, $z = 2$

Conversion steps:

- θ in radians: $120^\circ \times \pi/180 = 2\pi/3 \approx 2.0944$ radians
- $x = 3 \cos(2.0944) \approx 3 (-0.5) = -1.5$
- $y = 3 \sin(2.0944) \approx 3 (\sqrt{3}/2) \approx 3 \cdot 0.8660 \approx 2.598$
- $z = 2$

Rectangular coordinates: $(-1.5, 2.598, 2)$

Example 2:

Given rectangular coordinates: $(x, y, z) = (4, -4, 5)$

Conversion to cylindrical:

- $r = \sqrt{4^2 + (-4)^2} = \sqrt{16 + 16} = \sqrt{32} \approx 5.657$
- $\theta = \arctangent(y / x) = \arctangent(-4 / 4) = \arctangent(-1) \approx -45^\circ$ or $-\pi/4$ radians
- $z = 5$

Note: Since $x > 0$ and $y < 0$, θ is in the fourth quadrant, so:

- $\theta \approx 360^\circ - 45^\circ = 315^\circ$, or in radians: $7\pi/4$

Final Thoughts

Mastering the process of plotting points in cylindrical coordinates and converting to rectangular coordinates enhances your ability to visualize complex spatial relationships and simplifies solving many physics and engineering problems. Remember that the key is understanding the geometric interpretation of each coordinate system and practicing the conversion formulas until they become second nature.

Whether you're working on a homework problem, designing a mechanical component, or analyzing electromagnetic fields, being proficient in these conversions and visualizations will serve as a vital tool in your mathematical toolkit.

Happy plotting!

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