Polygon ABCD With Vertices At A(4, 6), B(2, 2), C(4, 2), And D(4, 4) Is Dilated Using A Scale Factor

Polygon ABCD With Vertices At A(4, 6), B(2, 2), C(4, 2), And D(4, 4) Is Dilated Using A Scale Factor

Understanding the properties and transformations of polygons is a fundamental aspect of geometry. Among these transformations, dilation plays a crucial role in changing the size of a figure while maintaining its shape and proportionality. In this article, we explore the dilation of a specific quadrilateral, Polygon ABCD, with vertices at A(4, 6), B(2, 2), C(4, 2), and D(4, 4), using a specified scale factor. We will delve into the concept of dilation, how to perform it on polygons, and analyze the resulting figures through a step-by-step approach.

Understanding Polygon ABCD and Its Coordinates

Before discussing the dilation process, it is essential to understand the initial polygon’s structure and coordinates.

Vertices of Polygon ABCD

- A(4, 6): Located at x=4, y=6
- B(2, 2): Located at x=2, y=2
- C(4, 2): Located at x=4, y=2
- D(4, 4): Located at x=4, y=4

Visualizing the Polygon

- The points B and C are horizontally aligned at y=2.
- Points A and D are vertically aligned at x=4.
- The shape is a quadrilateral with a specific orientation, resembling a "L" shape or a right-angled figure, depending on the order of vertices.

Determining the Polygon’s Shape

- Connecting the vertices in order A → B → C → D → A results in a shape that can be analyzed for its properties.
- The shape's dimensions and angles can be deduced from the coordinates to understand how dilation affects it.

Concept of Dilation in Geometry

Dilation is a transformation that enlarges or reduces a figure by a common scale factor relative to a fixed point called the center of dilation.

What is Dilation?

- A transformation that produces a similar figure but with a different size.
- The shape remains congruent; only the size changes.
- The scale factor determines whether the figure is enlarged (>1) or reduced (<1).

Center of Dilation

- The fixed point from which the figure is dilated.
- Commonly chosen as the origin (0,0), but can be any point in the plane.

Scale Factor

- A positive real number.
- Scale Factor > 1: Enlargement of the figure.
- Scale Factor < 1: Reduction of the figure.
- Scale Factor = 1: No change.

Mathematical Formula for Dilation

Given a point $P(x, y)$, dilated from a center $C(h, k)$ with scale factor r, the new point $P'(x', y')$ is calculated as:

$$
x' = h + r(x - h)
$$
$$
y' = k + r(y - k)
$$

Performing the Dilation of Polygon ABCD

To dilate Polygon ABCD, follow a systematic approach that involves applying the dilation formula to each vertex.

Step 1: Choose the Center of Dilation

- The most common choice is the origin $(0,0)$.
- Alternatively, the centroid or any specific point can be selected based on the problem's context.
- For simplicity, we assume the center of dilation is at the origin (0, 0).

Step 2: Select the Scale Factor

- The scale factor r can be any positive real number.
- For this example, let's assume a scale factor of 2 (doubling the size).

Step 3: Apply the Dilation Formula to Each Vertex

- Calculate the new coordinates for each vertex using the formula:

$$
x' = h + r(x - h)
$$
$$
y' = k + r(y - k)
$$

- Since the center is at (0, 0):

$$
x' = r \times x
$$
$$
y' = r \times y
$$

- Applying to each vertex:

Vertex	Original Coordinates	Dilated Coordinates (r=2)
A	(4, 6)	(8, 12)
B	(2, 2)	(4, 4)
C	(4, 2)	(8, 4)
D	(4, 4)	(8, 8)

Analyzing the Resulting Dilation

After performing the dilation, the new polygon $A'B'C'D'$ has vertices at the following points:

- A'(8, 12)

- B'(4, 4)
- C'(8, 4)
- D'(8, 8)

Shape and Size Comparison

- The new figure is similar to the original, preserving angles and side proportions.
- The size is scaled by the factor of 2.
- The shape's orientation and shape remain unchanged.

Properties of the Dilated Polygon

- The edges are twice as long as the original.
- The coordinates are proportionally increased.
- The polygon remains convex if the original was convex.

Visual Representation

- Plotting both the original and dilated polygons on a coordinate plane illustrates the effect of dilation.
- It demonstrates how the figure expands outward from the origin.

Alternative Centers of Dilation

While the previous example assumes the origin as the center, other choices can be used to change the dilation's effect.

Center at a Vertex

- For example, dilating with the center at point A(4, 6):

$$
x' = 4 + r(x - 4)
$$
$$
y' = 6 + r(y - 6)
$$

- This results in the figure expanding or contracting relative to point A, affecting the overall shape's position.

Center at the Polygon’s Centroid

- The centroid (x_c, y_c) can be computed as the average of the vertices:

$$
x_c = \frac{x_A + x_B + x_C + x_D}{4}
$$
$$
y_c = \frac{y_A + y_B + y_C + y_D}{4}
$$

- The centroid for the original polygon:

$$
x_c = \frac{4 + 2 + 4 + 4}{4} = \frac{14}{4} = 3.5
$$
$$
y_c = \frac{6 + 2 + 2 + 4}{4} = \frac{14}{4} = 3.5
$$

- Dilation about the centroid:

$$
x' = x_c + r(x - x_c)
$$
$$
y' = y_c + r(y - y_c)
$$

- Using this method provides a different perspective on how the figure scales and moves.

Applications of Dilation in Geometry and Real Life

Understanding dilation is not only fundamental in academic settings but also has practical applications in various fields.

In Geometry and Mathematics

- Similar figures and their properties.
- Coordinate geometry and transformation analysis.
- Construction of geometric proofs and models.

In Engineering and Design

- Scaling blueprints and models.

- Computer graphics and image resizing.
- Architectural planning and model scaling.

In Art and Animation

- Creating proportional enlargements.
- Transforming objects while maintaining shape integrity.

Educational Uses

- Teaching proportional reasoning.
- Visualizing geometric transformations.

Conclusion

Dilation is a vital concept in understanding how figures can be scaled proportionally while maintaining their shape. In the case of Polygon ABCD with vertices at A(4, 6), B(2, 2), C(4, 2), and D(4, 4), applying a dilation with a specific scale factor—whether centered at the origin, a vertex, or the centroid—alters its size but preserves its form. The process involves applying the dilation formula to each coordinate, leading to a similar, larger or smaller polygon depending on the scale factor used. Mastery of this transformation enhances spatial reasoning and provides essential tools for advanced geometry, engineering, art, and design.

By understanding the properties and methods of dilation, students and professionals can manipulate figures accurately and efficiently, whether for academic purposes or real-world applications.

Frequently Asked Questions

What are the coordinates of the vertices of polygon ABCD?

The vertices are A(4, 6), B(2, 2), C(4, 2), and D(4, 4).

What does it mean to dilate a polygon using a scale factor?

Dilating a polygon using a scale factor involves enlarging or reducing the polygon proportionally from a fixed point, called the center of dilation, by a specific scale factor.

Which point is typically used as the center of dilation for polygon ABCD?

The problem does not specify a center of dilation, so it is often assumed to be the origin (0,0) unless otherwise stated.

How does a scale factor affect the size of polygon ABCD?

A scale factor greater than 1 enlarges the polygon, while a scale factor between 0 and 1 reduces its size proportionally.

How can you find the coordinates of the dilated polygon's vertices?

Multiply the distance from each vertex to the center of dilation by the scale factor along the line connecting the vertex and the center.

If the scale factor is 2, what will be the coordinates of the dilated vertices?

Assuming the center of dilation is at the origin, the vertices will be A(8, 12), B(4, 4), C(8, 4), and D(8, 8).

What is the importance of knowing the original vertices when dilating a polygon?

Knowing the original vertices allows you to accurately calculate the new vertices after dilation, ensuring the shape is proportionally enlarged or reduced.

Can the dilation change the shape of polygon ABCD?

No, dilation is a similarity transformation that preserves the shape and angles, only changing the size.

How would you apply a scale factor of 0.5 to polygon ABCD?

Multiply the distance from each vertex to the center by 0.5 to find the new, smaller coordinates, assuming a center of dilation at the origin.

What are some real-world applications of dilating polygons in geometry?

Dilations are used in art, engineering, computer graphics, and map scaling to resize objects proportionally without distorting their shape.

Additional Resources

Understanding the transformation of polygons is fundamental in various fields such as geometry, computer graphics, engineering, and design. One common transformation is polygon ABCD with vertices at A(4, 6), B(2, 2), C(4, 2), and D(4, 4) being dilated using a scale factor. This process involves resizing the polygon either larger or smaller while maintaining its shape and proportions. In this comprehensive guide, we will explore the principles behind dilation, analyze the specific polygon in question, and step through the dilation process with detailed explanations. Whether you're a student, educator, or professional, this article aims to clarify the concepts and procedures involved in dilating a polygon.

Understanding Polygon ABCD and Its Coordinates

Before delving into the dilation process, it’s essential to understand the initial shape and coordinates of the polygon.

Vertices of Polygon ABCD

- A(4, 6)
- B(2, 2)
- C(4, 2)
- D(4, 4)

These points define a quadrilateral in the Cartesian plane. To visualize it:

- Point A is located at (4, 6), which is relatively high on the y-axis.
- Point B is at (2, 2), lower and to the left.
- Point C is at (4, 2), directly below A, aligned vertically.
- Point D is at (4, 4), somewhere between A and C vertically.

Plotting these points helps in understanding the shape. The shape is somewhat irregular, but with the given coordinates, we can analyze its properties.

Analyzing the Original Polygon

Shape and Orientation

The vertices suggest a quadrilateral with the following approximate shape:

- The left side is at x = 2 (B).
- The right side is at x = 4 (A, C, D).
- The topmost point is A at (4,6).
- The bottommost points are at y=2 (B and C).

Connecting the points in order (A, B, C, D) forms the polygon. The order of vertices is important for understanding the shape and performing transformations.

Calculating Side Lengths

To grasp the scale of the shape, compute some side lengths:

- AB: Distance between A(4,6) and B(2,2)

$$
\sqrt{(4-2)^2 + (6-2)^2} = \sqrt{2^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} \approx 4.47
$$

- BC: Distance between B(2,2) and C(4,2)

$$
\sqrt{(4-2)^2 + (2-2)^2} = \sqrt{2^2 + 0} = 2
$$

- CD: Distance between C(4,2) and D(4,4)

$$
\sqrt{(4-4)^2 + (4-2)^2} = \sqrt{0 + 2^2} = 2
$$

- DA: Distance between D(4,4) and A(4,6)

$$
\sqrt{(4-4)^2 + (6-4)^2} = \sqrt{0 + 2^2} = 2
$$

This indicates that the shape has a longer side (AB) and three shorter sides of length 2, suggesting a shape with a longer diagonal and three sides of equal length.

Understanding Dilation in Geometry

What Is Dilation?

Dilation is a transformation that resizes a figure proportionally from a fixed point called the center of dilation. The size of the figure changes based on the scale factor:

- If the scale factor > 1, the figure enlarges.
- If the scale factor < 1, the figure shrinks.
- If the scale factor = 1, the figure remains unchanged.

The dilation preserves the shape and angles of the figure but alters its size.

Center of Dilation

The center of dilation is the fixed point about which all points are moved during the transformation. Common choices are the origin (0,0) or a specific point like the centroid of the polygon.

Mathematical Formula

For a point $P(x, y)$, the dilated point $P'(x', y')$ with center $C(h, k)$ and scale factor k is:

$$
x' = h + k(x - h)
$$
$$
y' = k(y - k)
$$

If the center is at the origin (0,0), the formula simplifies to:

$$
x' = kx
$$
$$
y' = ky
$$

Performing the Dilation of Polygon ABCD

Now that we understand the basics, let's outline the steps to dilate polygon ABCD with a given scale factor.

Step 1: Determine the Center of Dilation

The choice of center depends on the problem's context. Common centers include:

- The origin (0,0)
- The centroid of the polygon
- A vertex or any specific point

For simplicity, we'll assume the center of dilation is the origin (0,0), unless specified otherwise.

Step 2: Choose the Scale Factor

Suppose the scale factor is k. For example, if the problem states "using a scale factor of 2," then:

\[

k = 2
\]
This means the polygon will double in size relative to the origin.

Step 3: Apply the Dilation Formula to Each Vertex

Using the formula for dilation about the origin:

\[
x' = kx
\]
\[
y' = ky
\]

Apply this to each vertex:

- A(4, 6):
\[
A' = (4 \times 2, 6 \times 2) = (8, 12)
\]

- B(2, 2):
\[
B' = (2 \times 2, 2 \times 2) = (4, 4)
\]

- C(4, 2):
\[
C' = (4 \times 2, 2 \times 2) = (8, 4)
\]

- D(4, 4):
\[
D' = (4 \times 2, 4 \times 2) = (8, 8)
\]

Interpreting the Dilated Polygon

After applying the scale factor, the new vertices are:

- A'(8, 12)
- B'(4, 4)
- C'(8, 4)

- D'(8, 8)

Plotting these points:

- The shape has expanded proportionally.
- The vertices are now farther from the origin, maintaining the same relative proportions.
- The shape's angles and side ratios remain consistent with the original.

Optional: Dilation About a Different Center

If the problem specifies a different center, such as a point $C(h, k)$, the formula adjusts accordingly:

$$
x' = h + k(x - h)
$$
$$
y' = k(y - k)
$$

For example, dilating about point $C(4, 2)$ with scale factor 2:

- For A(4, 6):
$$
x' = 4 + 2(4 - 4) = 4 + 0 = 4
$$
$$
y' = 2 + 2(6 - 2) = 2 + 2 \times 4 = 2 + 8 = 10
$$
So, A' = (4, 10)

- Proceed similarly for other vertices.

Visualizing the Transformation

Graphing both the original and dilated polygons provides valuable insight:

- The original polygon ABCD has vertices at the specified coordinates.
- The dilated polygon appears proportionally larger (or smaller, depending on scale factor).
- The shape is preserved, but all points move away from or toward the center.

Visualization aids in understanding how dilation impacts the overall geometry.

Summary of Key Steps in Dilating a Polygon

1. Identify the vertices of the original polygon.
2. Select the center of dilation (origin, centroid, or specific point).
3. Choose the scale factor (k) based on the desired size change.
4. Apply the dilation formula to each vertex:
- About the origin: multiply each coordinate by \(k\).
- About a specific center: use \(x' = h + k(x - h)\), \(y' = k(y - k)\).
5. Plot the new vertices to visualize the dilated polygon.
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Polygon Abcd With Vertices At A 4 6 B 2 2 C 4 2 And D 4 4 Is Dilated Using A Scale Factor

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