Produce The Equation Of The Line Tangent Of The Given Function At The Specified Point. Y=x2ex P(1 E)

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When studying calculus and its applications, one of the fundamental concepts is understanding the tangent line to a curve at a particular point. The tangent line provides a linear approximation of the function near that point and plays a crucial role in differential calculus. In this article, we will explore how to produce the equation of the tangent line to the given function $y = x^2 e^x$ at the specific point $P(1, e)$. We will discuss the necessary steps, including calculating the derivative, evaluating it at the point, and applying the point-slope form of a line. This comprehensive guide aims to deepen your understanding of tangents and their significance in mathematical analysis.

Understanding the Problem

Before diving into calculations, it's essential to interpret the problem correctly:

- Function: $y = x^2 e^x$
- Point: $P(1, e)$

The goal is to determine the equation of the tangent line to the curve $y = x^2 e^x$ at the point where $x = 1$, which corresponds to the point $(1, e)$.

Key Concepts to Review:

- Derivative: Measures the rate of change of the function at a specific point.
- Tangent line: A straight line that touches the curve at a point and has the same slope as the curve at that point.
- Point-slope form: Equation of a line given a point (x_0, y_0) and slope m: $y - y_0 = m(x - x_0)$.

Step-by-Step Process to Find the Tangent Line Equation

To produce the equation of the tangent line, follow these fundamental steps:

1. Verify the Point Lies on the Function
2. Calculate the Derivative y' of the Function
3. Evaluate the Derivative at $x=1$ to Find the Slope m
4. Use the Point-Slope Form to Write the Equation of the Tangent Line

Let's explore each step in detail.

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1. Confirm the Point is on the Curve

Given $P(1, e)$, verify that this point lies on the curve:

$$
y = x^2 e^x
$$

At $x=1$:

$$
y = (1)^2 \times e^{1} = 1 \times e = e
$$

Since the point $(1, e)$ matches the function value, it indeed lies on the curve.

---

2. Calculate the Derivative $y' = \frac{dy}{dx}$

The function:

$$
y = x^2 e^x
$$

is a product of two functions: $u = x^2$ and $v = e^x$. Use the product rule:

$$
\frac{d}{dx} [u \times v] = u' v + u v'
$$

Compute derivatives:

- $u' = 2x$
- $v' = e^x$

Thus,

$$
y' = u' v + u v' = 2x e^x + x^2 e^x
$$

Factor out e^x:

$$
y' = e^x (2x + x^2)
$$

Simplify:

$$
y' = e^x (x^2 + 2x)
$$

This expression gives the slope of the tangent line at any point x on the curve.

---

3. Find the Slope m at $x=1$

Evaluate y' at $x=1$:

$$
m = y'(1) = e^{1} (1^2 + 2 \times 1) = e (1 + 2) = 3e
$$

So, the slope of the tangent line at $P(1, e)$ is $3e$.

---

4. Write the Equation of the Tangent Line

Using the point-slope form:

$$
y - y_0 = m (x - x_0)
$$

where $(x_0, y_0) = (1, e)$ and $m = 3e$:

$$
y - e = 3e (x - 1)
$$

Expanding:

$$
y = 3e (x - 1) + e
$$

Distribute:

$$
y = 3e x - 3e + e = 3e x - 2e
$$

Final Equation:

$$
\boxed{
\textbf{y} = 3e x - 2e
}
$$

Additional Insights and Applications

Understanding how to produce the tangent line to a function at a specific point is a fundamental skill in calculus with numerous applications:

- Linear Approximation: The tangent line provides a local linear approximation to the function near the point.
- Optimization: Helps identify maxima and minima by analyzing slope changes.
- Curve Sketching: Assists in understanding the behavior of functions visually.
- Physics Applications: Slope corresponds to velocity or rate of change in physical systems.

Common Mistakes and How to Avoid Them

When working with tangent lines and derivatives, be mindful of the following pitfalls:

- Forgetting the Chain Rule or Product Rule: Always verify which rule applies when differentiating complex functions.
- Incorrectly Evaluating Derivatives: Double-check calculations at the specific point.
- Mixing Coordinates: Ensure the point used in the tangent line corresponds to the derivative's evaluation point.
- Misapplying the Point-Slope Formula: Remember to insert the correct point and slope.

Practice Problems for Mastery

To solidify your understanding, try solving these problems:

1. Find the equation of the tangent line to $y = \sin x$ at $x = \pi/2$.

2. Determine the tangent line to $y = \ln x$ at $x=1$.

3. Calculate the tangent line to $y = \frac{1}{x}$ at $x=2$.

Conclusion

Producing the equation of a tangent line to a function at a specific point involves understanding the function's derivative, evaluating it at the point of interest, and applying the point-slope form of a line. For the function $y = x^2 e^x$ at $P(1, e)$, we found the derivative, evaluated it to obtain the slope $3e$, and then derived the tangent line equation $y = 3e x - 2e$.

Mastering this process enhances your calculus proficiency, enabling you to analyze function behavior, approximate values, and solve real-world problems involving rates of change. Remember, practice is key to becoming confident in calculating tangent lines for various functions.

Keywords: tangent line, calculus, derivatives, point-slope form, function analysis, linear approximation, differential calculus, curve tangent, slope calculation, mathematical analysis

Frequently Asked Questions

How do I find the equation of the tangent line to the function $y = x^2e^x$ at the point P(1, e)?

First, compute the derivative $y' = d/dx[x^2e^x]$ using the product rule. Then, evaluate y' at $x=1$ to find the slope m. Next, find y at $x=1$, which is $y= e$. Finally, use point-slope form: $y - e = m(x - 1)$.

What is the derivative of $y = x^2e^x$ and how does it help in finding the tangent line?

The derivative $y' = 2x\, e^x + x^2\, e^x = e^x (2x + x^2)$. It gives the slope of the tangent line at any point x, allowing us to find the tangent line's equation at a specific point.

At the point P(1, e), what is the slope of the tangent line to $y = x^2e^x$?

Substitute $x=1$ into $y' = e^x (2x + x^2)$: $y' = e^1(21 + 1^2) = e(2 + 1) = 3e$. So, the slope is 3e.

How do I find the y-coordinate at x=1 for the function $y = x^2e^x$?

Substitute $x=1$ into $y = x^2e^x$: $y = 1^2\, e^1 = e$. The point P(1, e) confirms this value.

What is the step-by-step process to write the equation of the tangent line at P(1, e)?

1. Find the derivative y' and evaluate at $x=1$ to get the slope $m = 3e$. 2. Use the point P(1, e). 3. Apply point-slope form: $y - e = 3e(x - 1)$.

Can you provide the final equation of the tangent line to $y = x^2e^x$ at P(1, e)?

Yes. Using $y - e = 3e(x - 1)$, the equation of the tangent line is $y = 3e(x - 1) + e$.

Why is the product rule necessary when differentiating $y = x^2e^x$?

Because $y = x^2e^x$ is a product of two functions, x^2 and e^x, so the product rule is needed to correctly differentiate it.

How do I verify that the tangent line touches the curve at the given point?

Ensure the point P(1, e) lies on both the original function and the tangent line. Plug $x=1$ into the tangent line equation; it should give $y=e$, matching the point on the curve.

Additional Resources

Producing the Equation of the Tangent Line to a Function at a Specific Point: A Comprehensive Guide

Understanding how to find the equation of a tangent line to a curve at a given point is a fundamental skill in calculus. It combines concepts of differentiation, point-slope forms, and the properties of functions to produce a precise linear approximation of the function at a specific point. This detailed guide will walk through the process step-by-step, using the function \(y = x^2 e^x \) at the point \(P(1, e) \), to illustrate each phase thoroughly.

Introduction to Tangent Lines and Their Significance

Before diving into calculations, it's crucial to understand what a tangent line represents and why it is important.

What Is a Tangent Line?

- The tangent line to a curve at a specific point is the straight line that touches the curve precisely at that point.
- It has the same slope as the curve's instantaneous rate of change at that point.
- Geometrically, the tangent line "just touches" the curve and does not cross it at the point of tangency (though it may cross elsewhere).

Why Find the Equation of the Tangent Line?

- To approximate the behavior of a function near a point.
- To understand the function's local linearity.
- To solve optimization problems, analyze rates, or develop linear models in applied mathematics.

Core Concepts and Mathematical Foundations

Successfully deriving a tangent line equation involves some key mathematical concepts:

Derivative as the Slope of the Tangent

- The derivative $y' = \frac{dy}{dx}$ gives the slope of the tangent line at any point x.
- For a function $y = f(x)$, the derivative is computed using differentiation rules suited to the function's form.

Point-Slope Form of a Line

- The general equation for a line with slope m passing through (x_0, y_0):

$$
y - y_0 = m (x - x_0)
$$

- When finding the tangent line, m is the derivative at x_0, and (x_0, y_0) is the point of tangency.

Step-by-Step Derivation for the Given Function

Using the function $y = x^2 e^x$ at the point $P(1, e)$:

Step 1: Confirm the Point of Tangency

- The point given is $P(1, e)$.
- Verify that the function passes through this point:

$$
y(1) = (1)^2 \times e^{1} = 1 \times e = e
$$

Yes, the point $(1, e)$ lies on the curve.

Step 2: Compute the Derivative y'

- The function:

$$
y = x^2 e^x
$$

- Differentiation involves the product rule:

$$
\frac{d}{dx}[u \times v] = u' v + u v'
$$

- Here, $u = x^2$ and $v = e^x$.

- Compute derivatives:

$$
u' = 2x
$$

$$
v' = e^x
$$

- Apply the product rule:

$$
y' = u' v + u v' = 2x e^x + x^2 e^x
$$

- Simplify:

$$
y' = e^x (2x + x^2)

\]

Step 3: Find the Slope at $x = 1$

- Plug $x = 1$ into the derivative:

$$
y'(1) = e^{1} (2 \times 1 + 1^2) = e (2 + 1) = 3e
$$

- Therefore, the slope of the tangent line at $x = 1$ is:

$$
m = 3e
$$

Step 4: Write the Equation of the Tangent Line

- Use the point-slope form:

$$
y - y_0 = m (x - x_0)
$$

- Substituting $(x_0, y_0) = (1, e)$ and $m = 3e$:

$$
y - e = 3e (x - 1)
$$

- Final form:

$$
y = e + 3e(x - 1)
$$

- Simplify:

$$
y = e + 3e x - 3e
$$

$$
y = 3e x - 2e
$$

Therefore, the equation of the tangent line to $y = x^2 e^x$ at the point $P(1, e)$ is:

$$
\boxed{
\text{Tangent Line: } y = 3e x - 2e
}
$$

Additional Insights and Applications

Beyond simply deriving the tangent line, several additional considerations and applications enrich this understanding.

Linear Approximation of the Function Near the Point

- The tangent line provides the best linear approximation of the function near $x = 1$.
- For small h, $y \approx y(1) + y'(1) h = e + 3e h$, where $h = x - 1$.

Graphical Interpretation

- When plotted, the tangent line will touch the curve at $(1, e)$ and have the same slope $3e$.
- The tangent line visually demonstrates the rate at which y changes at that point.

Extensions to Other Points and Functions

- The same process applies to any differentiable function at any point:
- Find the derivative.
- Evaluate at the point.
- Use point-slope form for the tangent.

Incorporating Higher-Order Approximations

- For greater accuracy near $x = 1$, consider second derivatives or Taylor series expansions.

Common Mistakes and Troubleshooting

When tackling such problems, awareness of potential pitfalls ensures accuracy.

- Misapplication of differentiation rules: Ensure proper use of product rule, chain rule, etc.
- Incorrect evaluation of derivatives: Double-check calculations at the specific point.
- Confusing points: Confirm that the (x_0, y_0) point indeed lies on the curve.
- Sign errors: Be cautious with signs, especially when simplifying expressions.

Practical Applications and Real-World Contexts

Understanding tangent lines transcends pure mathematics, impacting fields such as:

- Physics: Analyzing velocity and acceleration as slopes of position-time graphs.
- Engineering: Approximating complex system behaviors locally.

- Economics: Linearization of cost functions around specific points.
- Biology and Medicine: Modeling rates of change in populations or concentrations.

Summary and Final Thoughts

Producing the equation of the tangent line to a function at a specified point involves a systematic approach:

1. Verify the point lies on the function.
2. Find the derivative to determine the slope.
3. Evaluate the derivative at the point.
4. Use point-slope form to produce the line equation.

In the case of $y = x^2 e^x$ at $P(1, e)$, the process is straightforward once differentiation rules are correctly applied. The resulting tangent line:

$$
\boxed{
y = 3e x - 2e
}
$$

serves as a powerful local approximation and a foundation for further explorations in calculus and applied mathematics.

By mastering this process, students and professionals can analyze complex functions more intuitively, model real-world phenomena with linear approximations, and deepen their understanding of the dynamic behavior of functions at specific points.

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