

Q: The Perimeter Of A Rectangle Is 54cm. One Side Is 6cm Longer Than The Other. Find The Area Of The

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Understanding how to find the area of a rectangle when given the perimeter and the relationship between its sides is a fundamental skill in geometry. This problem involves setting up algebraic expressions based on the given conditions and solving for the dimensions before calculating the area. In this article, we will explore the steps to solve this problem comprehensively, including defining variables, setting up equations, solving for unknowns, and finally calculating the area.

Understanding the Problem and Key Concepts

What is a Rectangle's Perimeter?

- The perimeter of a rectangle is the total distance around it.
- It is calculated as $P = 2 \times (\text{length} + \text{width})$.

Given Data in the Problem

- Perimeter $(P = 54\text{cm})$.
- One side is 6cm longer than the other, implying a relationship between the sides.

What are we asked to find?

- The area of the rectangle, which is calculated as $\text{Area} = \text{length} \times \text{width}$.

Step 1: Assign Variables to the Sides

Define the shorter side

- Let's denote the shorter side as (x) centimeters.

Express the longer side in terms of (x)

- Since one side is 6cm longer, the longer side will be $(x + 6)$ centimeters.

Step 2: Write an Equation Using the Perimeter

Perimeter formula for a rectangle

- $P = 2 \times (\text{length} + \text{width})$.

Substitute the known values

- $54 = 2 \times (x + x + 6)$.

Simplify the equation

- $54 = 2 \times (2x + 6)$,
- $54 = 4x + 12$.

Solve for (x)

- Subtract 12 from both sides:
- $54 - 12 = 4x$,
- $42 = 4x$,
- Divide both sides by 4:
- $x = \frac{42}{4} = 10.5$.

Thus, the shorter side is 10.5cm.

Step 3: Find the Longer Side

- Longer side = $(x + 6 = 10.5 + 6 = 16.5)$ cm.

So, the sides of the rectangle are 10.5cm and 16.5cm.

Step 4: Calculate the Area of the Rectangle

Use the area formula

- $(\text{Area} = \text{length} \times \text{width})$.

Substitute the side lengths

- $(\text{Area} = 10.5 \times 16.5)$.

Perform the multiplication

- $(10.5 \times 16.5 = (10 + 0.5) \times (16 + 0.5))$.

Apply distributive property

- $((10 \times 16) + (10 \times 0.5) + (0.5 \times 16) + (0.5 \times 0.5))$,
- $(160 + 5 + 8 + 0.25)$,
- Sum these up:
- $(160 + 5 = 165)$,
- $(165 + 8 = 173)$,
- $(173 + 0.25 = 173.25)$.

Therefore, the area of the rectangle is 173.25 square centimeters.

Summary of the Solution

- Step 1: Assign variables to sides based on the relationship given.
- Step 2: Use the perimeter formula to set up and solve for the smaller side.
- Step 3: Calculate the longer side using the relationship.
- Step 4: Apply the area formula to find the rectangle's area.

Final answer: The area of the rectangle is 173.25 cm².

Additional Tips and Considerations

Verifying the Solution

- Double-check calculations for accuracy.
- Confirm that the sides satisfy the perimeter condition:
- $2 \times (10.5 + 16.5) = 2 \times 27 = 54$ cm, which matches the given perimeter.

Understanding the Importance of Variables

- Assigning variables allows for easy manipulation of algebraic expressions and simplifies solving problems with unknowns.

Extensions of the Problem

- Similar problems can involve different relationships between sides or different perimeters.
- Practice setting up equations based on word problems involving geometric figures.

Conclusion

Solving for the area of a rectangle with a given perimeter and a specific relationship between its sides involves a clear understanding of the properties of rectangles and algebraic skills. By carefully defining variables, setting up equations, and performing arithmetic accurately, you can confidently determine unknown dimensions and compute the area. This problem exemplifies how algebra and geometry work hand in hand to solve real-world math problems, reinforcing the importance of methodical problem-solving techniques in mathematics.

Frequently Asked Questions

What are the steps to find the length and width of a rectangle when the perimeter is given, and one side is longer than the other?

First, assign variables to the sides, for example, let the shorter side be x and the longer side be $x + 6$ cm. Then, use the perimeter formula $P = 2(\text{length} + \text{width})$.

+ width) and substitute the known perimeter to form an equation. Solve for x to find the sides.

How do you set up the equation for the perimeter of a rectangle when one side is 6 cm longer than the other?

If the shorter side is x cm, then the longer side is $x + 6$ cm. The perimeter $P = 2(x + x + 6) = 54$ cm. Simplify and solve this equation to find x .

What is the value of the shorter side of the rectangle if the perimeter is 54 cm and one side is 6 cm longer?

Let x be the shorter side. Set up the equation $2(x + x + 6) = 54$. Simplify to $2(2x + 6) = 54$, then $4x + 12 = 54$. Subtract 12: $4x = 42$. Divide by 4: $x = 10.5$ cm. So, the shorter side is 10.5 cm.

How do you find the area of the rectangle once you know the sides?

Calculate the area by multiplying the length and width. Since the shorter side is 10.5 cm and the longer side is $10.5 + 6 = 16.5$ cm, the area = $10.5 \text{ cm} \times 16.5 \text{ cm} = 173.25$ square centimeters.

What is the area of the rectangle with perimeter 54 cm, where one side is 6 cm longer than the other?

The area is 173.25 cm^2 , calculated as 10.5 cm (shorter side) multiplied by 16.5 cm (longer side).

Can you verify the solution by checking the perimeter with the found sides?

Yes. Perimeter = $2(10.5 + 16.5) = 2 \times 27 = 54$ cm, which matches the given perimeter, confirming the solution is correct.

Additional Resources

Q: The Perimeter Of A Rectangle Is 54cm. One Side Is 6cm Longer Than The Other. Find The Area Of The

Mathematics often presents us with intriguing problems that challenge our understanding of geometric concepts. One such problem involves calculating the area of a rectangle based on its perimeter and the relationship between its sides. The question, "The perimeter of a rectangle is 54cm. One side is 6cm longer than the other. Find the area of the rectangle," not only tests basic algebraic skills but also offers insights into problem-solving strategies, geometric properties, and real-world applications.

This comprehensive review aims to dissect this problem thoroughly, exploring its foundational principles, solution methodology, common pitfalls, and broader implications. Through this investigation, readers will gain a deeper appreciation of how algebra and geometry intertwine to solve practical problems, as well as develop critical thinking skills essential for tackling similar challenges.

Understanding the Problem Statement

Before delving into calculations, it is essential to fully understand the parameters and what is being asked.

Key Details:

- The perimeter (P) of the rectangle is 54cm.
- One side (length) is 6cm longer than the other (width).
- The goal is to find the area.

The problem involves two main variables:

- Let the shorter side be x cm.
- Then, the longer side is $(x + 6)$ cm.

The perimeter relates to the sides as follows:

$$P = 2 \times (\text{length} + \text{width})$$

Given the perimeter, this becomes:

$$54 = 2 \times (x + (x + 6))$$

Our task is to find the area A , which is:

$$A = \text{length} \times \text{width} = x \times (x + 6)$$

Mathematical Framework and Solution Strategy

The problem combines algebra and geometry, requiring the formulation of an equation based on the perimeter and the relationship between the sides.

Step 1: Set Up the Equations

- Let:

$$x = \text{shorter side}$$

$$\backslash [x + 6 = \text{longer side} \backslash]$$

- Perimeter equation:

$$\backslash [54 = 2(x + x + 6) \backslash]$$

- Simplify:

$$\backslash [54 = 2(2x + 6) \backslash]$$

$$\backslash [54 = 4x + 12 \backslash]$$

Step 2: Solve for (x)

- Subtract 12 from both sides:

$$\backslash [54 - 12 = 4x \backslash]$$

$$\backslash [42 = 4x \backslash]$$

- Divide both sides by 4:

$$\backslash [x = \frac{42}{4} = 10.5 \backslash]$$

Thus, the shorter side is 10.5 cm.

- The longer side is:

$$\backslash [x + 6 = 10.5 + 6 = 16.5 \backslash , \text{cm} \backslash]$$

Step 3: Find the Area

- Area (A) :

$$\backslash [A = x \times (x + 6) = 10.5 \times 16.5 \backslash]$$

- Calculate:

$$\backslash [10.5 \times 16.5 \backslash]$$

Using multiplication:

$$\backslash [10.5 \times 16.5 = (10 + 0.5) \times (16 + 0.5) \backslash]$$

Apply distributive property:

$$\backslash [= 10 \times 16 + 10 \times 0.5 + 0.5 \times 16 + 0.5 \times 0.5 \backslash]$$

Calculate each term:

$$\backslash [10 \times 16 = 160 \backslash]$$

$$\backslash [10 \times 0.5 = 5 \backslash]$$

$$\backslash [0.5 \times 16 = 8 \backslash]$$

$$\backslash [0.5 \times 0.5 = 0.25 \backslash]$$

Sum:

$$\backslash [160 + 5 + 8 + 0.25 = 173.25 \backslash]$$

Area = 173.25 cm²

Deepening the Understanding: Geometric and Algebraic Insights

The Relationship Between Sides and Perimeter

The problem exemplifies a classic algebraic approach to geometric figures. The perimeter formula for rectangles:

$$P = 2(l + w)$$

connects directly to the algebraic expressions for the sides. Recognizing the relationship between the sides (one being 6cm longer than the other) allows us to set up a single-variable quadratic equation, simplifying the problem significantly.

The Significance of the Side Difference

The fact that one side is longer by a fixed amount (6cm) introduces a linear relationship, which simplifies the algebra. Problems involving differences between dimensions often reduce to quadratic equations, enabling systematic solutions.

Possible Variations and Generalizations

- If the difference between sides were variable or unknown, the approach would involve more complex equations.
- For perimeters given in different units or shapes (e.g., rectangles with varying side-length relationships), similar principles could be applied with adjustments.

Common Challenges and Misconceptions

While the solution appears straightforward, students and practitioners sometimes encounter pitfalls:

1. Incorrect setting up of the equations:
 - Confusing the order of sides or mislabeling variables.
2. Forgetting to double the sum of sides when calculating perimeter:
 - The perimeter formula must be meticulously applied.
3. Neglecting the absolute value of the sides:
 - Sides cannot be negative; algebraic solutions should be checked for practicality.
4. Miscalculations during arithmetic:
 - Multiplication mistakes can lead to incorrect area values.

By meticulously setting up the equations and verifying the calculations, these issues can be mitigated.

Broader Implications and Applications

Understanding such problems extends beyond academic exercises into real-world contexts:

- Architecture and Engineering: Precise measurements are vital in designing structures with specified perimeters and area constraints.
- Manufacturing: Material estimation often relies on calculations similar to this problem.
- Education: Developing problem-solving skills through such exercises enhances critical thinking and quantitative reasoning.

Furthermore, this problem exemplifies the importance of algebraic literacy for interpreting and solving geometric problems efficiently.

Alternative Strategies and Extensions

While algebra provides a direct approach, other methods or extensions could include:

- Graphical representation: plotting the relationship between sides and perimeter.
- Using systems of equations: especially if additional constraints are introduced.
- Applying coordinate geometry: to visualize and verify solutions.

Extensions could involve:

- Changing the perimeter value.
- Altering the side difference.
- Considering non-rectangular quadrilaterals with similar properties.

Conclusion

The problem, "The perimeter of a rectangle is 54cm. One side is 6cm longer than the other. Find the area," demonstrates the power of algebraic methods combined with geometric understanding. By translating the problem into a set of equations, solving step-by-step, and verifying the results, we arrive at a comprehensive solution that not only solves the immediate question but also reinforces foundational mathematical principles.

The calculated area of 173.25 cm^2 underscores the importance of precise algebraic reasoning in geometric contexts. Such problems serve as excellent pedagogical tools for developing problem-solving skills and appreciating the interconnectedness of mathematical concepts. Whether in academic settings or

practical applications, mastering these techniques equips individuals with valuable tools for tackling complex real-world problems with confidence and clarity.

References

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- National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. NCTM.

Note: The calculations and explanations provided in this article aim to offer a comprehensive understanding of the problem-solving process for the given mathematical question.

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